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Research on Coupled Systems of Chaotic Oscillators and Noisy Oscillators

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Abstract—In this study, the breakdown of synchronization observed from four coupled chaotic oscillators is investigated. In order to understand the phenomenon, the model of coupled modified van der Pol oscillators with noise is considered. The comparison of the coupled chaotic oscillators with the coupled modified van der Pol oscillators with noise gives us some interesting results.

1. Introduction

Synchronization is one of typical nonlinear phenomena observed in the field of natural science. There have been many investigations on mutual synchronization of oscillators. Breakdown of synchronization is a kind of cooperative phenomenon for dissipated assembly oscillators and is important to clarify its mechanism for better understanding of higher-dimensional complicated phenomena [1]-[6]. In the case of coupled chaotic oscillators, chaotic fluctuations of their waveforms are supposed to play a role to break the synchronization and the breakdown sometimes causes chaotic wandering phenomenon.

On the other hand, there is certainly noise in actual physical systems. In these years, investigations on systems including noise attract many researchers' attentions. We are interested in the difference between chaotic fluctuation and noise, because sometimes chaos exhibits better performance than random noise in information processing tasks [7].

In our previous research, we have investigated the breakdown of synchronization observed from four coupled chaotic oscillators. In order to understand the phenomenon, the model of coupled modified van der Pol oscillators with the additive white Gaussian noise (AWGN) was proposed. By computer simulations, we have confirmed that chaotic systems were synchronized more stably than modified van der Pol oscillators with AWGN [8].

In this study, we propose other two types of noise for adding the van der Pol oscillators to confirm the different between chaotic systems and van der Pol systems with AWGN. First noise has two band characteristic, and second noise is a scaled AWGN. By adding the scaled AWGN, modified van der Pol oscillators are close to coupled chaotic systems is confirmed.

2. Circuit Model

2.1. Chaotic Oscillators

Figure 1 shows the circuit model.

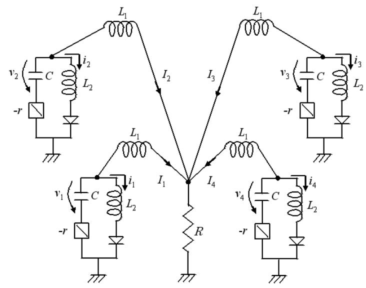

Figure 1: Coupled chaotic oscillators.

In this circuit, four identical chaotic oscillators are coupled by one resistor. First the $i - v$ characteristics of the diodes are approximated by two-segment piecewise-linear functions as

$$v_d(i_k) = \frac{1}{2}(r_d i_k + E - |r_d i_k - E|). \tag{1}$$

By changing the variables and the parameters,

$$\begin{aligned} &I_k = \sqrt{\frac{C}{L_1}}Ex_k,\ \ i_k = \sqrt{\frac{C}{L_1}}Ey_k,\ \ v_k = Ez_k, \\ &t = \sqrt{L_1 C}\tau,\ \ \alpha = \frac{L_1}{L_2},\ \ \beta = r\sqrt{\frac{C}{L_1}}, \\ &\gamma = R\sqrt{\frac{C}{L_1}},\ \ \delta = r_d\sqrt{\frac{C}{L_1}}, \end{aligned} \tag{2}$$

the normalized circuit equations of the circuit are described as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \beta(x_k + y_k) - z_k - \gamma\displaystyle\sum_{j=1}^{4} x_j \\ \dfrac{dy_k}{d\tau} = \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\} \\ \dfrac{dz_k}{d\tau} = x_k + y_k \qquad\qquad (k = 1, 2, 3, 4) \end{cases} \tag{3}$$

where

$$f(y_k) = 0.5(\delta y_k + 1 - |\delta y_k - 1|). \tag{4}$$

Figure 2 shows an example of the observed four-phase quasi-synchronization of chaos. In the figures the phase differences of x_2, x_3 and x_4 with respect to x_1 are almost 90°, 180° and 270°, respectively.

Because of symmetry of the coupling structure, six different combinations of phase states coexist.

As the coupling resistance R (γ) decreases, we can observe that the synchronization states become unstable and that the self-switching phenomenon of the states occurs [6]. The breakdown of the synchronization means the start of the self-switching, hence it is important to investigate how the synchronization breaks down.

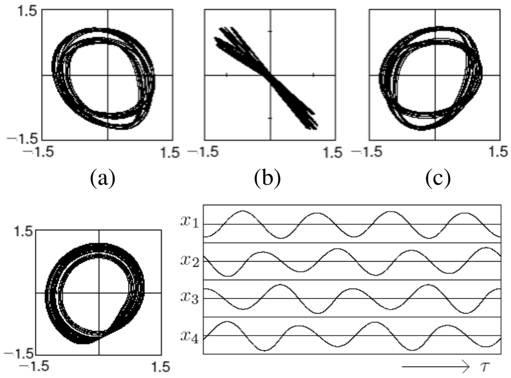

Figure 2: Four-phase quasi-synchronization of chaos (computer calculated result). $\alpha = 7.0$, $\beta = 0.10$, $\gamma = 0.10$ and $\delta = 100.0$. (a) x_1 vs. x_2. (b) x_1 vs. x_3. (c) x_1 vs. x_4. (d) x_1 vs. z_1. (e) Time waveforms.

2.2. Modified van der Pol Oscillators

Next, we consider four coupled van der Pol oscillators.

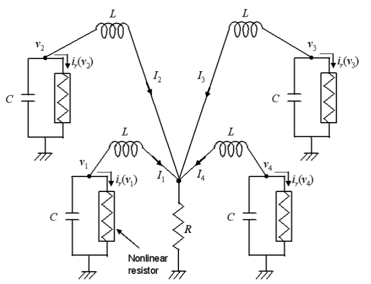

Figure 3: Coupled van der Pol oscillators.

In order to obtain the waveforms similar to those of the chaotic oscillator, we modify the van der Pol oscillator with the nonlinear resistor whose $v - i$ characteristics are described by the following asymmetric function

$$i_r(v_k) = -g_1 v_k + g_2 v_k^2 + g_3 v_k^3 \quad (g_1, g_2, g_3 > 0). \tag{5}$$

By changing the variables and the parameters,

$$\begin{aligned} v_k &= \sqrt{\frac{g_1}{g_3}} x_k, \quad i_k = \sqrt{\frac{C g_1}{L g_3}} y_k, \quad t = \sqrt{LC}\tau, \\ \nu &= \frac{g_2}{\sqrt{g_1 g_3}}, \quad \gamma = R\sqrt{\frac{C}{L}}, \quad \varepsilon = g_1\sqrt{\frac{L}{C}}, \end{aligned} \tag{6}$$

the normalized circuit equations are given as

$$\left\{ \begin{aligned} \frac{dx_k}{d\tau} &= \xi\left\{-y_k + \varepsilon(x_k - \nu x_k^2 - x_k^3)\right\} \\ \frac{dy_k}{d\tau} &= x_k - \gamma \sum_{j=1}^{4} y_j \\ (k &= 1, 2, 3, 4) \end{aligned} \right. \tag{7}$$

where ξ is the parameter added to tune the period of the waveform.

Figure 4 shows an example of the observed four-phase synchronization of the modified van der Pol oscillators.

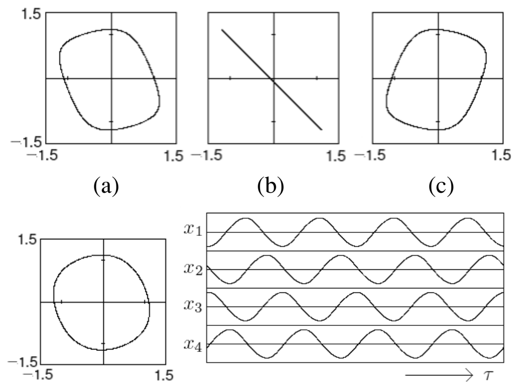

Figure 4: Four-phase synchronization of modified van der Pol oscillators (computer calculated result). $\varepsilon = 0.50$, $\gamma = 0.60$, $\xi = 1.07$ and $\nu = 0.1035$. (a) x_1 vs. x_2. (b) x_1 vs. x_3. (c) x_1 vs. x_4. (d) x_1 vs. y_1. (e) Time waveforms.

In this study, we consider the case that the noise is added to the modified van der Pol oscillator in order to simulate the breakdown of the synchronization in the coupled chaotic oscillators caused by chaotic fluctuations of their waveforms.

When we add the noise to the voltage amplitude of the modified van der Pol oscillator, the circuit equation of the coupled oscillators are described as

$$\left\{ \begin{aligned} \frac{dx_k}{d\tau} &= \xi[-y_k + \varepsilon\{(1 + \rho_k n_k(\tau))x_k \\ &\quad - \nu((1 + \rho_k n_k(\tau))x_k)^2 - ((1 + \rho_k n_k(\tau))x_k)^3\}] \\ \frac{dy_k}{d\tau} &= (1 + \rho_k n_k(\tau))x_k - \gamma \sum_{j=1}^{4} y_j \\ (k &= 1, 2, 3, 4) \end{aligned} \right. \tag{8}$$

where $n_k(\tau)$ is the added noise and ρ_k is constant to tune the amplitude of the noise.

While when we add the noise to the voltage period of the modified van der Pol oscillator, the circuit equation of the coupled oscillators are described as

$$\begin{cases} \dfrac{dx_k}{d\tau} = (1 + \rho_k n_k(\tau))\xi\{-y_k + \varepsilon(x_k - \nu x_k^2 - x_k^3)\} \\ \dfrac{dy_k}{d\tau} = x_k - \gamma \displaystyle\sum_{j=1}^{4} y_j \qquad (k = 1, 2, 3, 4). \end{cases} \tag{9}$$

The noise $n_k(\tau)$ is the additive white Gaussian noise (AWGN) with the average 0 and the variance σ^2.

In the next section, the computer calculated results are described. Moreover, for all of the computer calculations, the fourth-order Runge-Kutta method is used with step size $h = 0.005$.

3. Breakdown of Synchronization

When the coupling parameter γ is relatively large, both the coupled chaotic oscillators and the modified van der Pol oscillators with noise exhibit four phase synchronizations. While for relatively smaller γ, the synchronizations break down and we observe the switchings of phase states. This means that a critical value of the coupling parameter exists which divides the parameter space into the synchronization region and the self-switching region. We define this critical coupling parameter as γ_c and investigate how γ_c changes when the strength of chaos or noise increases.

First, we define the breakdown of synchronization as at least one switching during 50,000 periods [8]. Next, in order to compare the coupled chaotic oscillators with the modified van der Pol oscillators, we set the parameters as follows. We investigate the critical coupling parameter γ_c for the coupled chaotic oscillators when the chaotic oscillator exhibits the first period-doubling bifurcation (β=0.0425). This value γ_c=1.189 is set as the standard value. We tune the parameter of the modified van der Pol oscillators without noise in order to the synchronization breaks down at this standard value of γ_c. By this setting of the parameter, the chaotic oscillators without chaotic fluctuation and the modified van der Pol oscillators without noise are equalized in the sense of the stability of the synchronization.

Figures 5 and 6 show one-parameter bifurcation diagram of the chaotic oscillator and the critical coupling parameter γ_c, respectively. In Fig. 6, the region over the curve corresponds to the synchronization region and the region under the curve corresponds to the self-switching region. We can observe that γ_c increases as β increases. This means that the synchronization becomes easier to be broken down for larger β, namely larger chaotic fluctuation.

The breakdown of the synchronization between chaotic oscillators and modified van der Pol oscillators with AWGN was rather different. Therefore we propose two types of noise for adding to modified van der Pol oscillators.

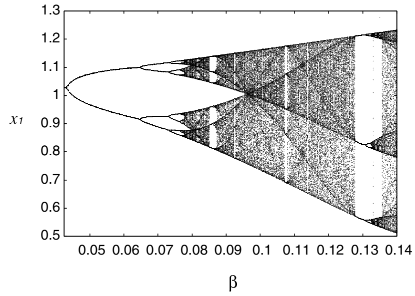

Figure 5: One-parameter bifurcation diagram of chaotic oscillator on the Poincaré section as $z_1 = 0$ and $x_1 < 0$ ($\alpha = 7.0$, $\gamma = 0.0$ and $\delta = 100.0$).

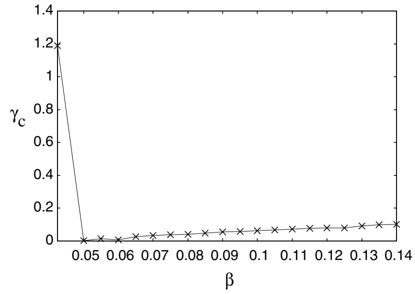

Figure 6: Breakdown of synchronization of coupled chaotic oscillators $\alpha = 7.0$ and $\delta = 100.0$).

3.1. Noise with Two Band Characteristics

Figure 7 shows distribution of voltage amplitude of chaotic oscillators. The chaotic oscillator shows a kind of two-band characteristics.

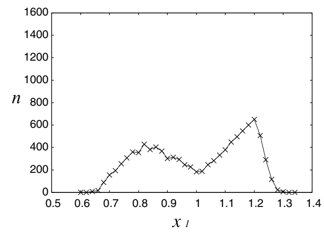

Figure 7: Distribution of voltage amplitude of chaotic oscillators ($\alpha = 7.0, \beta = 0.1, \gamma = 0.065$ and $\delta = 100.0$).

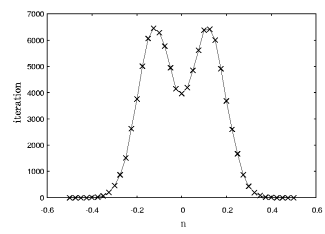

Figure 8: Distribution of noise with two band characteristics.

In order to take this feature into account, we consider to add the noise with two band characteristics into the modified van der Pol oscillators. Figure 8 shows the distribution of noise with two band characteristics. The noise is created by combining two AWGN with the same variance and different averages.

Figures 9 and 10 show the breakdown of synchronization of modified van der Pol oscillators with noise with

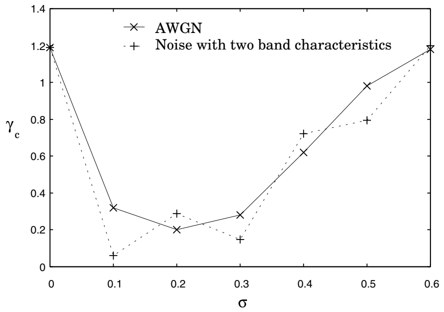

Figure 9: The breakdown of synchronization of modified van der Pol oscillators with noise with two band characteristics (noise is added to the amplitude) ($\gamma = 1.336$, $\epsilon = 0.5$, $\xi = 1.07$, $\nu = 0.3$, $\mu = 2.795$, $\sigma = 1.0$ and $\rho_k = 1.0$).

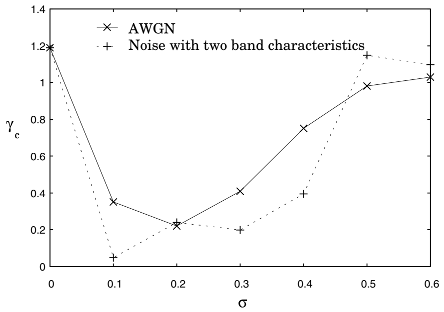

Figure 10: The breakdown of synchronization of modified van der Pol oscillators with noise with two band characteristics (noise is added to the period) ($\gamma = 1.336$, $\epsilon = 0.5$, $\xi = 1.07$, $\nu = 0.3$, $\mu = 2.795$, $\sigma = 1.0$ and $\rho_k = 1.0$).

two band characteristics. In this case, It is seen that two band characteristics noise gives similar result to the case of AWGN. From these results, the breakdown of synchronization is lightly affected by the distribution of noise.

3.2. Scaled AWGN

Next, the scaled AWGN is added to the modified van der Pol oscillators ($n_1 = n_2 = n_3 = n_4$, ρ_k is changed). This is because we consider that the chaotic fluctuation occurred during chaos synchronization has a certain amount of correlation between the subcircuits.

Figures 11 and 12 show the breakdown of synchronization of modified van der Pol oscillators with the scaled AWGN (γ_c). It is seen that non-scaled AWGN has a smaller area of stable synchronization. From these results, we can say that by adding the scaled AWGN the modified van der Pol oscillators behave more similar to the coupled chaotic systems.

4. Conclusions

In this study, the breakdown of synchronization observed from four coupled chaotic oscillators has been investigated. In order to understand the phenomenon, the model of coupled modified van der Pol oscillators with noise was considered. By adding the scaled AWGN, the modified van der Pol oscillators are confirmed to be closer to the coupled chaotic circuits.

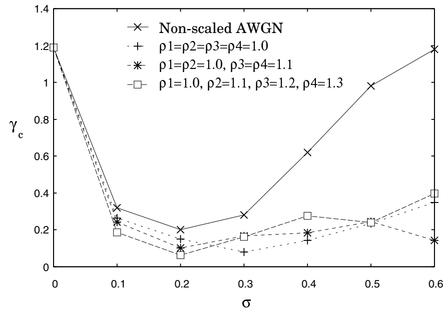

Figure 11: The breakdown of synchronization of modified van der Pol oscillators with scaled AWGN (noise is added to the amplitude) ($\gamma = 1.336$, $\epsilon = 0.5$, $\xi = 1.07$, $\nu = 0.3$, $\mu = 2.795$ and $\sigma = 1.0$).

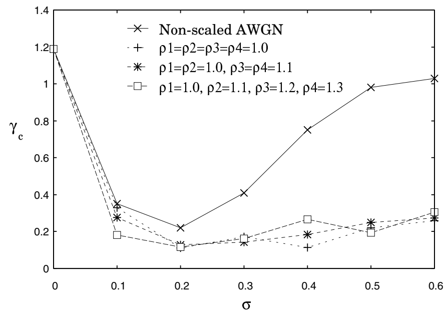

Figure 12: The breakdown of synchronization of modified van der Pol oscillators with scaled AWGN (noise is added to the period) ($\gamma = 1.336$, $\epsilon = 0.5$, $\xi = 1.07$, $\nu = 0.3$, $\mu = 2.795$ and $\sigma = 1.0$).

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