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Meta-Heuristic Algorithms with Chaotic Neuro-Dynamics for Solving Combinatorial Optimization Problems

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Abstract—In science and engineering, we are often asked to realize optimization: for example, job scheduling, delivery planning, VLSI design, circuit drilling, computer wiring, and so on. To solve these problems, an heuristic algorithm with chaotic neuro-dynamics has already been proposed. In this paper, we first reviewed the algorithm for solving combinatorial optimization problems by the chaotic neuro-dynamics. We also mentioned how the chaotic neuro-dynamics is effective for combinatorial optimization in real-world problems.

1. Introduction

In our daily life, we are often confronted with difficulties of realizing optimization: for example, job scheduling, delivery planning, circuit designing and drilling, computer wiring, packet routing and so on. These problems are classified into discrete optimization. To solve these problems, we may try to find a solution intuitively. However, such an intuition generally does not work well, then, it is important to design algorithms systematically, even if they are heuristic.

To solve NP-hard combinatorial, such as traveling salesman problem (TSP) [1] , quadratic assignment problem (QAP) [2] , packet routing problem (PRP) [3] , motif extraction problem (MEP) [4] , and vehicle routing problem (VRP) [5, 6] , various heuristic methods have already been proposed. Among them, a method with chaotic neuro-dynamics, one of meta-heuristics, solves the problems efficently.

In this paper, we reviewed the the approaches for solving combinatorial optimization problems using the chaotic neuro-dynamics. Firstly we discussed how to realize chaotic neuro-dynamics to solve the problems. Then, we also mentioned how the approach with chaotic neuro-dynamics is applicable to various types of real-world problems.

2. Chaotic neuro-dynamics

2.1. Mutual connection neural network

The basic concept of using steepest descent down hill dynamics of a mutual connection neural network was formulated by Hopfield and Tank [18]. Although this approach was theoretically attractive, two major drawbacks exist if the approach is applied to real-world problems.

The first one is that many undesirable local minima exist, then the searching states get stuck at the local minima. The second one is that the neural network often offer unfeasible firing patterns which cannot be encoded to a closed-tour of TSP. Thus, possible size of the problems is too small to go beyond the toy problems as real engineering applications.

2.2. Mutual connection chaotic neural network

To solve combinatorial optimization problems with a heuristic algorithm, one of the most important issue is how to avoid undesirable local minima. Although many algorithms have already been proposed to resolve such an issue, it has been shown that a new approach with chaotic neuro-dynamics is very effective. The idea is based on the fact that chaotic dynamics could produce a fractal attractor. Because such fractal attractors have zero-Lebesgue measure, it is possible to reduce volume of the searching state space, if the optimum solution is embedded in the fractal attractors.

The results based on the above concept was reported by H. Nozawa [33]. Nozawa modified a mutual connection neural network by the Euler method to obtain a neural network model with negative self-feedback connections, which is equivalent to the chaotic neural network proposed by Aihara et al. [7].

Even if the chaotic dynamics is implemented in the dynamics of the mutual connection neural network, it is not so easy to obtain feasible solution with this framework. In Refs.[22, 35], an algorithm for obtaining feasible solutions from the firing patterns of the mutual connection neural networks was proposed. This algorithm is widely used to realize feasible solutions under the framework of the mutual connection neural network, for example a chaotic neuro-computer system [19], and an additive chaotic-noise approach to the mutual connection neural network [17, 16, 37, 36].

3. Local search controlled by chaotic neuro-dynamics

Although the local minimum problem can be avoided by chaotic dynamics, the framework with mutual connection neural networks requires n^2 neurons and n^4 mutual connections in case of solving an n-city TSP. If the problem size n increases, the number of mutual connection becomes larger, consequently calculation gets difficult.

To solve TSP, several heuristic algorithms have been studied. These methods are effective to obtain near optimal solutions. However, these algorithms also have steepest descent down hill dynamics. Thus, the issue of undesirable local minima still exists.

In Ref.[11], Hasegawa, Ikeguchi and Aihara proposed a novel heuristic algorithm which combines chaotic dynamics and the 2-opt, one of the local search algorithms for TSP (Fig.1). In their approach, execution of the 2-opt algorithm is controlled by memory effect in chaotic neuron. It was shown that the algorithm can be applied to much larger-size problems than the conventional neural network approach with high performance.

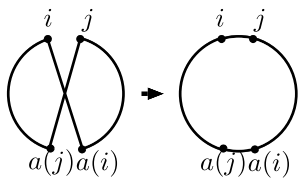

Figure 1: Execution of the 2-opt algorithm: $a(i)$ is the next city of the city i in the current tour.

This basic idea [11] has been extended to include the tabu search, one of the strongest heuristic methods[9, 10]. The tabu search memorizes the states that have been already searched and forbid to return to the states [9, 10]. The chaotic neural network has refractoriness, which is one of the important characteristics of real biological neurons [7]. Then the chaotic neural network model avoids to search the same state due to the refractory effects. which is similar effect as the tabu effect [9, 10]. However, an exponential decay in the refractory effect of the chaotic neuron brings better results than the conventional tabu search. [12, 15].

4. Application to other local searches

Although the simple local search, or the 2-opt, is used in the chaotic search [11, 14, 13], it is natural to expect that the chaotic search can search much better solutions if we use a more sophisticated local search.

In.Ref.[30], a new chaotic algorithm with another simple local search, or the Or-opt[34], has been proposed. The Or-opt attempts to improve a current tour by moving a partial path of maximum three consecutive cities in a different location (Fig. 2). As a result, the proposed method[30] obtains better solutions than conventional chaotic searches [11, 14, 13].

The Or-opt algorithm could obtain solutions comparable to the 3-opt. In general, if k increases, performance of the k-opt algorithm becomes better. However, calculating costs becomes huge as k increases. To avoid such unnecessary calculation, Lin and Kernighan proposed an algorithm [26], one of the most powerful local search for TSP. In Ref.[32], an effective algorithm of controlling the Lin-Kernighan method by chaotic neuro-dynamics is proposed.

In Table 1, performance for several problem instances are summarized. From these results, if we use a sophisticated algorithm for chaotic neuro-dynamics, we could obtain better solutions, even though the computational time increases.

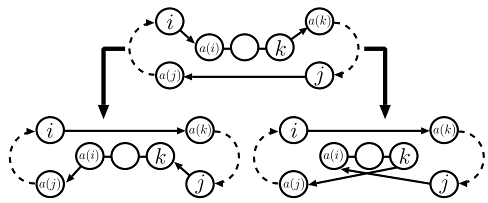

Figure 2: Execution of the Or-opt algorithm. In this example, $a(i)$ is the next city to i, and a partial tour $a(i)$-$\cdots$-k is inserted into another path (j-$a(j)$).

5. Applications to real-world problems

5.1. Motif extraction problem

The Human Genome Project completed in April 2003 generated approximately three billion base pairs and 25,000 genes. One of the primary issues in bioinformatics is identification of biologically important parts, called motif, in DNA, RNA and protein sequences. To realize the issue, many approximation algorithms have been proposed [8].

To solve MEP, a new motif extraction method with chaotic neuro-dynamics, called Chaotic Motif Sampler (CMS), [31] has already been proposed. Avoiding undesirable local minima, the CMS detects the motifs by firing of chaotic neurons.

Table 1: Results of (1) the 2-opt based chaotic search [14], (2) the results of the 2-opt based chaotic search with double bridge [13], (3) the adaptive k-opt based chaotic search with double bridge [13], (4) the 2-opt and Or-opt chaotic search, and (5) the LK based chaotic search. Results are expressed by percentages of gaps between obtained solutions and the optimal solutions.

	(1)	(2)	(3)	(4)	(5)
kroA100	0.287	0.045	0.045	**0.002**	*0.037*
kroA200	0.599	0.413	0.415	*0.372*	**0.208**
pcb442	1.034	0.982	0.825	*0.409*	**0.243**
pcb1173	1.692	1.748	1.569	*0.804*	**0.577**
pr2392	1.952	2.000	1.839	*1.153*	**1.107**
rl5915	2.395	2.273	1.742	*1.291*	**1.119**
rl11849	2.223	1.730	1.186	*1.139*	**1.076**

In Refs.[28, 27], analysis on the refractoriness produced from chaotic neurons is discussed from the effectiveness of searching solutions of problems. Using the method of surrogate data, it is shown that the chaotic search has significantlly better performance than stochastic noise

During a searching process, chaotic neurons in CMS generate complicated spike time-series. In Ref.[29], to analyze characteristic property of chaotic neurons in CMS, the spike time-series is analyzed by statistical measures, coefficient of variation C_V and local variation of inter-spike intervals L_V. As a result, chaotic neurons corresponding to correct motif-position have higher C_V values and lower L_V values.

5.2. Vehicle routing problem

Although VRP is a typical combinatorial optimization problems as well as TSP and QAP, it has a different feature from TSP and QAP: the object of VRP is constrained by a temporal frame defined by customers. Then, it becomes an important factor to consider feasibility of solutions in exploring state spaces.

In Ref.[20], a method using chaotic dynamics for VRP has been proposed. The results show that chaotic method exhibits better performance than the conventional tabu search for VRP of Solomon's benchmark problems [5].

In Ref.[21], an extended chaotic search with two simple local searches was also proposed. It is shown that the proposed method [21] has good performance for large-scale VRP benchmark problems[6].

5.3. Packet routing problem

How to find shortest paths between two nodes in a network is an important issue in science and engineering. In the simplest case, weights of links of the network are static, which can be easily solved by the Dijkstra algorithm. However, under practical situation, the weights of the links usually depend on the state of the network. For example, in computer networks, they depend on flowing packets or queuing packets at nodes. Thus, it becomes difficult to find the shortest paths. Such a network is referred as a dynamic and stochastic network.

In Refs. [24, 25, 23], Kimura and Ikeguchi proposed a new heuristic algorithm using chaotic dynamics to solve the dynamic and stochastic shortest path problems in the computer networks. The results show that chaotic rouging could be a powerful tool to reduce congestion in computer networks which have complex topology, such as small-world or scale-free structures.

6. Conclusions

In this paper, we have reviewed a novel approach for solving combinatorial optimization problems: the chaotic neuro-dynamics controls execution of local search.The chaotic dynamics could be a useful, practical and powerful tool to solve the combinatorial optimization problems. It is an important future problem to theoretically analyze the reason why these methods using chaotic dynamics show high solving ability, and the method to tune parameters for better performance.

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