2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

Study of the Effects of Initial Conditions on an Ideal Memristor Model

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Abstract– This paper presents an analysis of the effects of the initial conditions on the I-V characteristics of an ideal memristor. Using Leon Chua's voltage controlled ideal memristor, a simple piecewise linear charge flux relationship has been modeled that facilitates changes to a number of initial conditions, such as the amplitude of the initial voltage function and the initial flux of the system. Using extensive simulation, the significant effects of the variables on the pinched hysteresis loop that is characteristics of memristors has been presented.

1. Introduction

The theoretical existence of the memristor was first postulated by Leon Chua in 1971 [1] and stands alongside the resistor, capacitor and inductor as the fourth fundamental two terminal passive circuit element. His theory that there may be a fourth missing fundamental circuit element was based on the already widely accepted relationships between the four fundamental circuit variables: current i, voltage v, charge q and flux φ. Hewlett Packard's 2008 physical implementation of a memristor device [2] using partially doped titanium oxide revived interest in the field of memristor study and since then many potential applications have been proposed for these new devices such as in creating a cross bar memory system [3], in modeling neuromorphic systems [4] and even in implementing digital logic [5]. Many models have been proposed for the memristor [6-10] each with varying degrees of complexity and applicability to a physical implementation; however, the fundamentals still remain grounded in Chua's initial work. More recently, Chua has extended his theory to define a number of more specific categories of memristors, such as the ideal memristor model [11, 12]. Some analysis of these models has been presented, but one particular area that has not been comprehensively explored is the affects that the initial conditions have on the behavior. A better understanding of these affects could lead to more comprehensive models and development of further theoretical relationships.

2. Modeling of Memristors

The memristor is characterized by the relation of the type $g(\varphi, q) = 0$ [1]. The memristor is controlled by charge q or flux φ and can be expressed as a single-valued function of either. The voltage across a charge-controlled memristor is given by:

$$v(t) = M\big(q(t)\big)i(t) \tag{1a}$$

Where the memristance M is given by:

$$M(q) \equiv d\varphi(q)/dq \tag{1b}$$

A flux-controlled memristor is defined by a similar method, where the current across a flux controlled memristor is given by:

$$i(t) = G(\varphi(t))v(t) \tag{2a}$$

Where the memductance G is given by:

$$G(\varphi) \equiv dq(\varphi)/d\varphi \tag{2b}$$

2.1. Ideal Memristor Model

Chua also defined an ideal memristor model [11] which looks at the very fundamentals of a memristor. A voltage controlled ideal memristor is defined by the relationship:

$$q = \hat{q}(\varphi) \tag{3a}$$

or

$$i = G(\varphi)v \tag{3b}$$

$$\frac{d\varphi}{dt} = v \tag{3c}$$

$$\text{where } G(\varphi) \triangleq \frac{d\hat{q}(\varphi)}{d\varphi} \tag{3d}$$

Modeling a voltage controlled ideal memristor requires defining a driving voltage with amplitude A, a charge flux relationship and initial flux $\varphi(0)$.

2.1.1. Piecewise Linear Model

The charge flux relationship is the primary function that defines the memristors behavior. In order to simplify the analysis, a piecewise linear model can be used and thus the charge flux relationship is defined in (4) and seen in Figure 1.

$$q = 0.01\varphi + 0.1|\varphi + 0.1| - 0.1|\varphi - 0.1| + 0.1|\varphi + 0.3| - 0.1|\varphi - 0.3| \quad (4)$$

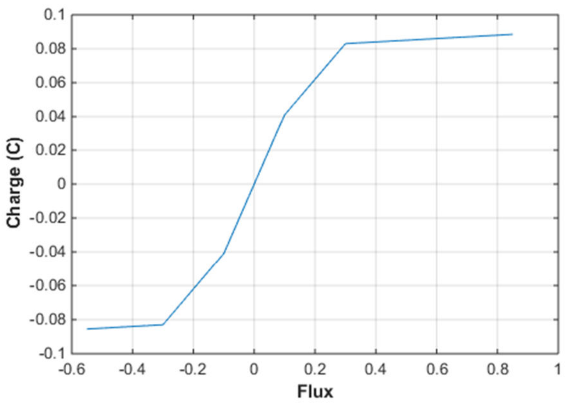

Fig 1. Charge Flux Relationship for the function given in (4)

2.1.2. Driving Voltage

Choosing a simple sinusoidal driving voltage allows for the first initial condition to be defined: the amplitude A of the voltage function. Thus the voltage function is defined as:

$$v(t) = Asin(t) \quad (5)$$

2.1.3. Initial Flux

A function for time dependent flux can be derived by solving the equation in (3c) using the relationships defined in (4) and (5):

$$\varphi(t) \triangleq \varphi(0) + \int_0^t v(\tau)d\tau \quad (6)$$

Initial flux $\varphi(0)$ is now a second initial condition that has an effect on the behavior of the voltage controlled ideal memristor model.

2.1.4. Current

Current is the last variable of interest to be defined. This can be easily found by differentiating the time dependent charge function and using the definition of current as being the rate of charge flow:

$$\frac{dq(t)}{dt} = i(t) \quad (7)$$

2.2. Modeling the Memristor

Applying this method to a system with the initial conditions of $A = 0.7$ and $\varphi(0) = -0.55$ allows both the current $i(t)$ and voltage $v(t)$ to be defined as functions of time as seen in Figure 2. This has been achieved by using the piecewise linear charge flux relationship defined in (4) which is represented in Figure 1. Together, these functions create the pinched hysteresis loop in the V-I plot shown in Figure 3 which is a unique trait displayed by memristors [9].

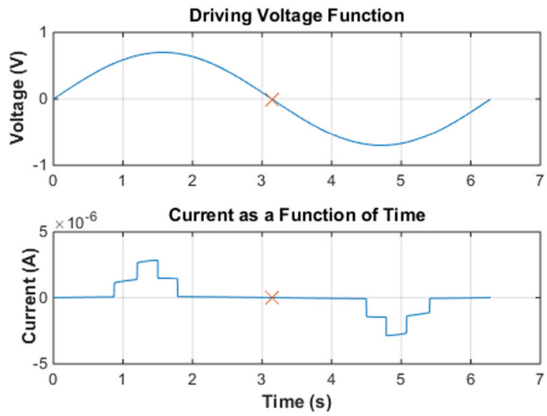

Fig 2. Voltage and current functions for $\boldsymbol{\varphi(0) = -0.55}$, the charge flux relationship defined in (4) and $\boldsymbol{A = 0.7V}$

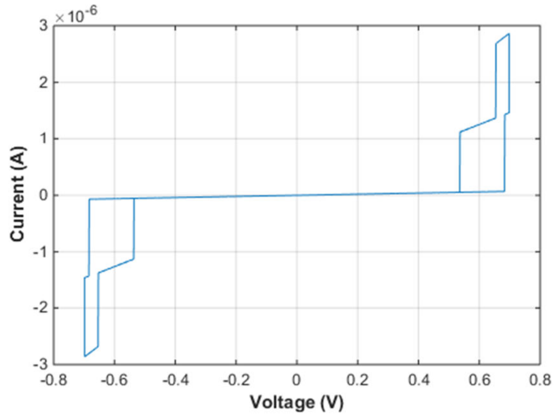

Fig 3. Pinched hysteresis loop of a voltage controlled ideal memristor model using voltage amplitude $\boldsymbol{A = 0.7V}$ and an initial flux of $\boldsymbol{\varphi(0) = -0.55}$.

3. Varying Voltage Amplitude

By varying amplitude, a number of different pinched hysteresis loops can be found over a relatively small variation of A. Figure 4 shows a variation of $0.1V$ in A in increments of $0.05V$ which results in three significantly different pinched hysteresis loops.

Each plot has a drastically different shape in regards to the pinched hysteresis loop. Whilst some of the uncertainty can be attributed to the piecewise linear charge flux model used, a significant amount of sensitivity is still present in the system. By further altering the amplitude between the bounds of $A = [0.05, 1]$ whilst maintaining $\varphi(0) = -0.55$, the plots begin to standardize in shape indicating a larger degree of sensitivity for lower levels of A.

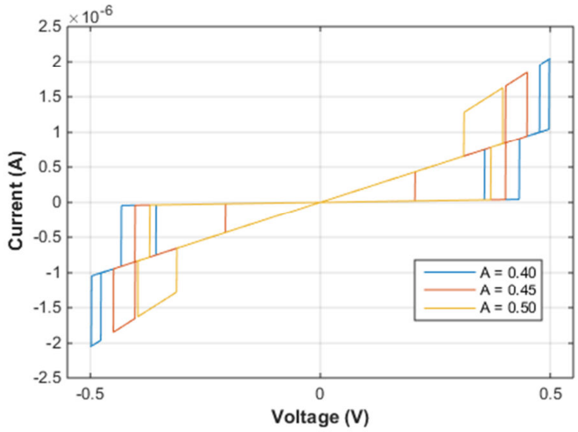

Fig 4. Pinched hysteresis loops for $\boldsymbol{\varphi(0) = -0.55}$, the charge flux relationship defined in (4) and (a) $\boldsymbol{A = 0.4V}$ (b) $\boldsymbol{A = 0.45V}$ (c) $\boldsymbol{A = 0.5V}$

Another interesting result is found when A is increased to a relatively large value, such as around 50V as shown in Figure 5. The tails of the hysteresis loop increase to the full -50V to 50V range, but the actual qualitative characteristic of the memristor stay the same i.e. the area enclosed by the hysteresis loop remains the same after a certain point. Simulations with the amplitude incremented from 0V to 50V confirmed these results.

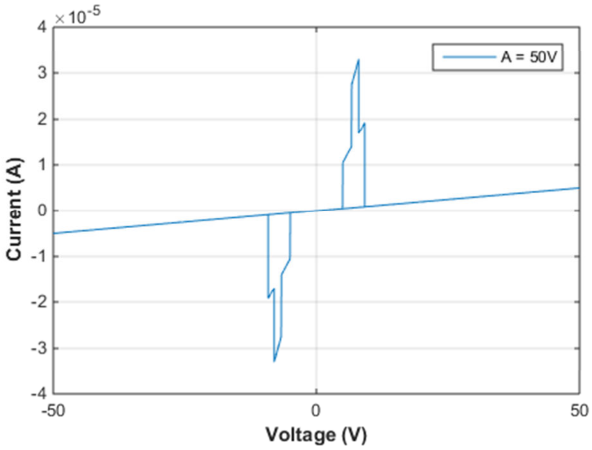

Fig. 5. Pinched hysteresis loop of a voltage controlled ideal memristor model using $\boldsymbol{\varphi(0) = -0.55}$ and $\boldsymbol{A = 50V}$

3. Varying Initial Flux

The second initial condition that can be varied using this model is that of initial flux. Like A, varying initial flux produces pronounced results over a relatively short range. Figure 6 shows this result, with four different hysteresis loops produced by varying $\varphi(0)$ in 0.5 increments from -0.95 to -0.5. Again, the shapes of the pinched hysteresis loops vary dramatically, demonstrating the significant sensitivity that the V-I characteristics display towards $\varphi(0)$.

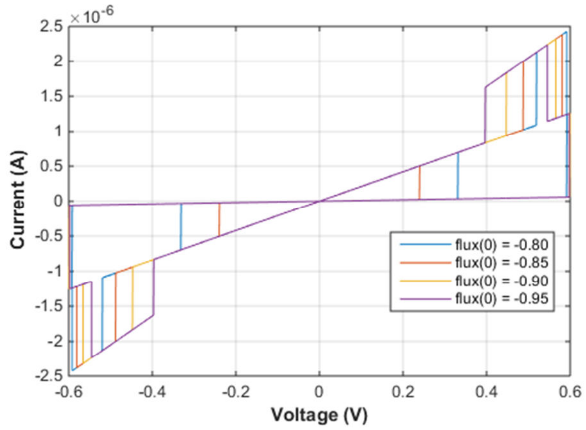

Fig. 6. Pinched hysteresis loops for $\boldsymbol{A = 0.6V}$, the charge flux relationship defined in (4) and (a) $\boldsymbol{\varphi(0) = -0.95}$ (b) $\boldsymbol{\varphi(0) = -0.90}$ (c) $\boldsymbol{\varphi(0) = -0.85}$ (d) $\boldsymbol{\varphi(0) = -0.80}$

The effect on the pinched hysteresis loop when the initial flux is given an extreme value produces another interesting result. This is shown in Figure 5.

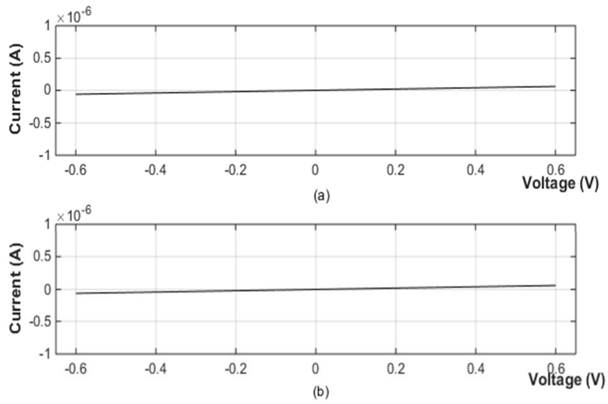

Fig. 7. Pinched hysteresis loop of a voltage controlled ideal memristor model using $\boldsymbol{A = 0.6V}$ and (a) $\boldsymbol{\varphi(0) = -5}$ (b) $\boldsymbol{\varphi(0) = 5}$

These graphs show that as $\varphi(0)$ tends towards a relatively large absolute value, the pinched hysteresis loop effect degenerates and the V-I characteristics tends toward a linear representation. Similar to the analysis of varying A, simulation has been undertaken to confirm that the pinched hysteresis loop tends to a straight line representation as the absolute value of the initial flux is increased.

This behavior appears similar to what occurs to a memristor system under high frequencies, as these conditions also cause the pinched hysteresis loop to degenerate to a linear representation [11].

4. Charge Flux Relationship

Changing the charge flux relationship that is used in the model allows a better overall representation of the system.

Higher order charge flux relationships can also be used in order to more effectively demonstrate real world behavior of a memristor. Using the quadratic charge flux relationship seen in (8), the variables of A and $\varphi(0)$ can be swept.

$$q(t) = \varphi(t)^2 \tag{8}$$

Figure 8 and 9 display the results and highlight the significant change that occurs due to changes in the initial conditions. For $\varphi(0)$, a somewhat constant change is apparent but for A the hysteresis loops appear to invert around the point of $A = 0V$.

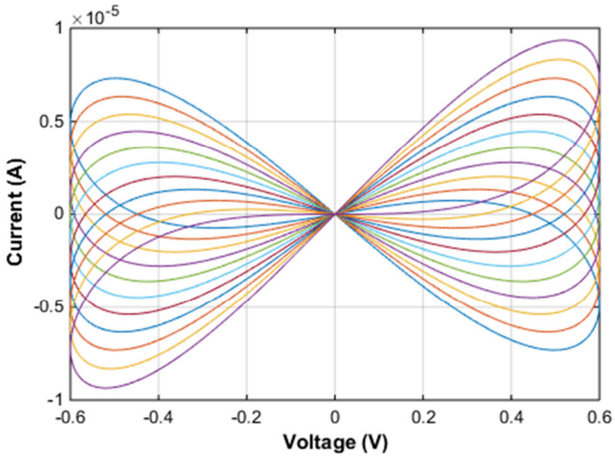

Fig. 8. Pinched hysteresis loops of a voltage controlled ideal memristor model using the charge flux relationship in (8), $\boldsymbol{A} = \mathbf{0.6}\boldsymbol{V}$ and sweeping $\boldsymbol{\varphi}(\mathbf{0})$ from -1 to 0 in 0.1 increments.

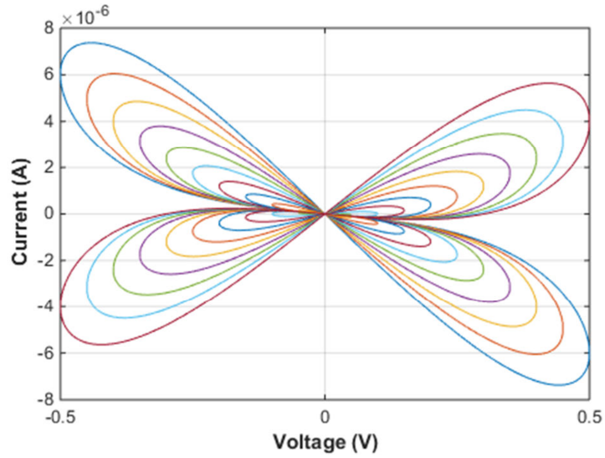

Fig. 9. Pinched hysteresis loops of a voltage controlled ideal memristor model using the charge flux relationship in (8), $\boldsymbol{\varphi}(\mathbf{0}) = -\mathbf{0.1}$ and sweeping $\boldsymbol{A}$ from -0.5V to 0.5V in 0.05V increments.

5. Discussion and Further Study

These results indicate that indeed, the initial conditions of an ideal memristor model (and by extension, the majority of the memristor models) have a very significant impact on the pinched hysteresis loop that is characteristic of a memristor system. Further qualitative analysis has been undertaken and the results have hinted at relationships that may be able to be defined quantitatively. The evolution of the pinched hysteresis loop when run in an animation environment is evidence of this and as demonstrated it is somewhat more obvious when a quadratic charge flux relationship is used. It appears that there is some link between the effects of the initial conditions, highlighted by the similarity between a high frequency memristor system and a system that has a large absolute value of initial flux.

6. Conclusion

This paper has presented simulations of Chua's ideal memristor model with the intent of analyzing the effects of the initial conditions of the system, namely the amplitude A of the driving voltage function and also the initial flux $\varphi(0)$. A number of interesting results have been presented, such as a memristors pinched hysteresis loop showing a high sensitivity to even slight variations in the above variables. The effect of large values of these variables has also been presented.

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