2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

Entropy enhancement in a semiconductor laser with double optical feedback

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Abstract—We compute the Lyapunov spectrum and Kolmogorov-Sinai entropy for a semiconductor laser subject to double optical feedback in numerical simulation. The spectrum and entropy are utilized for evaluating complexity of chaos and are important for some applications such as physical random number generation and secure optical communication based on optical chaos. In particular, we focus on the dependence of their measure on the relationship between the two delay times for double optical feedback. The entropy is enhanced when one of the delay times is longer than amounts of the time when characteristics of the shorter delay time in the auto-correlation function of the time-series become vanished.

1. Introduction

Semiconductor lasers with optical feedback from an external mirror can generate chaotic outputs. This systems have theoretically infinite dimension since they have time delayed feedback. Therefore, optical chaos generated from them has high complexity. This complex chaos has been studied for some applications, such as high-speed physical random number generation [1] and chaotic secure communications [2]. In these applications, high complexity of chaotic temporal waveforms is required for randomness of generated bits in physical random number generation and for secure communications.

To enhance the complexity of optical chaos, we consider to add additive optical feedback to a semiconductor laser with optical feedback. Figure 1 shows a model for a semiconductor laser with double optical feedback for generation of complex chaos. In this model, it is necessary to select the two feedback delay times $\tau_{1,2}$ and the two reflectivities of the external mirrors $r_{1,2}$ to enhance the complexity. The semiconductor laser with double optical feedback has been already studied for eliminating time delay signature [3]. However, the enhancement of complexity in the semiconductor laser with double optical feedback has not been studied yet.

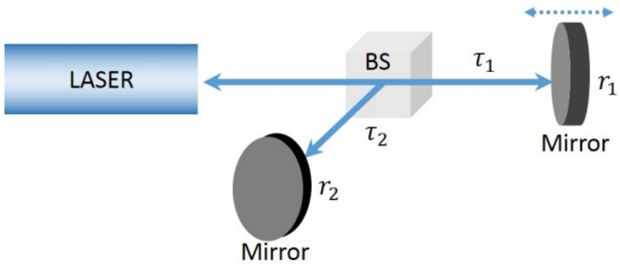

Figure 1. Model for investigation of chaotic dynamics in a semiconductor laser with double optical feedback. $r_{1,2}$ are the reflectivity of the external mirror, $\tau_{1,2}$ is the round-trip time of feedback light in the external cavity, and BS is the beam splitter.

In this research , we numerically evaluate the complexity of optical chaos by using Lyapunov exponents in a semiconductor laser with double optical feedback. Lyapunov exponents represent a growth rate of perturbations to a trajectory in phase space of nonlinear dynamical systems and measure the complexity of chaos. We also calculate Kolmogrov-Sinai entropy which is obtained from the sum of all the positive Lyapunov exponents. We investigate the delay times and the reflectivities for double optical feedback which enhance KS entropy.

2. Method

2.1. Lang-Kobayashi equation with double optical feedback

In our numerical simulation, we use the Lang-Kobayashi equations for a semiconductor laser with double optical feedback [4]. The Lang-Kobayashi equations consist of two equations for the complex slowly varying electric-field amplitude $\boldsymbol{E}$ and the carrier density N as follows,

$$\frac{d\boldsymbol{E}}{dt} = \frac{1+i\alpha}{2}\left[\frac{G_N(N(t)-N_0)}{1+\varepsilon|\boldsymbol{E}(t)|^2} - \frac{1}{\tau_p}\right]\boldsymbol{E}(t) + \kappa_1\boldsymbol{E}(t-\tau_1)\exp(-i\omega\tau_1) + \kappa_2\boldsymbol{E}(t-\tau_2)\exp(-i\omega\tau_2) \tag{1}$$

$$\frac{dN}{dt} = J - \frac{N(t)}{\tau_s} - \frac{G_N(N(t)-N_0)}{1+\varepsilon|\boldsymbol{E}(t)|^2}|\boldsymbol{E}(t)|^2 \tag{2}$$

The parameters in Eqs (1) and (2) are following: $G_N = 8.4\times10^{-13}$ is the gain coefficient, $N_0 = 1.4\times10^{24}$ is the carrier density at transparency, $\alpha = 3$ is the linewidth enhancement factor, $\varepsilon = 2.0\times10^{-23}$ is the saturation coefficient, $\tau_p = 1.927$ ps and $\tau_s = 2.04$ ns are the photon and carrier lifetimes, $\omega = 2\pi c/\lambda$, where c is the speed of light and $\lambda = 1537$ nm is the optical wavelength, is the optical angular frequency, and $J = 1.3J_{th}$, where J_{th} is the injection current at lasing threshold, is the injection current. The feedback strengths for two optical feedback are represented as $\kappa_{1,2} = (1 - r_f^2) r_{1,2} / (r_f \tau_{in})$, where $r_f = 0.556$ is the reflectivity of the laser facet, $\tau_{in} = 8.0$ ps is the round-trip time of light in the internal cavity, and $r_{1,2}$ are the reflectivities of the external mirrors. The delay times of light in the external cavities are represented as $\tau_{1,2}$. We fix $r_2 = 0.05$ and $\tau_2 = 1.0$ ns, and change r_1 and τ_1 in our study.

2.2. Calculation methods of Lyapunov exponents and Lyapunov spectrum

One of the important characteristics of chaos is known as sensitive dependence on initial conditions. The characteristic can be quantitatively measured by using Lyapunov exponent, which is one of the most widely acceptable evidences of chaos.

The Lyapunov exponents are calculated from a perturbation $\delta \boldsymbol{x}$ to a trajectory in phase space of dynamical systems. By linearizing the Lang-Kobayashi equations, we can obtain linearized equations for $\delta \boldsymbol{x}$.

We consider phase space of the semiconductor laser with double optical feedback. We can generally configure phase space from variables which show the state of dynamical systems. On the other hand, it is necessary to regard all variables in the delay time as the state in time delayed dynamical systems [3]. In addition we should note that our system have two time delayed feedback. Therefore, phase space must be configured from all variables included in two feedback. But, variables included in feedback with a short delay time are included in feedback with a long delay time. Accordingly, we can configure the phase space of the laser with double optical feedback from variables included in feedback with a long delay time.

To calculate the norm of the perturbation, it is necessary to temporally discretize $\delta \boldsymbol{x}$ by using an integration time step h since $\delta \boldsymbol{x}$ is a continuous time variable. Here, the number of $\delta \boldsymbol{x}$ in the delay time is given as $M = \tau/h + 1$. The norm D of the perturbation is calculated from the following equation:

$$D(\mathrm{t}) = \sqrt{\sum_{k=0}^{M} |\delta \boldsymbol{x}(t + kh)|^2} \qquad (3)$$

The Lyapunov exponent can be obtained from the following equation:

$$\lambda = \frac{1}{Nh} \sum_{i=1}^{N} ln \frac{D(t - (i-1)h)}{D(t - ih)} \qquad (4)$$

The same number of Lyapunov exponents as the number of variables of dynamical systems can be calculated. A set of these Lyapunov exponents is called Lyapunov spectrum. In this study, it should be notice that the semiconductor laser with double optical feedback is theoretically infinite dimensional system since the laser has time delayed feedback. Therefore, we can theoretically infinite number of Lyapunov exponents.

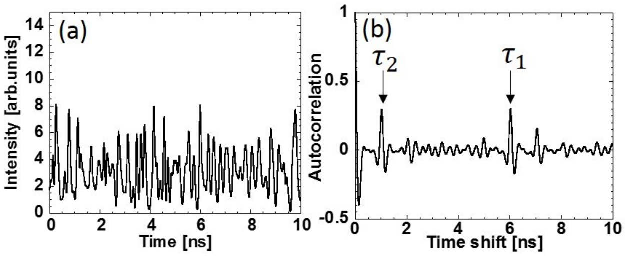

Figure 2. A temporal waveform (a) and an auto-correlation function (b) of the semiconductor laser with double optical feedback. The delay times are $\tau_1 = 6.0$ ns and $\tau_2 = 1.0$. The reflectivities are $r_1 = r_2 = 0.05$.

2.3. Kolmogorov-Sinai entropy

One of important characteristics of chaos is unpredictability for a long-term duration due to the sensitive dependence on initial conditions. This characteristic can quantitatively evaluated by Kolmogorov-Sinai (KS) entropy. KS entropy is calculated from the sum of all the positive Lyapunov exponents.

$$h_{KS} = \sum_{i=1}^{N} \lambda_i \qquad (5)$$

The Lyapunov exponents are ordered as $\lambda_i > 0 (i = 1, 2, \cdots, N)$.

3. Numerical results

3.1. Temporal waveform and auto-correlation function

A temporal waveform of the semiconductor laser with double optical feedback and the corresponding auto-correlation function are shown in this section. Figure 2 shows the temporal waveform and the auto-correlation function. The two delay times are set to $\tau_1 = 6.0$ ns and $\tau_2 = 1.0$ ns, respectively. The two reflectivities of the external mirrors are set to $r_1 = r_2 = 0.05$. The temporal waveform is shown in Fig. 2(a). We can observe complex oscillation. The auto-correlation function obtained from the temporal waveform shown in Fig. 2(a) is shown in Fig. 2(b). We observe the two peaks at time shifts of 1.0 ns and 6.0 ns, which are approximately equal to the two delay times $\tau_1 = 6.0$ ns and $\tau_2 = 1.0$ ns. These peaks are called time delay signature.

3.2. Dependence of KS entropy on the reflectivity r_1

KS entropy as a function of the reflectivity r_1 is shown in Fig. 3. When $r_1 = 0$, the semiconductor laser has only single feedback for the reflectivity r_2. The laser with single feedback has the maximum value of KS entropy at $r_2 = 0.05$. On the other hand, the laser has double feedback for $r_1 > 0$. KS entropy increases except for $0.010 \leq r_1 \leq 0.015$ as increases r_1 until $r_1 = 0.05$. The maximum value of KS entropy is obtained at $r_1 = 0.05$, which is

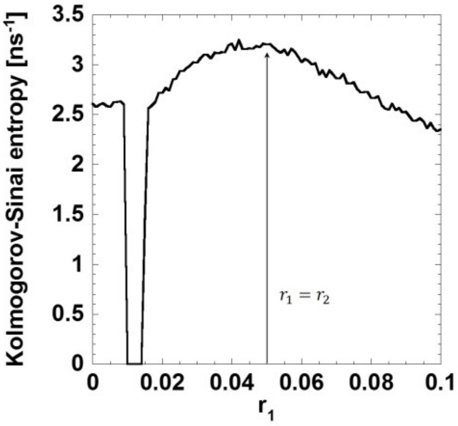

Figure 3. Kolmogorov-Sinai entropy as a function of the reflectivity r_1. The delay times are $\tau_1 = 6.0$ ns and $\tau_2 = 1.0$. The reflectivity of another mirror is $r_2 = 0.05$.

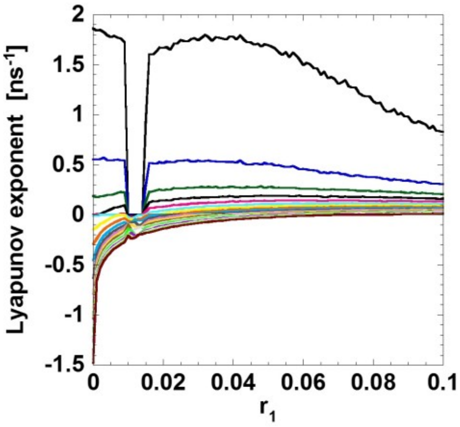

Figure 4. The Lyapunov spectrum as a function of the reflectivity r_1. The delay times are $\tau_1 = 6.0$ ns and $\tau_2 = 1.0$. The reflectivity of another mirror is $r_2 = 0.05$.

equal to $r_2 (= 0.05)$ for another external mirror. When r_1 is further increased, KS entropy monotonically decreases. From these results, we find that the semiconductor laser with double feedback can produce more complex chaos than one for the laser with single feedback. Furthermore, we find that a high value of KS entropy is obtained for nearly equal reflectivities $r_1 \approx r_2$ for the two external mirrors.

We show Lyapunov spectrum used for the calculation of KS entropy in Fig. 3 to investigate the reason why KS entropy increases. The Lyapunov spectrum as a function of the reflectivity r_1 is shown in Fig. 4. The number of the Lyapunov exponent shown in Fig. 4 is 20. As similarly shown in Fig. 3, $r_1 = 0$ means that the semiconductor laser has single feedback for r_2, and $r_1 > 0$ means that the laser has double feedback. The curve with the largest value corresponds to the maximum Lyapunov exponent λ_1, and the curve with the second-largest value is λ_2, and so on. We find that the maximum Lyapunov exponent is not so

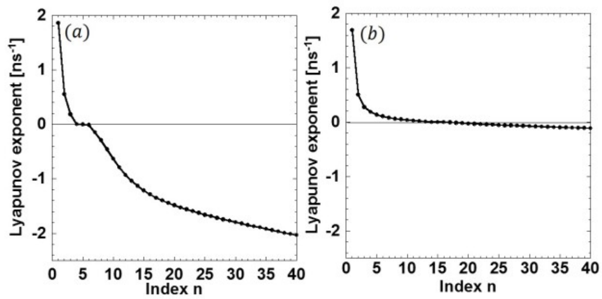

Figure 5. Lyapunov exponents λ_n are protted as a function of the index n up to $n = 40$. The reflectivity $r_1 = 0$ for (a) and the reflectivity $r_1 = 0.05$ for (b). Therefore, the laser has single feedback in (a). The delay times are $\tau_1 = 6.0$ ns and $\tau_2 = 1.0$. The reflectivity of another mirror is $r_2 = 0.05$.

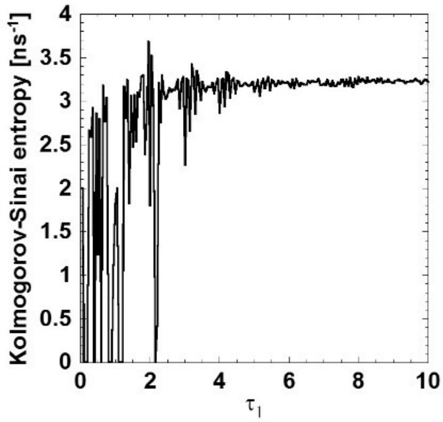

Figure 6. KS entropy as a function of the feedback delay time τ_1. The reflectivities are $r_1 = r_2 = 0.05$. The delay time of another feedback is $\tau_2 = 1.0$ ns.

changed except for $0.010 \leq r_1 \leq 0.015$ and the number of positive Lyapunov exponents is increased.

For more detail comparison of single feedback with double feedback, we show the Lyapunov exponents λ_n as a function of the index n up to $n = 40$ in Fig. 5. The semiconductor laser has single feedback $(r_1 = 0.0)$ for Fig. 5(a) and double feedback $r_1 = 0.05$ for Fig. 5(b). Four positive Lyapunov exponents are obtained for the laser with single feedback in Fig. 5(a). On the other hand, the laser with double feedback in Fig. 5(b) has sixteen positive, which number is larger than Fig. 5(a). KS entropy is calculated from the sum of all positive Lyapunov exponents. The laser with double feedback can produce more complex chaos since the laser has large number of positive Lyapunov exponents.

3.3. Dependence of KS entropy on the delay time τ_1

Figure 6 shows KS entropy as a function of the delay time τ_1. The delay time for another feedback is $\tau_2 = 1.0$ ns. The two reflectivities for the two external mirrors are set to $r_1 = r_2 = 0.05$. We find that KS entropy intensely fluctuates at the integral multiple of τ_2, such as $\tau_1 = 1.0$ ns,

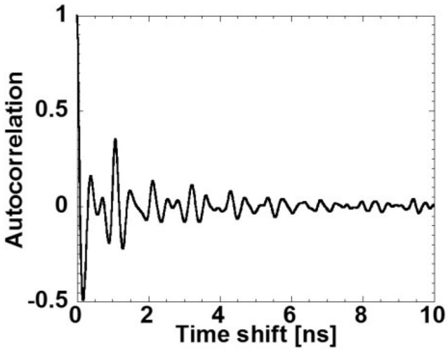

Figure 7. An auto-correlation function of a semiconductor laser with single feedback. The delay time τ is 1.0 ns and the reflectively r is 0.05.

2.0 ns, 3.0 ns, and so on. However, KS entropy is almost constant for $\tau_1 > 6.0$ ns.

We think about the reason why KS entropy is almost constant for $\tau_1 > 6.0$ ns. Figure 7 shows the auto-correlation function calculated from a temporal waveform of the semiconductor laser with single feedback. The delay time for the feedback is 1.0 ns and the reflectivity is 0.05. We can observe the peak at the time shift of 1.0 ns in the autocorrelation function. We also can see some peaks at the integral multiple of the delay time. However, the peaks decay for larger time shift than 6.0 ns. Therefore, if the delay time for additive feedback is shorter than 6.0 ns, the additive feedback can enhance the auto-correlation, which can degenerate KS entropy. Otherwise, additive feedback with a longer delay time than 6.0 ns can enhance KS entropy.

3.4 Kolmogorov-Sinai entropy as a function of the feedback phase

Finally, we show the dependence of KS entropy on the phase ψ for optical feedback from the external cavity 1. Figure 8 shows KS entropy as a function of ψ. In Fig. 8(a), $\tau_1 = 2.0$ ns and $\tau_2 = 1.0$ ns are used. It should be noted that τ_1 is the integral multiple of $\tau_2 (\tau_1 = \tau_2)$. We find that KS entropy increases and decreases by changing the feedback phase. We can obtain larger values of KS entropy if the feedback phase can be properly fixed. But the feedback phase easily fluctuates due to mechanical vibration [1]. Therefore, if the feedback phase cannot be fixed, the dependence should be avoided.

On the other hand, $\tau_1 = 3.8$ ns and $\tau_2 = 1.0$ ns (τ_1 is not integral multiple of τ_2) are used in Fig. 8(b). We cannot observe any dependence of KS entropy on the feedback phase. This means that the laser is stable for the fluctuation of the feedback phase due to mechanical vibration. Therefore, it should be avoided that the two delay times have a relationship of integral multiple.

4. Conclusion

In this study, we numerically calculated KS entropy, which is obtained from the sum of all the positive Lyapunov

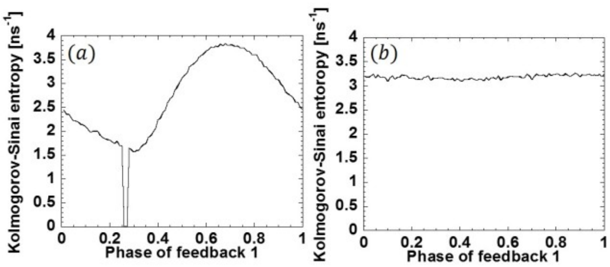

Figure 8. KS entropy as a function of the feedback phase for the external cavity 1. The delay time for the external cavity 1 is $\tau_1 = 2.0$ ns for (a). The delay time for the external cavity 1 is $\tau_1 = 3.8$ ns for (b). The delay time of another feedback is $\tau_2 = 1.0$ ns. The reflectivities are $r_1 = r_2 = 0.05$.

exponents, for evaluating complexity of chaos in a semiconductor laser with double feedback. We found that KS entropy for the laser with double feedback becomes larger than the laser with single feedback when the reflectivities of both external mirrors are almost identical. In addition, long one of the delay times should be selected to be enough longer than the time when the characteristic of short one of the delay times in auto-correlation functions disappears. We also should avoid the relationship of integral multiple of the two delay times if the feedback phase cannot be stabilized. Otherwise, we can enhance KS entropy by properly selecting the feedback phase.

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