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# Finite $a\%$ Settling Time Control of Discrete Time Nonlinear Constrained Systems

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**Abstract**—In this paper, we study the constraint control of nonlinear systems. We propose a generalization of static relatively optimal control (ROC) methods so that it is applicable to nonlinear servo systems. ROC methods use a piecewise linear feedback for linear systems to achieve a deadbeat control. To utilize the idea of ROC, we assume that nonlinear systems are incrementally polytopic uncertain systems, and consider the convergence to a region such that the error between the output and the constant reference command is less than or equal to $a\%$ of the constant reference command.

## 1. Introduction

For almost all practical control systems, we need to take into account constraints on state and/or control input caused by amplitude limitation of state variables, saturation property of actuators and so on. If we ignore these constraints, then the real performance of the system degrades because of the wind-up phenomena, or in worst cases the control system becomes unstable. In these respects, extensive researches have been done to cope with such constraints (see e.g. [1]–[10] and references herein).

In this paper, we consider a nonlinear servo system tracking a constant reference command. We propose a generalization of static relatively optimal control (ROC) methods proposed in [9], [10] so that it is applicable to nonlinear servo systems. ROC methods use piecewise linear feedback control for linear systems to achieve a deadbeat (finite time convergence to an equilibrium). To utilize the idea of ROC, we assume that nonlinear systems are incrementally polytopic uncertain systems. In this case, we can not achieve a deadbeat because of uncertainty. Instead, we consider the convergence to a region $\Psi_0$ such that the error between the output and the constant reference command is less than or equal to $a\%$ of the constant reference command. We propose a new algorithm to compute $\Psi_0$ and regions $\Psi_k$ to achieve finite time $a\%$ settling time control. The proposing method reduces on-line process time than traditional ROC methods [9], [10].

**Notation** For a matrix $L$, $L_i$ is the $i$-th row vector of $L$. For a vector $m$, $m_i$ is the $i$-th element of $m$. For a polytope (a bounded polyhedral set) $P$, $\mathcal{N}(P)$ and $\mathcal{F}(P)$ denote the set of nodes and facets of $P$, respectively. $\operatorname{conv}\{\cdot\}$ denotes the convex hull. For a set $\mathcal{X}$, $\operatorname{int}\mathcal{X}$ denotes the interior of the sets $\mathcal{X}$.

## 2. Servo System and Problem Setting

### 2.1. Servo System

Let us consider a discrete time nonlinear system given by

$$x_P[k+1] = f_P(x_P[k], u[k]), \quad y[k] = g_P(x_P[k]), \quad (1)$$

where $x_P[k] \in \mathbf{R}^{n_P}$, $u[k] \in \mathbf{R}$, and $y[k] \in \mathbf{R}$ are, respectively, the state, the control input and the output of the plant at time $k \in \mathbf{Z}_+$, and $\mathbf{Z}_+$ is the set of nonnegative integers. Functions $f_P : \mathbf{R}^{n_P} \times \mathbf{R} \to \mathbf{R}^{n_P}$ and $g_P : \mathbf{R}^{n_P} \to \mathbf{R}$ are continuously differentiable.

We consider an integral type servo system. The control law is given by

$$x_I[k+1] = x_I[k] + r[k] - y[k], \quad (2)$$
$$u[k] = K_P x_P[k] + K_I x_I[k], \quad (3)$$

where $x_I[k] \in \mathbf{R}$ is the state of the integrator, $r[k]$ is the reference input to be managed which we will state later.

Define

$$x = \begin{bmatrix} x_P \\ x_I \end{bmatrix} \in \mathbf{R}^{n_x}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \in \mathbf{R}^{n_x}, \quad g(x) = g_P(x_P),$$
$$f(x) = \begin{bmatrix} f_P(x_P, K_P x_P + K_I x_I) \\ -g_P(x_p) + x_I \end{bmatrix}. \quad (4)$$

Then the closed system is given

$$x[k+1] = f(x[k]) + Br[k], \quad y[k] = g(x[k]). \quad (5)$$

System (5) has constrains given by

$$Lx[k] + dr[k] - m \leqq 0 \quad \forall k \geqq 0, \quad (6)$$

where $L \in \mathbf{R}^{n_c \times n_x}$ and $d, m \in \mathbf{R}^{n_c}$.

A typical constraint is the constraint on the magnitude of $u[k] = Kx[k]$ such that $u_{\min} \leqq u[k] \leqq u_{\max}$, where $K = \begin{bmatrix} K_P & K_I \end{bmatrix}$. In this case, $n_c = 2$, and $L$, $d$ and $m$ are given by

$$L = \begin{bmatrix} K \\ -K \end{bmatrix}, \quad d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad m = \begin{bmatrix} u_{\max} \\ -u_{\min} \end{bmatrix}.$$

### 2.2. Equilibrium and Stability

For (1), we assume the following.

**Assumption 1** *Let $\mathcal{R} \subseteq \mathbf{R}$ be a bounded closed interval. For each $\hat{r} \in \mathcal{R}$, there exist $\hat{x}_P(\hat{r}) \in \mathcal{X}_P$ and $\hat{u}(\hat{r}) \in \mathcal{U} = [u_{\min}, u_{\max}]$ such that*

$$f_P(\hat{x}_P(\hat{r}), \hat{u}(\hat{r})) = \hat{x}_P(\hat{r}), \quad g_P(\hat{x}_P(\hat{r})) = \hat{r}, \tag{7}$$

*where $\mathcal{X}_P \subseteq \mathbf{R}^{n_P}$ is a polytope such that $0 \in \operatorname{int} \mathcal{X}_P$ and $\mathcal{U} \subseteq \mathbf{R}$ is a bounded interval such that $u_{\min} \leqq 0 \leqq u_{\max}$.*

We note the following.

**Lemma 1** *Assume that Assumption 1 is satisfied, and $K_I$ in (3) is not 0. Then, for each $\hat{r} \in \mathcal{R}$, set*

$$\hat{x}_I(\hat{r}) = \frac{\hat{u}(\hat{r}) - K_P \hat{x}_P(\hat{r})}{K_I}, \quad \hat{x}(\hat{r}) = \begin{bmatrix} \hat{x}_P(\hat{r}) \\ \hat{x}_I(\hat{r}) \end{bmatrix}. \tag{8}$$

*Then, $\hat{x}(\hat{r})$ is an equilibrium of (5) and we have*

$$K_P \hat{x}_P(\hat{r}) + K_I \hat{x}_I(\hat{r}) = \hat{u}(\hat{r}), \tag{9}$$
$$f(\hat{x}(\hat{r})) + B\hat{r} = \hat{x}(\hat{r}), \quad g(\hat{x}(\hat{r})) = \hat{r}. \tag{10}$$

■

**Assumption 2** *The nonlinear system (1) is an incrementally polytopic uncertain system, that is, there exist matrices $\{(A_{p,q},\, B_{p,q},\, C_{p,q})\}_{q=1}^{Q}$ such that*

$$\begin{bmatrix} \frac{\partial f_p}{\partial x_p}(x_p, u) & \frac{\partial f_p}{\partial u}(x_p, u) \\ \frac{\partial g_p}{\partial x_p}(x_p) & 0 \end{bmatrix} \in \operatorname{conv} \left\{ \begin{bmatrix} A_{p,q} & B_{p,q} \\ C_{p,q} & 0 \end{bmatrix} \right\}_{q=1}^{Q}$$
$$\forall x_P \in \mathcal{X}_P + \hat{x}_P, \quad \forall u \in \mathcal{U}, \tag{11}$$

*where*

$$\mathcal{X}_P + \hat{x}_P(r) = \{x_P = \tilde{x}_P + \hat{x}_P(r) : \tilde{x}_P \in \mathcal{X}_P\}. \tag{12}$$

Applying the mean value theorem [11], we have the following.

**Lemma 2** *Assume that Assumptions 1 and 2 are satisfied. For a given $\hat{r} \in \mathcal{R}$, let us consider errors*

$$\tilde{x}[k] = x[k] - \hat{x}(\hat{r}) = f(\tilde{x}[k] + \hat{x}(\hat{r})) - f(\hat{x}(\hat{r})), \tag{13}$$
$$e[k] = \hat{r} - y[k] = g(\hat{x}(\hat{r})) - g(x[k]). \tag{14}$$

*Then we have*

$$\tilde{x}[k] = A[k]\tilde{x}[k], \quad A[k] \in \operatorname{conv}\{A_q,\ q \in \mathcal{Q}\}, \tag{15}$$
$$e[k] = C[k]\tilde{x}[k], \quad C[k] \in \operatorname{conv}\{C_q, q \in \mathcal{Q}\}, \tag{16}$$

*where $\mathcal{Q} = \{1, 2, \ldots, Q\}$,*

$$A_q = \begin{bmatrix} A_{p,q} + B_{p,q}K_P & B_{p,q}K_I \\ -C_{p,q} & 1 \end{bmatrix}, \quad q \in \mathcal{Q}, \tag{17}$$
$$C_q = \begin{bmatrix} C_{p,q} & 0 \end{bmatrix}, \quad q \in \mathcal{Q}. \tag{18}$$

■

**Assumption 3** *There exists a Lyapunov function $V : \mathbf{R}^{n_x} \to \mathbf{R}_+$ satisfying*

$$\alpha|x| \leqq V(x) \leqq \beta|x| \quad \forall x \in \mathbf{R}^{n_x}, \tag{19}$$

*for some positive numbers $\alpha$ and $\beta$, and the following properties hold.*

$$V(x + x') \leqq V(x) + V(x') \quad \forall x, x' \in \mathbf{R}^{n_x}, \tag{20}$$
$$V(\tau x) = \tau V(x) \quad \forall \tau \geqq 0, \quad \forall x \in \mathbf{R}^{n_x}. \tag{21}$$

*Moreover there exists a number $\gamma \in [0, 1)$ such that*

$$V(A_q x) \leqq \gamma V(x) \quad \forall q \in \mathcal{Q}, \quad \forall x \in \mathbf{R}^{n_x}. \tag{22}$$

Applying standard Lyapunov theory, we have the following.

**Lemma 3** *Assume that Assumptions 1 - 3 are satisfied. Then, a linear time varying system given by*

$$\tilde{x}[k+1] = A[k]\tilde{x}[k], \quad A[k] \in \operatorname{conv}\{A_q,\ q \in \mathcal{Q}\} \tag{23}$$

*is exponentially stable.*

*Let*

$$\mathcal{X} = \{x \in \mathbf{R}^{n_x} :\ x \in \mathcal{X}_P \times \mathbf{R}\}, \tag{24}$$
$$\Omega(\rho) = \{x \in \mathbf{R}^{n_x} :\ V(x) \leqq \rho\}, \tag{25}$$
$$\rho_{\max} = \max\{\rho > 0 : \Omega(\rho) \subseteq \mathcal{X}\}, \tag{26}$$
$$\Omega(\rho) + \hat{x} = \{x = \tilde{x} + \hat{x} :\ \hat{x} \in \Omega(\rho)\}. \tag{27}$$

*Then, under the absence of constraints, for each $\hat{r} \in \mathcal{R}$, the equilibrium $\hat{x}(\hat{r})$ of (5) is exponentially stable and for each $x_{k_o} \in \Omega(\rho_{\max}) + \hat{x}(\hat{r})$, the solution $x[k; k_0, x_{k_0}, \hat{r}]$ stays in $\Omega(\rho_{\max}) + \hat{x}(\hat{r})$ and converges to $\hat{x}(\hat{r})$, where $x[k; k_0, x_{k_0}, \hat{r}]$ denotes the solution of (5) with the initial condition $x[k_0] = x_{k_0}$ and the reference input $r[k] \equiv \hat{r}$.* ■

### 2.3. $a$% Settling Time and Problem Setting

Since $\hat{r} = g(\hat{x}(\hat{r}))$, $y[k] = g(x[k])$ converges to $\hat{r}$. For a given $a > 0$, we define $a$% settling time $k_s$ by

$$k_s = \min\{k' : |g(x[k; 0, x_0, r]) - \hat{r}| \leqq a_r|\hat{r}| \quad \forall k \geqq k'\}, \tag{28}$$

where $a_r = a/100$, $\mathcal{N}_c = \{1, 2, \ldots, n_c\}$, and $r[k] = \hat{r}$ for $k \geqq k'$ and $r[k]$ is managed so that (6) holds for $k \geqq 0$.

We will compute a stability region $\Psi_0(\hat{r})$ such that $x_{k_0} \in \Psi_0(\hat{r})$ means

$$x[k; k_0, x_{k_0}, \hat{r}] \in \Psi_0(\hat{r}) \quad \forall\, k \geqq k_0, \tag{29}$$

$$x[k; k_0, x_{k_0}, \hat{r}] \to \hat{x}(\hat{r}), \quad k \to \infty, \tag{30}$$

$$|C_q[x[k; k_0, x_{k_0}, \hat{r}] - \hat{x}(\hat{r})]| \leqq a_r|\hat{r}| \qquad \forall k \geqq k_0,\ \forall q \in \mathcal{Q}, \tag{31}$$

$$L_i x[k; k_0, x_{k_0}, \hat{r}] + d_i \hat{r} \leqq m_i \ \forall i \in \mathcal{N}_c,\ \forall k \geqq k_0. \tag{32}$$

In this paper, we propose a nonlinear feedback control law to manage $r[k]$ so that the constraint (6) is satisfied and that $x[k; 0, x_0, r]$ reaches $\Psi_0(\hat{r})$ at most $k_0$ steps, where $k_0$ is the integer such that $x_0 \in \Psi_{k_0}(\hat{r})$ and $\Psi_k(\hat{r})$ is defined later.

## 3. Computation $\Psi_0(\hat{r})$

Let $\mathcal{P} \subseteq \mathbf{R}^{n_x}$ be a polytope such that $0 \in \operatorname{int}\mathcal{P}$, and let $\mathcal{N}_F = \{1, 2, \cdots, N_F\}$. The normal vector $\eta_j$ of $\mathcal{F}_j \in \mathcal{F}(P)$, $j \in \mathcal{N}_F$, is normalized in the sense that $\eta_j^T \tilde{x} = 1$ for all $\tilde{x} \in \mathcal{F}_j$, and $\mathcal{P}$ is represented by

$$\mathcal{P} = \{x \in \mathbf{R}^{n_x} : \eta_j^T x \leqq 1, \ \ j \in \mathcal{N}_F\}. \tag{33}$$

A Polytopic Lyapunov Function (PLF) $V(x)$ determined by $\mathcal{P}$ is given by

$$V(\tilde{x}) = \max_{j \in \mathcal{N}_F} \ \eta_j^T \tilde{x}. \tag{34}$$

Let us denote the boundary of the set $\mathcal{P}$ by $\partial\mathcal{P}$. Then $V(x) = 1$ for all $x \in \partial\mathcal{P}$, and, hence, when we define $\Omega(\rho)$ by (25), we have $\Omega(\rho) = \rho\mathcal{P}$.

By the definition, $V(x)$ satisfies (19) - (21) . Relating to (22), we have the following.

**Lemma 4** *Suppose that $\mathcal{P}$ is the polytope given by (33). Let us consider (23). If*

$$\frac{\eta_\ell^T A_q}{\gamma} \in \operatorname{conv}\{\eta_j^T,\ j \in \mathcal{N}_F\} \ \ \forall \ell \in \mathcal{N}_F,\ \forall q \in \mathcal{Q} \tag{35}$$

*holds for some $\gamma > 0$, then (22) is satisfied.* ■

When $\mathcal{Q} = \{1\}$, that is, $Q = 1$, it is a good idea to choose $\gamma = \max\{|\lambda_j(A_1)|, j = 1, 2, \cdots, n_x\} + \varepsilon < 1$, where $\varepsilon$ is a small positive number. When $Q \geqq 2$, an initial approximation of $\gamma > 0$ can be computed by solving an LMI.

**Theorem 1** *Let $\mathcal{P}$ be the polytope given by (33). Assume that Assumptions 1 - 3 are satisfied. Moreover, we assume that (35) is satisfied for some $\gamma \in (0, 1)$, and that following conditions hold.*

$$\pm \tilde{C}_q = \frac{\pm C_q}{a_r|\hat{r}|} \in \operatorname{conv}\{\eta_j^T,\ j \in \mathcal{N}_F\} \ \ \forall q \in \mathcal{Q}, \tag{36}$$

$$\tilde{m}_i(\hat{r}) = m_i - L_i\hat{x}(\hat{r}) - D_i\hat{r} > 0 \ \ \forall i \in \mathcal{N}_c, \tag{37}$$

$$\tilde{L}_i = \frac{L_i}{\tilde{m}_i(\hat{r})} \in \operatorname{conv}\{\eta_j^T,\ j \in \mathcal{N}_F\} \ \ \forall i \in \mathcal{N}_c. \tag{38}$$

*Then, (29) - (32) are satisfied for $\Psi_0(\hat{r}) = \mathcal{P} + \hat{x}(\hat{r})$.* ■

When a polytope $\mathcal{P}$ which includes 0 as an interior point is given by (33), the polytope $\mathcal{P}^D = \operatorname{conv}\{\eta_j,\ j \in \mathcal{N}_F\}$ is the dual polytope of $\mathcal{P}$, that is, normalized normal vectors of $\mathcal{P}$ are vertexes of $\mathcal{P}^D$. Conversely, normalized normal vectors of $\mathcal{P}^D$ are vertices of $\mathcal{P}$.

From Theorem 1, we have an algorithm to construct the dual polytope $\mathcal{P}^D$ of $\mathcal{P}$. Let the polytope $\mathcal{X}$ in Assumption 2 be given by $\mathcal{X} = \{x : \eta_{i,0}^T x \leqq 1, \ \ i = 1, 2, \cdots, n_X\}$.

Procedure comp_Ψ

1. begin
2. k:=0;
3. $Stack1 := \{\eta_{i,0}\}_{i=1}^{n_X} \cup \{\tilde{C}_q, -\tilde{C}_q\}_{q=1}^{Q} \cup \{\tilde{L}_i\}_{i=1}^{n_c}$;
4. $\mathcal{P}^D := \operatorname{conv}\{Stack1\}$; $\mathcal{F} := \mathcal{N}(\mathcal{P}^D)$;
5. $Node1 := \mathcal{F}$; $Stack1 := \emptyset$;
6. for $\eta \in Node1$ begin
6.1. for $A_q \in \mathcal{A}$ begin
6.1.1. if $(A_q^T\eta/\gamma \notin \mathcal{P}^D)$ begin
6.1.1.1. $Stack1 := Stack1 \cup A_q^T\eta_\ell/\gamma$;
6.1.2. end;
6.2. end;
7. end;
8. if $(Stack1 \neq \emptyset)$ begin;
8.1. $\mathcal{P}^D := \operatorname{conv}\{Stack1, \mathcal{F}\}$; $\mathcal{F} := \mathcal{N}(\mathcal{P}^D)$;
8.2. $Stack1 := \emptyset$; $k := k + 1$; goto 5
9. end;
10. end;

In comp_Ψ, $\mathcal{A} = \{A_q,\ q \in \mathcal{Q}\}$.

It is guaranteed that comp_Ψ stops in a finite iteration under the condition (35).

## 4. Nonlinear Feedback Control Law

The polytope computed by applying comp_Ψ depends on $\hat{r} \in \mathcal{R}$. In the following, we fix a $\hat{r} \in \mathcal{R}$, and for simplicity of notation we drop $(\hat{r})$ in representing $\mathcal{P}(\hat{r})$, $\Psi_0(\hat{r})$ and $\hat{x}(\hat{r})$.

When $\Psi_0 = \mathcal{P} + \hat{x}$ was computed, compute $\Phi_k$ and $\Psi_k$ for $k = 1, 2, \ldots, k_{\max}$ in the following way.

$$\Phi_k = \{\tilde{x} = \begin{bmatrix} x^T & r \end{bmatrix}^T : (A_q x + Br) \in \Psi_{k-1} \ \ \forall A_q \in \mathcal{A}, \qquad Lx + dr \leqq m\}. \tag{39}$$

Let $\mathcal{N}(\Phi_k) = \{\tilde{x}_{k,\ell} = \begin{bmatrix} x_{k,\ell}^T & r_{k,\ell} \end{bmatrix}^T\}_{\ell=1}^{N_{N_k}}$, and compute $\Psi_k = \text{conv}\{x_{k,\ell}\}_{\ell=1}^{N_{N_k}}$. For each $k$, let $\mathcal{F}(\Psi_k) = \{F_{k,j}\}_{j=1}^{N_{F_k}}$, and for each $F_{k,j}$, let $\mathcal{N}(F_{k,j}) = \{x_{k,j,i}\}_{i=1}^{N_{N_{k,j}}}$. Define a pyramid $\Psi_{k,j} = \text{conv}\{\{\hat{x}\} \cup \mathcal{N}(F_{k,j})\}$. Then

$$\text{int}\,\Psi_{k,j} \cap \text{int}\,\Psi_{k,j'}, \quad j \neq j', \quad \Psi_k = \bigcup_{j=1}^{N_{F_k}} \Psi_{k,j}. \tag{40}$$

Each $x_{k,j,i}$ corresponds to an $x_{k,\ell} \in \mathcal{N}(\Psi_k)$, where $\ell$ depends on $(j,i)$, and we denote $\ell$ as $\ell(j,i)$ and we represent $x_{k,\ell}$ as $x_{k,\ell(j,i)}$. Let $r_{k,\ell(j,i)}$ correspond to $x_{k,\ell(j,i)}$, that is, $\tilde{x}_{k,\ell(j,i)} = \begin{bmatrix} x_{k,\ell(j,i)}^T & r_{k,\ell(j,i)} \end{bmatrix}^T \in \mathcal{N}(\Phi_k)$. For convenience, let $x_{k,\ell(j,0)} = \hat{x}$ and $r_{k,\ell(j,0)} = \hat{r}$.

We propose a piecewise linear control law $r(x)$. When $x \in \Psi_{k,j}$, there exists $\{\lambda_i \in [0,1]\}_{i=0}^{N_{N_{k,j}}}$ such that $\sum_{i=0}^{N_{N_{k,j}}} \lambda_i = 1$ and $x = \sum_{i=0}^{N_{N_{k,j}}} \lambda_i x_{k,\ell(j,i)}$, and $r(x)$ is given by

$$r(x) = \begin{cases} \sum_{i=0}^{N_{N_{k,j}}} \lambda_i r_{k,\ell(j,i)}, & \text{if } x \in (\Psi_{k,j} \backslash \Psi_0), \\ \hat{r}, & \text{if } x \in \Psi_0. \end{cases} \tag{41}$$

**Theorem 2** *Assume that $\Psi_0 = \mathcal{P} + \hat{x}$, $\{\Phi_k, \Psi_k\}_{k=1}^{k_{\max}}$ are computed. Then, we have*

$$\Psi_0 \subseteq \Psi_1 \subseteq \ldots \subseteq \Psi_{k_{\max}}. \tag{42}$$

*If $x_0 \in \Psi_{k_0}$ for some $k_0 < k_{\max}$ and if we apply the control law (41), then constraints (6) hold and a% settling time $k_0$ is achieved.* ∎

If $\Psi_{k,j}$ is a simplex, then $N_{N_{k.j}} = n_x$, $\mathbf{rank}\, X_{k,j} = n_x$ and $r(x)$ in (41) is given by $r(x) = R_{k,j} X_{k,j}^{-1} x + \hat{r}$, where $X_{k,j} = \begin{bmatrix} x_{k,j,1} - \hat{x} & x_{k,j,2} - \hat{x} & \cdots & x_{k,j,n_x} - \hat{x} \end{bmatrix}$ and $R_{k,j} = \begin{bmatrix} r_{k,j,1} - \hat{r} & r_{k,j,2} - \hat{r} & \cdots & r_{k,j,n_x} - \hat{r} \end{bmatrix}$.

## 5. Concluding Remark

In this paper, we studied the constraint control of nonlinear servo systems. The contribution of this paper are to derive Theorems 1 and 2 and to propose a new control law (41), This control law is quite similar to those in [9], [10], but it is different because we do not consider decompositions of $\Psi_k \backslash \Psi_{k-1}$ into union of simplexes. As a result, our implementation reduces on-line process time than the previous implementation in [9], [10].

Because of the lack of spaces, we do not include proofs of lemmas and theorems. The proofs can be obtained by requiring them by email to tcs.y.ohta@people.kobe-u.ac.jp .

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