2022 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2022, Virtual, December 12-15, 2022

Tunnel Conductance Modeling of Spintronics Devices Based on Device Temperature Dynamics

Yushi Kikuchi†, Yoshihiko Horio†, Shunsuke Fukami†, and Hiroyasu Ando‡

†Research Institute of Electrical Communication, Tohoku University
2–2–1 Katahira, Aoba–ku, Sendai, Miyagi, 980-8577, Japan
‡Advanced Institute for Materials Research,Tohoku University
2–2–1 Katahira, Aoba–ku, Sendai, Miyagi, 980-8577, Japan
Email: yushi.kikuchi.p6@dc.tohoku.ac.jp

Abstract—Spintronics devices reproduce important functions of synapses, such as spike-timing-dependent plasticity, and neurons, such as leaky integration of input spikes. However, only a few models have been investigated for the use of spintronics devices in neural network circuits. In this study, a mathematical model of spintronics devices for the application to spiking neural network circuits is developed. The proposed model describes the relationship between the input voltage and value of the tunneling conductance of the spintronics device.

1. Introduction

Spintronics devices to reproduce the functions of neurons (e.g., leaky integration of input pulses) and synapses (e.g., spike-timing-dependent plasticity (STDP)) have been developed [1]. Such spintronics devices promote the development of spiking neural network circuits.

For the application of spintronics devices to spiking neural networks [2], a mathematical model describing the behavior of spintronics devices is essential. A spintronics device serving as an artificial synapse comprises numerous magnetic domains. The previous studies have modeled the behavior of all magnetic domains to accurately describe the behavior of spintronics devices [3][4]. However, for devices containing numerous magnetic domains, the computational cost of computing their behavior limits the application. Moreover, if we only consider applications to spiking neural network circuits, knowledge of the relationship between the input voltage and value of the tunneling conductance is sufficient.

In this study, the relationship between the input voltage to the spintronics device and the value of the tunneling conductance is mathematically modeled. In this model, the behavior of each magnetic domain inside the spintronics device is ignored, and only the tunneling conductance is determined by averaging the behavior of all magnetic domains.

ORCID iDs Yushi Kikuchi: 0000-0002-8514-3363, Yoshihiko Horio: 0000-0003-0115-3095, Shunsuke Fukami: 0000-0001-5750-2990, Hiroyasu Ando: 0000-0003-1102-2291

The rest of this paper is structured as follows. Section 2 provides an overview of spintronics devices. Section 3 discusses the development of the proposed mathematical model for spintronics devices. Finally, Section 4 presents the conclusions of this study.

2. Spintronics Devices

Figure 1 illustrates a three-terminal spin-orbit torque (SOT) device modeled in this paper. The SOT device consists of four layers: channel, free, insulating, and reference. The channel layer is composed of a heavy metal, the insulating layer is composed of an insulator, and the free and reference layers are composed of a ferromagnetic material.

Figure 2 shows the distribution of the magnetic domains in the free layer. In ferromagnetic materials, there are regions where the directions of the neighboring magnetic moments are aligned. Such regions constitute the magnetic domains. The reference layer has a single magnetic domain, whereas the free layer of synaptic devices has multiple magnetic domains with either parallel or antiparallel magnetization to the reference layer [1]. Hereinafter, the magnetization state parallel to the magnetization direction of the reference layer is called “state P”, and the antiparallel magnetization state “state AP”.

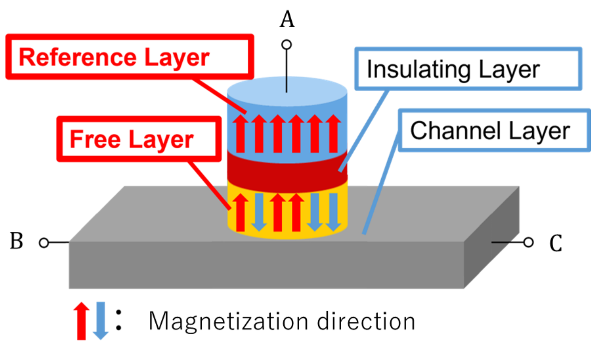

Figure 1: Schematic of the structure of a spintronics device. The reference and free layers are ferromagnetic.

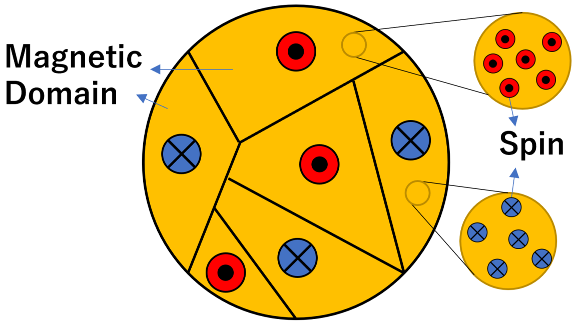

Figure 2: Distribution of magnetic domains in the free layer and its magnified view ⊙ indicates the direction parallel to the magnetization state of the reference layer, and ⊗ indicates the antiparallel direction.

Let us consider the tunneling current flowing through an insulating layer when a voltage is applied between terminals A and C shown in Fig. 1. The magnitude of the tunneling current varies depending on the magnetization state of the two ferromagnetic materials; this phenomenon is called the tunnel magnetoresistance effect [5]. The tunneling conductance is determined by the magnetization state of the two ferromagnets; its maximum value is obtained when all magnetic domains in the free layer are in state P, and its minimum value is obtained when all magnetic domains are in state AP. A finite number of magnetic domains exists in a ferromagnetic body. Therefore, the low tunnel conductances that occur in each magnetic domain in the free layer can be considered to be connected in parallel. So, the sum of the tunnel conductance values generated in each magnetic domain gives the overall tunnel conductance value $g(t)$ (see Section 3.5).

We consider controlling the value of tunnel conductance with an input voltage. The value of the tunneling conductance can be controlled by changing the state of the magnetic domains in the free layer, which is achieved by inputting a current between terminals B and C in Fig. 1. When current flows through the channel layer composed of a heavy metal, a spin current is generated from the channel layer in the vertical direction and through the free layer owing to the spin–orbit interaction [6]. This spin current causes the magnetic domains in the free layer to receive torque, which induces magnetization reversal. Therefore, the magnetic moment in the free layer can be controlled by applying current to the free layer.

The direction of the spin current is determined by the direction of the applied current. The current producing the spin current that reverses the magnetic domain state in the free layer from state AP to state P is defined as the positive current. Moreover, the current in the direction producing the spin current that reverses the magnetic domain state from state AP to state P is defined as the negative current.

When a large current flows through the channel layer, the magnetic domains in the free layer undergo magnetization reversal by SOT alone. The threshold value of the current that causes magnetization reversal is called magnetization reversal current, or magnetization reversal voltage if the voltage is the input. When the current is lower than the magnetization reversal current, magnetization reversal occurs stochastically by thermal noise and SOT, with a higher probability of magnetization reversal to state P when positive current flows and to state AP when negative current flows. Also, the higher the temperature of the magnetic domain, the more likely magnetization reversal will occur.

3. Mathematical Model

In this section, we derive a mathematical model of the spintronics device for pulsed neural network circuit. Figure 3 illustrates a schematic of the time variations of each variable.

3.1. Input Signal

Figure 4 shows a schematic of the circuit of the spintronics device used to derive the mathematical model. Considering the use of spintronics devices in a pulsed neural network circuit, we input voltage pulses. Here, only pulse inputs from a single input source are considered. The pulse input voltage $v_{\rm ch}(t)$ between terminals B and C of the channel layer of the spintronics device is given as

$$v_{\rm ch}(t) = \begin{cases} V_k, & t_k^{\rm s} \le t < t_k^{\rm e}, \\ 0, & \text{else}, \end{cases} \tag{1}$$

where $V_k(\neq 0)$, $t_k^{\rm s}$, and $t_k^{\rm e}$ are the amplitude, time when input goes from 0 to V_k, and time when input goes from V_k to 0 of the $k(\in \mathbb{Z})$th pulse, respectively, as shown in Fig. 3(a).

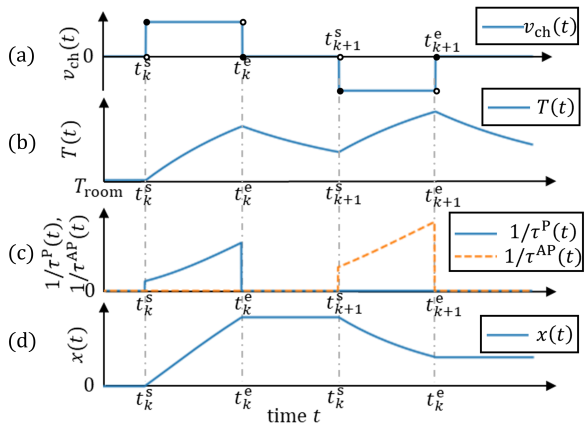

Figure 3: Schematic of the time variations of each variable: (a) channel voltage $v_{\rm ch}(t)$, (b) device temperature $T(t)$, (c) Néel relaxation time values in magnetization reversal from state AP to state P, $\tau^{\rm P}(t)$, and from state P to state AP, $\tau^{\rm AP}(t)$, (d) the fraction of magnetic domains in state P, $x(t)$.

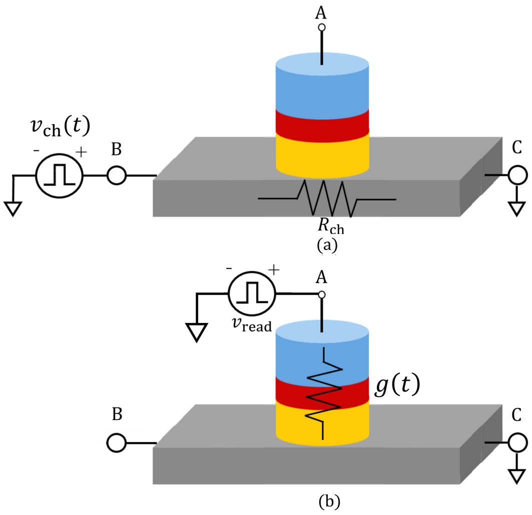

Figure 4: Schematic of the circuit used to derive the mathematical model: (a) when controlling the magnetization state, and (b) when measuring the tunneling conductance.

3.2. Device Temperature

When the voltage is input into the channel layer, the device temperature $T(t)$ increases owing to the Joule heat generated by channel resistance R_{ch} in Fig. 4. In addition, when the device temperature is higher than room temperature, the device heat is dissipated into air. Here, the device temperature is mathematically modeled by assuming that the temperature of the spintronics device is uniform, and that the room temperature dose not change.

When $t_k^s \leq t < t_k^e$, $T(t)$ can be obtained as follows (changes in R_{ch} owing to the device temperature are ignored here).

$$T(t) = T(t_{k-}^s) + \left(\mathrm{T_{room}} + \frac{\tau_T V_k^2}{CR_{ch}} - T\left(t_{k-}^s\right)\right)\left(1 - \exp\left(-\frac{t - t_k^s}{\tau_T}\right)\right), \tag{2}$$

where t_{k-}^s is the time immediately before t_k^s, $\mathrm{T_{room}}$ is room temperature,τ_T is the device temperature time constant determined by the device geometry and material, and C is the heat capacity of the device [3][4].

When $t_k^e \leq t < t_{k+1}^s$, $T(t)$ decreases to room temperature; therefore, $T(t)$ is expressed as

$$T(t) = T\left(t_{k-}^e\right) - \left(T\left(t_{k-}^e\right) - \mathrm{T_{room}}\right)\left(1 - \exp\left(-\frac{t - t_k^e}{\tau_T}\right)\right). \tag{3}$$

Equations (2) and (3) enable the determination of $T(t)$ at any time t from the initial state of $T(t)$. Fig. 3(b) shows the schematic of $T(t)$.

3.3. Néel Relaxation Time

Each magnetic domain is assumed to change its magnetization state through Néel relaxation [7]. The relaxation time is known as the Néel relaxation time and has different values for each magnetic domain. Let $\tau^P(t)$ be the mean value of the Néel relaxation time values for all magnetic domains at time t in magnetization reversal from state AP to state P at time t. In addition, let $\tau^{AP}(t)$ be the mean value of the Néel relaxation time values for all magnetic domains at time t in magnetization reversal from state P to state AP. These are then expressed as follows:

$$\tau^P(t) = \tau_0 \exp\left(\frac{E_b}{k_B T(t)}\left(1 - \frac{v_{ch}(t)}{V_{sw0}}\right)^2\right), \tag{4}$$

$$\tau^{AP}(t) = \tau_0 \exp\left(\frac{E_b}{k_B T(t)}\left(1 + \frac{v_{ch}(t)}{V_{sw0}}\right)^2\right), \tag{5}$$

where τ_0 is the attempt time, E_b is the energy barrier, k_B is Boltzmann's constant, and V_{sw0} is the average magnetization reversal voltage in the magnetic domain [6][8]. When $v_{ch}(t) > V_{sw0}$, $\tau^P(t) = \tau_0$, and when $v_{ch}(t) < -V_{sw0}$, $\tau^{AP}(t) = \tau_0$ as shown in Fig. 3(c).

It should be noted that while V_{sw0} varies among the multiple magnetic domains constituting the free layer of the synapce device in real devices [9], we do not consider this effect for simplicity.

3.4. Time Variation of the Magnetization State

A finite number of magnetic domains exists in the free layer, each of which independently achieves state P or AP. Let $x(t)$ be the fraction of magnetic domains in state P among those present in the device at time t. In the following, we assume that there are sufficient magnetic domains in the free layer, and that $x(t)$ takes continuous value.

The fraction of magnetic domains that undergoes magnetization reversal from state P to state AP per unit time is proportional to $x(t)$ and inversely proportional to $\tau^{AP}(t)$ [7]. Similarly, the fraction of magnetic domains that flips the magnetization from state AP to state P per unit time is proportional to $1 - x(t)$ and inversely proportional to $\tau^P(t)$ [7]. The following differential equation can be obtained for the time variation of $x(t)$.

$$\mathrm{d}x(t) = \frac{1 - x(t)}{\tau^P(t)}\mathrm{d}t - \frac{x(t)}{\tau^{AP}(t)}\mathrm{d}t. \tag{6}$$

When $t_k^s \leq t < t_k^e$, the magnetization state of the magnetic domain in the free layer changes because of Néel relaxation. When $V_k > 0$, let $\tau^{AP}(t) >> t_k^e - t_k^s$ and assume that magnetization reversal from state P to state AP does not occur. In this case, $x(t)$ is expressed as

$$\mathrm{d}x(t) = \frac{1 - x(t)}{\tau^P(t)}\mathrm{d}t. \tag{7}$$

By integrating both sides of Eq. (7), we obtain

$$\int_{x(t_k^s)}^{x(t)} \frac{1}{1 - x(t)}\mathrm{d}x(t) = \int_{t_k^s}^{t} \frac{1}{\tau^P(t)}\mathrm{d}t. \tag{8}$$

When $V_k < 0$, let $\tau^{\mathrm{P}}(t) >> t_k^{\mathrm{e}} - t_k^{\mathrm{s}}$ and assume that magnetization reversal from state AP to state P does not occur. In this case, $x(t)$ can be written as

$$\mathrm{d}x(t) = -\frac{x(t)}{\tau^{\mathrm{AP}}(t)}\mathrm{d}t. \tag{9}$$

By calculating Eq. (9) in the same manner as $V_k > 0$, the following equation is obtained:

$$\int_{x(t_k^{\mathrm{s}})}^{x(t)} \frac{1}{x(t)}\mathrm{d}x(t) = -\int_{t_k}^{t} \frac{1}{\tau^{\mathrm{AP}}(t)}\mathrm{d}t. \tag{10}$$

When $t_k^{\mathrm{e}} \leq t < t_{(k+1)}^{\mathrm{s}}$, we assume $\tau^{\mathrm{P}}(t), \tau^{\mathrm{AP}}(t) >> t_{(k+1)} - t_k^{\mathrm{s}}$. As the magnetization state does not change in this case, we obtain the following equation:

$$x(t) = x\left(t_{k-}^{\mathrm{e}}\right). \tag{11}$$

Using Eqs. (6) to (11), $x(t)$ for all t can be expressed. This allows us to determine $x(t)$ at any time t if the initial state of $x(t)$ is known as shown in Fig. 3(d).

3.5. Tunneling Conductance

To determine the tunneling conductance $g(t)$ of a spintronics device in Fig. 4(b), we consider the tunneling conductance of each magnetic domain in the free layer.

The magnitude of the tunnel conductance in a single magnetic domain varies depending on the state of the magnetic domain. Let us assume that all magnetic domains are equal in size, and let the value of the tunnel conductance of the magnetic domain in state P be G^{P} and the value of the tunnel conductance of the magnetic domain in state AP be G^{AP}. When M magnetic domains exist in the free layer, $g(t)$ is expressed using $x(t)$ as follows:

$$g(t) = M\left(\mathrm{G}^{\mathrm{P}}x(t) + \mathrm{G}^{\mathrm{AP}}(1 - x(t))\right). \tag{12}$$

From Eqs. (1)–(12), the time variation of $g(t)$ with respect to the voltage pulse train input can be modeled.

4. Conclusion

The mathematical model for the relationship between $v_{\mathrm{ch}}(t)$ and $g(t)$ for spintronics devices was developed. We considered pulse trains from a single input source. However, we must consider pulse trains from more input sources to use spintronics device as a neuronal device.

Also, the Néel relaxation times for each magnetic domain were assumed to be equal, but in practice these take different values. In the future, the distribution of magnetization reversal susceptibility of magnetic domains will be estimated and included in the model.

In addition to the proposed mathematical model, neuron and synapse models for neural networks and lumped models for integrated circuits are required to design spiking neural network circuits. We intend to construct these models based on the mathematical model constructed in this study.

Acknowledgments

This work was supported by JSPS KAKENHI Grant Number 20H00596, JST SPRING Grant Number JPMJSP2114, JST-CREST Grant Number JPMJCR19K3, the Cooperative Research Project of the Research Institute of Electrical Communication (R02/A31), Tohoku University, and the WISE Program for AI Electronics, Tohoku University.

References

[1] A. Kurenkov *et al.*, "Artificial neuron and synapse realized in an antiferromagnet/ferromagnet heterostructure using dynamics of spin-orbit torque switching," *Adv. Mater.*, vol. 31, no. 23, 1900636, 2019.

[2] W. Maass, "Networks of spiking neurons: The third generation of neural network models," *Neural Networks*, vol. 10, no. 9, pp. 1659–1671, 1997.

[3] Y. Kikuchi *et al.*, "Mathematical modeling of spintronics neuron devices based on thermal dynamics," *IEICE Technical Report*, vol. 119, no. 381, NLP2019–104, pp. 99–104, 2020 (in Japanese).

[4] Y. Kikuchi *et al.*, "Mathematical modeling of neuron/synapse-like spintronics devices," *in Proceedings of International Symposium on Nonlinear Theory and Its Applications*, pp. 77-80, 2020.

[5] T. Miyazaki, N. Tezuka, "Giant magnetic tunneling effect in Fe/Al2O3/Fe junction," *Journal of Magnetism and Magnetic Materials*, vol. 139, no. 3, pp. L231–L234, 1995.

[6] S. Fukami *et al.*, "A spin-orbit torque switching scheme with collinear magnetic easy axis and current configuration," *Nat. Nanotechnol.*, vol. 11, pp. 621–625, 2016.

[7] L. Néel, "Théorie du traînage magnétique des ferromagnétiques en grains fins avec applications aux terres cuites," Ann. Géophys, vol. 5, pp. 99–136 , 1949 (in French); an English translation is available in Selected Works of Louis Néel, edited by N. Kurti (Gordon and Breach, New York, 1988,), pp. 407–427.

[8] L. Liu *et al.*, "Spin-torque switching with the giant spin hall effect of tantalum," *Science*, vol. 336, pp. 555–558, 2012.

[9] G. Krishnaswamy *et al.*, "Multidomain memristive switching of Pt38Mn62/[Co/Ni]n Multilayers," *Phys. Rev. Applied*, vol. 14, 044036, 2020.