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Principal frequency band of cascaded single-mode semiconductor lasers injected with broadband random light

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Abstract—We numerically study characteristics of dynamical response of a cascaded semiconductor laser system driven by broadband light with randomly fluctuating phase and amplitude. The output light waveform of the cascaded laser system is thoroughly determined by the injected light when the system exhibits a consistent response. We examine the frequency components of the injected light that dominantly affect the output. It is shown that the output is dominated by components in a particular frequency band and the width of this band strongly depends on detunings between two consecutive lasers in the cascade. We clarify a mechanism underlying this dependence.

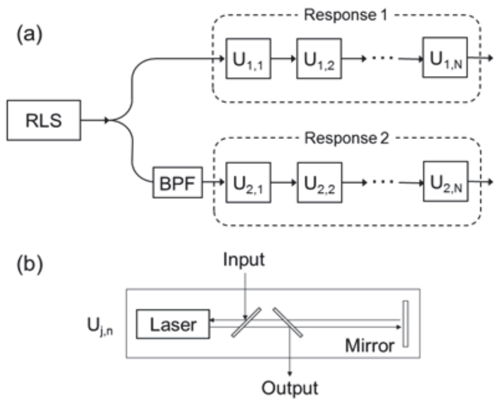

Figure 1: Illustration of system configuration (a) and laser unit (b).

1. Introduction

In a variety of dynamical systems, it is often observed that a system reproduces a consistent output in response to a repeatedly applied external input signal: i.e., the output waveform dose not depend on the initial condition but is thoroughly determined by the external signal. This property is called *consistency* [1].

The consistency property closely relates with synchronization between two independent dynamical systems which are driven by a common external input signal: two systems synchronize with each other if the system with a given input signal has consistency, and vice versa. This common-signal-induced synchronization (CSIS) has been of much interest and theoretically studied by several authors in the case of random input signal [2, 3, 4].

The CSIS in semiconductor lasers driven by common random light has potential application to secure communications. Recently, a secret key distribution scheme using correlated randomness in cascaded laser systems driven by injection of common random light with broad bandwidth, which has fast randomly fluctuating phase and/or amplitude, has been proposed [5, 6]. The security of this scheme relies on the difficulty of completely observing the broadband common random light with current technology. Such approach using the limits of observation technology is called *bounded observability* approach [7].

In order to achieve higher security in the above scheme, it is necessary to use a common random light with broader bandwidth, which is more difficult to completely observe. In addition, it is desirable that a large fraction of the frequency components of the random light effectively affects the output of the cascaded laser system, provided that the system has consistency. We call the frequency band of the injected random light that almost determines the output the *principal frequency band* (PFB).

It has been shown that a single-mode semiconductor laser has consistency for injection of a random light with broad bandwidth up to the order of THz [8, 9, 10]. The PFB has been numerically studied in such a case and shown to be much narrower than the bandwidth of the original random light [10]. This result indicates necessity of a method of extending the PFB for the secret key distribution application. In this paper, we investigate the PFB of a cascaded single-mode semiconductor laser system which is injected with a broadband random light, and propose a method of extending the PFB.

2. Model

Consider the system illustrated in Fig. 1(a), which is used in our simulation. A portion of light from a random light source (RLS) with broad bandwidth is injected into two response systems, which we call Response 1 and Response 2. Each response system consists of a cascade of N laser units, which are unidirectionally coupled via optical injection. We denote the nth unit in Response j with $\mathrm{U}_{j,n}$. Each unit consists of a semiconductor laser with an optical self-feedback loop as shown in Fig. 1(b). The light of the RLS is assumed to have fast and randomly fluctuating phase and amplitude. The RLS can be experimentally implemented by using a super luminescent diode. The optical coupling is unidirectional from the RLS to the response systems. A bandpass filter (BPF) is applied to the injected light for Response 2.

2.1. Laser

To model the semiconductor laser units in the response systems, we use the Lang-Kobayashi equation with optical injection:

$$\begin{aligned}\frac{dE_{j,n}}{dt} &= \left[i\Delta\omega_{j,n} + \frac{1+i\alpha}{2}G_N\left(N_{j,n}-N_{\mathrm{th}}\right)\right]E_{j,n} \\ &+ \frac{\kappa_{\mathrm{r}}}{\tau_{\mathrm{in}}}E_{j,n}(t-\tau)\exp[i\theta_{j,n}] + \frac{\kappa_{\mathrm{inj}}}{\tau_{\mathrm{in}}}E_{\mathrm{inj},j,n}, \quad (1)\\ \frac{dN_{j,n}}{dt} &= J - \frac{1}{\tau_{\mathrm{s}}}N_{j,n} - G_N\left(N_{j,n}-N_0\right)|E_{j,n}|^2, \quad (2)\end{aligned}$$

where $j = 1, 2$ indicate Response 1 and Response 2, respectively, the unit number is indicated by $n = 1, \ldots, N$. $E_{j,n}$ and $N_{j,n}$ represent the complex electric field and the carrier number density of unit $\mathrm{U}_{j,n}$, respectively, α the linewidth enhancement factor, κ_{r} the optical feedback strength, τ the external-cavity delay time, $E_{\mathrm{inj},j,n}$ the complex electric field of the injected light to $\mathrm{U}_{j,n}$, and κ_{inj} the injection strength. The injected light is given by $E_{\mathrm{inj},j,n} = E_{j,n-1}(t)$ for $n = 2, \ldots, N$, while we will describe $E_{\mathrm{inj},j,1}$ in the following subsections. Let $S(t)$ be the light generated by the RLS and ω_0 be its center optical angular frequency. We denote the optical angular eigenfrequency of $\mathrm{U}_{j,n}$ with $\omega_{j,n}$. The detuning parameter $\Delta\omega_{j,n}$ is defined by $\Delta\omega_{j,n} = \omega_{j,n} - \omega_0$. For later use, we define the frequencies $f_0 = \omega_0/2\pi$ and $f_{j,n} = \omega_{j,n}/2\pi$.

2.2. Random light

We describe a model for the RLS output $S(t)$. Let $\rho(t)$ and $\phi(t)$ be fluctuations in amplitude and phase of $S(t)$ defined by $S(t) = E_0|\,1 + \varepsilon\rho(t)\,|\exp[i\phi(t)]$, where E_0 and ε are positive constants. We assume $\rho(t)$ and $\phi(t)$ are described by the stochastic differential equations

$$\frac{d\rho}{dt} = -\rho/\tau_{\mathrm{m}} + \sqrt{2/\tau_{\mathrm{m}}}\,\xi(t) \quad (3)$$

and

$$\frac{d\phi}{dt} = \sqrt{2/\tau_{\mathrm{m}}}\,\eta(t), \quad (4)$$

where τ_{m} is a positive constant. In Eqs. (3) and (4), $\xi(t)$ and $\eta(t)$ are the normalized white Gaussian noise with the properties $\langle\xi(t)\rangle = \langle\eta(t)\rangle = 0$, $\langle\xi(t)\eta(s)\rangle = 0$, and $\langle\xi(t)\xi(s)\rangle = \langle\eta(t)\eta(s)\rangle = \delta(t-s)$, where $\langle\cdot\rangle$ denotes the ensemble average and δ is Dirac's delta function. The amplitude $\rho(t)$ is the Ornstein-Uhlenbeck process, and it has the properties $\langle\rho(t)\rangle = 0$ and $\langle\rho(t)\rho(s)\rangle = \exp[-|t-s|/\tau_{\mathrm{m}}]$. This indicates that the correlation time of $\rho(t)$ is given by τ_{m}. On the other hand, $\phi(t)$ has the property $\langle[\phi(t)-\phi(s)]^2\rangle = 2\tau_{\mathrm{m}}^{-1}|t-s|$. Since $\phi(t)$ has the diffusion constant τ_{m}^{-1}, its characteristic time for correlation decay can be defined by τ_{m}. Therefore, τ_{m} gives the time scale of fluctuation of $S(t)$. This implies that the bandwidth of $S(t)$ is of the order of τ_{m}^{-1}.

The injected light to the first unit is just given by $E_{\mathrm{inj},1,1}(t) = S(t)$ for Response 1. On the other hand, for Response 2, it is given by the light obtained by passing $S(t)$ through a bandpass filter.

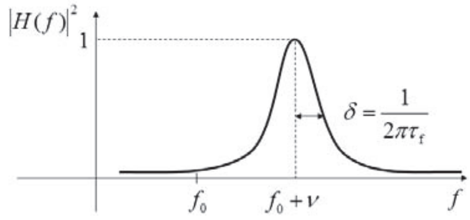

Figure 2: Illustration of $|H(f)|^2$ for the bandpass filter.

2.3. Bandpass filter

A bandpass filter is used for the injection to Response 2. We describe its details. Let ν and τ_{f} be positive constants, and $S_\nu(t) = S(t)\exp[-i2\pi\nu t]$. Using this S_ν, we define a new variable Y by the equation

$$\tau_{\mathrm{f}}\frac{dY}{dt} = -Y + S_\nu(t). \quad (5)$$

The injected light for Response 2 is given by $E_{\mathrm{inj},2,1}(t) = Y(t)\exp[i2\pi\nu t]$. The frequency response function $H(f)$ of this filter, which transforms S to $E_{\mathrm{inj},2,1}$, is obtained as $H(f) = 1/\{1 + i2\pi(f - f_0 - \nu)\tau_{\mathrm{f}}\}$. Figure 2 illustrates its modulus square $|H(f)|^2 = 1/[1 + \{2\pi\tau_{\mathrm{f}}(f - f_0 - \nu)\}^2]$. The center frequency of $|H(f)|^2$ is given by $f_0 + \nu$, and its half-maximum half width (HMHW) by $1/2\pi\tau_{\mathrm{f}}$, which we denote with δ.

2.4. Parameters

In our numerical simulations, the following parameter values were used: $G_N = 1.0\times10^{-12}\,\mathrm{m^3s^{-1}}$, $N_0 = 1.4\times10^{24}\,\mathrm{m^{-3}}$, $N_{\mathrm{th}} = 2.018\times10^{24}\,\mathrm{m^{-3}}$, $\tau_{\mathrm{in}} = 7\,\mathrm{ps}$, $\tau_{\mathrm{s}} = 2.04\,\mathrm{ns}$, $\tau = 0.3\,\mathrm{ns}$, $\theta_{j,n} = 0$, $\alpha = 5$, $\kappa_{\mathrm{r}} = 0.2$, $\kappa_{\mathrm{inj}} = 3$, and $J = 1.09\,J_{\mathrm{th}}$, where $J_{\mathrm{th}} = N_{\mathrm{th}}/\tau_{\mathrm{s}}$ is the lasing threshold of injection current. For this value of J, each laser in the response systems have the relaxation oscillation frequency 1.5 GHz. We assumed $\omega_{1,n} = \omega_{2,n}$

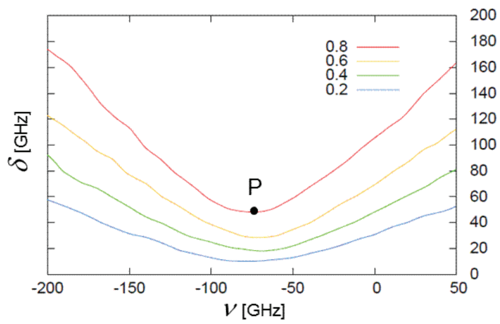

Figure 3: Contour plot of correlation C in (ν, δ) plane. Parameters are $N = 2$ and $\Delta\omega_{j,1} = \Delta\omega_{j,2} = 0$.

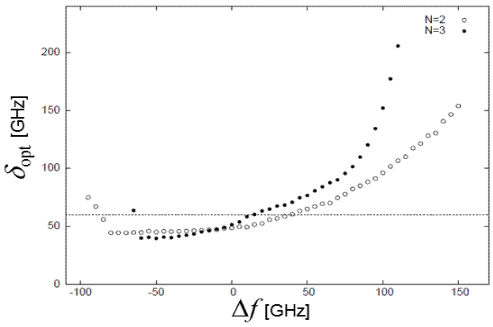

Figure 4: $\delta_{\rm opt}$ vs. Δf for cascaded laser systems: $N = 2$ (open circle) and $N = 3$ (filled circle). $\delta_{\rm opt}$ for $N = 1$ is also shown by dashed line.

for $n = 1, \ldots, N$ and $\Delta\omega_{j,1} = 0$ for $j = 1, 2$. As for the injected light, we set $\tau_{\rm m} = 1\,{\rm ps}$, $\varepsilon = 0.3$, and $E_0 = [0.16 J_{\rm th}/G_N(N_{\rm th} - N_0)]^{1/2}$.

3. Numerical simulation

We are interested in the frequency components of $S(t)$ that strongly affect the output of a response system, i.e., the PFB, provided that the system has consistency. For this purpose, we compare the outputs of both response systems. In order to measure the similarity of their outputs, we use the correlation between the output intensities $I_j(t)$ of the two response systems, where $I_j(t) = |E_{j,N}(t)|^2$. The correlation between $I_1(t)$ and $I_2(t)$ is defined by

$$C = \frac{\langle (I_1 - \mu_1)(I_2 - \mu_2)\rangle_T}{\sigma_1 \sigma_2}, \tag{6}$$

where μ_j and σ_j are the average and the standard deviation of I_j, respectively, and $\langle\cdot\rangle_T$ denotes the time average. By definition, C is in the range $-1 \leq C \leq 1$, and it takes the maximum $C = 1$ when the outputs are identical, i.e., $I_1(t) = I_2(t)$.

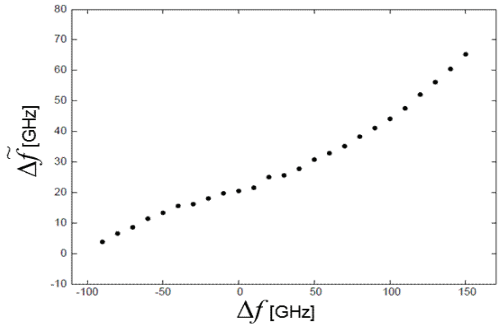

Figure 5: $\Delta\tilde{f}$ vs. Δf in cascaded laser system of $N = 2$.

It was confirmed that the response system has the consistency property in an appropriate parameter region. The following numerical calculations were carried out for parameter values in such a region.

We numerically calculated the correlation C for different values of the bandpass filter parameters (ν, δ) to examine which frequency components of $S(t)$ dominantly affect the response system output. Figure 3 shows an example of contour plot of C in (ν, δ) plane for the system of $N = 2$. Values of C close to unity imply that Response 2 can accurately reproduce the output of Response 1 although their injection lights are not identical with each other. If we use $C = 0.8$ as a reference value, accurate output reproducibility with $C \geq 0.8$ is achieved in a wedge-shaped region above red line in Fig. 3. This region takes the minimum value of δ at point P located at $(\nu, \delta) \simeq (-75, 50)$: the minimum HMHW needed for achieving $C \geq 0.8$ is $\delta \simeq 50$ GHz, and the optimal center frequency of the filtered band, which achieves $C \geq 0.8$ with this minimum HMHW, is given by $f_0 + \nu \simeq f_0 - 75$ GHz. This fact indicates that only the frequency components of $S(t)$ in the band of $\nu \simeq -75$ GHz and $\delta \simeq 50$ GHz almost determine the output of Response 1.

Let $\delta_{\rm opt}$ be the minimum value of δ, and $\nu_{\rm opt}$ be the value of ν at the point achieving $\delta_{\rm opt}$. We define the PFB by the frequency band characterized with $\nu_{\rm opt}$ and $\delta_{\rm opt}$. Contour plots of $C(\nu, \delta)$ for different N are qualitatively the same as Fig. 3, and thus $(\nu_{\rm opt}, \delta_{\rm opt})$ of the PFB can be defined analogously.

We investigate how the PFB depends on the detunings $f_{j,n-1} - f_{j,n}$ between two consecutive units and the number N of units. For simplicity, we assume that the detunings are the same for all $n = 2, \ldots, N$ and $j = 1, 2$: i.e., $f_{j,n-1} - f_{j,n} = \Delta f$. Figure 4 shows $\delta_{\rm opt}$ plotted as a function of Δf for different N. The value of $\delta_{\rm opt}$ for $N = 1$ is also shown for comparison. This figure shows that the dependence of $\delta_{\rm opt}$ on Δf is qualitatively the same in both cases of $N = 2$ and 3: $\delta_{\rm opt}$ is nearly constant in a region of negative and small positive Δf while it increases with increasing Δf for large positive Δf. It is clearly observed that systems with $N \geq 2$ can achieve larger values of $\delta_{\rm opt}$

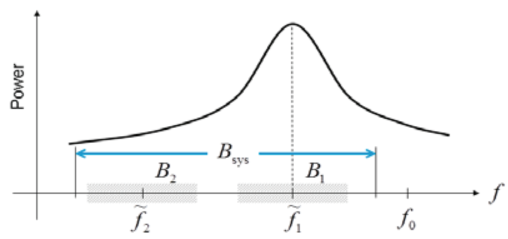

Figure 6: Illustration of mechanism for PFB in cascaded laser system.

than that of the single unit system, provided that $\Delta f > 0$ is sufficiently far from zero. In addition, δ_{opt} increases as N increases for a fixed $\Delta f > 0$. Based on these numerical results, it may be concluded that the PFB bandwidth can be extended by using a cascaded laser system with large N and appropriate Δf, compared with the single laser unit system.

In order to understand the Δf-dependence of δ_{opt}, we consider the simplest cascaded laser system of $N = 2$, and examine the frequency difference $\Delta \tilde{f} = \tilde{f}_1 - \tilde{f}_2$ between its first and second units, where $\tilde{f}_n$ is defined by

$$\tilde{f}_n = \lim_{T\to\infty} \frac{1}{2\pi T} \left[\arg E_{1,n}(T) - \arg E_{1,n}(0)\right]. \quad (7)$$

This $\tilde{f}_n$ is the mean frequency of nth unit of Response 1, which differs from its eigenfrequency $f_{1,n}$. Figure 5 shows $\Delta \tilde{f}$ plotted against Δf. It can be seen that $\Delta \tilde{f}$ is close to zero and only weakly depends on Δf, i.e., the frequency locking occurs, in a region approximately given by $\Delta f < 10$. This region approximately coincides with the region of near constant δ_{opt} observed in Fig. 4. Outside the frequency locking region, $\Delta \tilde{f}$ rapidly increases as Δf increases, and it should be noted that δ_{opt} also rapidly increases.

We discuss a mechanism underlying the Δf-dependence of δ_{opt}. Let us consider the mean frequencies f_0, $\tilde{f}_1$, and $\tilde{f}_2$ in the case of $N = 2$, and assume that they are different from each other and $\Delta \tilde{f} = \tilde{f}_1 - \tilde{f}_2 > 0$. This situation is illustrated in Fig. 6. The PFB of a single laser unit has been investigated in [10], and it has been shown that the PFB is centered at the mean frequency of the laser output. Based on this fact, the two laser units have their PFBs as shown by shaded bands in Fig. 6, which are labeled by B_1 and B_2. The first unit has an output spectrum shown in Fig. 6, which has most of its power near the peak at $\tilde{f}_1$. This is just the spectrum of input signal to the second unit. The second unit is sensitive to the input components within its PFB B_2, and then the output strongly depends on these components in spite of their small power. On the other hand, the second unit output also strongly depends on the input components around $\tilde{f}_1$ since they have large power, although the second unit is insensitive in this frequency range. This implies that the PFB of the cascaded laser system is given by the minimum frequency band of $S(t)$ that dominates the components in these two bands of the first unit output, as indicated by B_{sys} in Fig. 6. Thus, the bandwidth of B_{sys} increases with increasing $\Delta \tilde{f} = \tilde{f}_1 - \tilde{f}_2$. This mechanism explains the Δf-dependence of δ_{opt}: both δ_{opt} and $\Delta \tilde{f}$ are approximately constant in the frequency locking region in Δf while both of them increases as Δf increases outside the locking region.

4. Conclusions

We numerically studied the PFB of a cascaded semiconductor laser system injected with broadband random light in the parameter regime that the system has consistency. It has been shown that the PFB bandwidth can be extended by using cascade systems consisting of multiple laser units with large detunings, compared with the single unit system. It has been found that the PFB bandwidth δ_{opt} strongly depends on the detuning Δf between two consecutive units in the cascade: δ_{opt} is approximately constant in the frequency locking region while it rapidly increases with increasing Δf outside the region. We have clarified the mechanism underlying this Δf-dependence of δ_{opt}.

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