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Power Spectral Analysis of the Computation Process of Turing Machine on the Game of Life

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Abstract—It is known that the Game of Life, a two-dimensional cellular automaton can emulate Turing machine on its array. In this research we performed spectral analysis to investigate the dynamical aspect of the computation process carried out by Turing machine on the Game of Life. An actively evolving part of the whole area exhibits $1/f$ noise although the whole area does not. The deviation of power spectrum from $1/f$ noise that is commonly observed in the evolution from random configuration can be explained as a consequence of the 'regularity' contained in initial configuration capable of supporting computation.

1. Introduction

Cellular automata (CAs) is a d-dimensional array with a finite automaton residing at each site. Each automaton takes the states of neighboring sites as input and makes the transition of its state according to a set of transition rules. CAs are also considered to be spatially and temporally discrete dynamical systems with large degrees of freedom. It was proved that elementary CA (ECA), namely one-dimensional and two-state, three-neighbor CA rule 110 is computationally universal [1] that means any algorithms can be performed by preparing an appropriate initial configuration on the array. Another example of computationally universal CA is the Game of Life (LIFE) [2]. LIFE is a two-dimensional and two-state, nine-neighbor outer totalistic CA. These two computationally universal CAs exhibit $1/f$-type power spectrum [3] when they start from random configuration [4, 5]; moreover ECA rule 110 exhibits $1/f$ noise also in the computation process [6]. These results suggest that $1/f$ noise marks a kind of dynamics that can support universal computation.

However The power spectral analysis of computation process in LIFE has not been investigated yet. In this research we study the power spectra of the computation process of LIFE in which the evolution of a Turing machine (TM) is emulated.

2. Turing Machine on LIFE

Let $s_{x,y}(t) \in \{0, 1\}$ denote the state of the site (x, y) at time step t in LIFE. The transition function d of LIFE is defined as

$$s_{x,y}(t+1) = d(s_{x,y}(t), n_{x,y}(t)), \quad (1)$$

where $n_{x,y}(t)$ denotes the sum of the states of the eight nearest neighboring sites around the site (x, y) at time step t. The transition function d is defined by

$$d(0,3) = d(1,2) = d(1,3) = 1, \quad \textit{otherwise} \quad d = 0. \quad (2)$$

Table 1: Transition function of the Turing machine. The leftmost column is state and the first line is tape symbol.

	0	1	2
s_0	(s_2,0,L)	(s_1,2,L)	(s_0,2,R)
s_1	(s_0,2,R)	–	(s_1,2,L)
s_2	(s_H,0,L)	–	(s_2,1,R)

It is known that Turing machine (TM) can be constructed on the array of LIFE [7]. The TM dealt with this research has the set of states $Q = \{s_0, s_1, s_2, s_H\}$ and s_0 is the start state and s_H is the final state. The set of input symbols is $\Sigma = \{0, 1\}$ and the set of tape symbols is $\Gamma = \{0, 1, 2\}$ and '0' is the blank symbol that appears initially in all but the finite number of initial cells holding input symbols. The transition function is represented by $\delta(q, X) = (p, Y, D)$, where q is a state and X is a tape symbol , p is the next state, Y is the output symbol, and $D \in \{L, R\}$ is a direction of the tape head. δ is given by table 1. If this TM starts with a configuration $s_0 1^n, (n \geq 0)$, it halts with a configuration $s_H 001^{2n}$. So this TM is called 'string doubler'. Table 2 shows configurations in successive transition starting from a configuration $s_0 11$.

Figure 1 shows a pattern to emulate the computation process shown in Table 2 on the array of LIFE. This configuration file executable on Golly [8], a CA simulator, can be downloaded from the web site [9]. White squares represent site with state '0', black squares with state '1'. The components of TM on this array are depicted in Fig. 2. All the components are deployed in the area with the length and breadth of about 1600 × 1700 sites. The two parts of stack work as a tape of TM and the upper left contains the symbols written on the left-hand side of the tape head and the

Table 2: Example of the sequence of the transition of the string doubler Turing machine. The rightmost column is the time step on LIFE simulating the Turing machine.

seq. no.	configuration	time step
0	$s_0 11$	4,500
1	$s_1 021$	15,000
2	$2s_0 21$	26,000
3	$22s_0 1$	37,000
4	$2s_1 22$	49,000
5	$s_1 222$	60,000
6	$s_1 0222$	72, 000
7	$2s_0 222$	82,000
8	$22s_0 22$	93,000
9	$222s_0 2$	104,000
10	$2222s_0 0$	116,000
11	$222s_2 20$	127,000
12	$22s_2 210$	137,000
13	$2s_2 2110$	148,000
14	$s_2 21110$	160,000
15	$s_2 011110$	171,000
16	$s_H 0011110$	180,000

lower right contains the symbols written on the right-hand side of the tape head. The symbol beneath the tape head is on the signal line from the stack to the finite state machine.

3. Power Spectra of Computation Process

Spectral analysis is one of the useful methods to study the behaviour of dynamical systems. Therefore we apply it to the analysis of the computation process shown in Table 2. The discrete Fourier transformation of a time series of states $s_{x,y}(t)$ for $t = 0, 1, \cdots, T-1$ is given by

$$\hat{s}_{x,y}(f) = \frac{1}{T}\sum_{t=0}^{T-1} s_{x,y}(t)\exp(-i\frac{2\pi t f}{T}). \quad (3)$$

The power is defined as

$$S(f) = \frac{1}{N}\sum_{x,y} |\hat{s}_{xy}(f)|^2, \quad (4)$$

where the summation is taken in all N sites in consideration. The period of the component at a frequency f in a power spectrum is given by T/f.

We divide the area in which the string doubler TM predominantly works into 140 sections with 100×100 sites to clarify the regional difference among the array and calculate the power spectrum of each section for $T = 16,384$ time steps. The area employed to calculate the power spectra is depicted as a polygon in Fig. 3.

The exponent β of power spectrum is estimated by means of the least-squares fitting by $\ln(S) = \alpha + \beta\ln(f)$ in the range of $f = 1 \sim 50$.

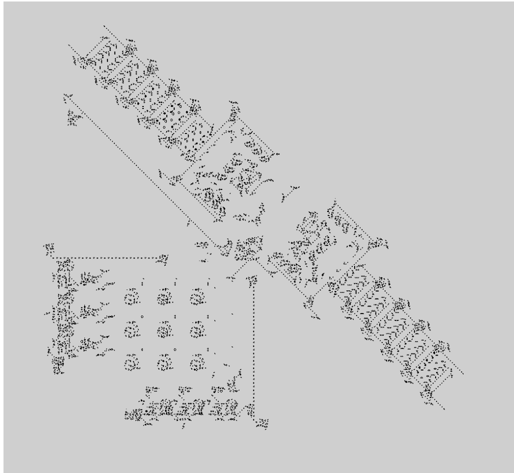

Figure 1: Initial configuration of string doubler Turing machine on the array of LIFE.

The residual sum of squares is given by

$$\sigma^2 = \frac{1}{f_b}\sum_{f=1}^{f_b} (\ln(S) - \alpha - \beta\ln(f))^2, \quad (5)$$

where f_r is the number of data used for the calculation of σ^2 and we set $f_r = 50$.

The top of Fig. 4 shows a typical example of power spectrum of the section with 100×100 sites marked with ‘A’ in Fig. 3. Both x and y axes are plotted on a logarithmic scale. The power spectrum is almost even at low frequencies and has the highest power at $f = 546$ corresponding to the periodic behaviour with period $16,384/546 \approx 30$. That is caused by the periodic patterns with period 30 called ‘queen bee shuttle’ shown in Fig. 5. The queen bee shuttle is the most commonly used oscillator in this realization of TM and is about the 18th and 19th most common naturally-occurring oscillators [10].

The second from the top of Fig. 4 shows the power spectra with the least value of $\beta = -0.90$ among those with $\sigma^2 < 0.2$. Since the power spectrum with $\sigma^2 \geq 0.2$ is not considered to follow the power law, we exclude those from consideration. This power spectrum presents $1/f$ characteristic and its evolution is in a section marked with ‘B’ in Fig. 3. This section is located in a stack cell that is most frequently rewritten during the transition. And the power spectrum with the second least value of β -0.67 (second from bottom of Fig. 4) is calculated in a section marked with ‘C’ beneath the section ‘B’ although this evolution is not considered to be $1/f$ noise.

The power spectrum averaged over 140 sections is shown at the bottom of Fig. 4 with the exponent $\beta = -0.31$. This result implies that the computation process does not exhibits $1/f$ noise as a whole. Figure 6 shows the power spectrum of the evolution for $T = 16,384$ time steps starting from a random configuration with array size of

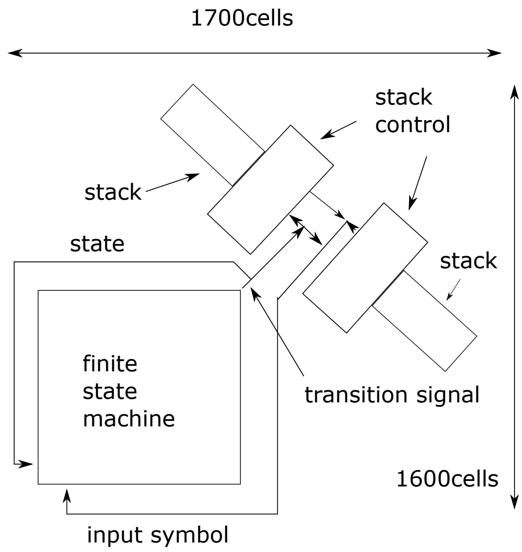

Figure 2: Components of Turing machine on the array of the Game of Life.

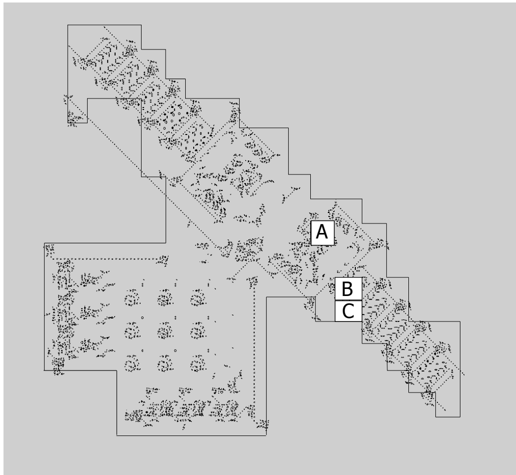

Figure 3: Area in which power spectra are calculated among the array of LIFE.

$1183 \times 1183 = 1,399,489$ with absorbing boundary conditions in which the sites beyond the boundary is fixed to state '0'. The exponent β is estimated at -1.32 and the one estimated under the periodic boundary conditions is -1.31. These results show that the evolution staring from random configuration exhibits $1/f$ noise.

4. Discussion

We calculated the power spectra of the computation process of TM emulated on LIFE. The sections containing a most frequently rewritten stack cell exhibit $1/f$ noise although the power spectrum averaged over whole area in which the TM works has the exponent value close to zero. These results imply that the $1/f$-type characteristics is localized in actively evolving area. On the other hand, the evolution starting from a random configuration with almost the same number of sites exhibits $1/f$ noise.

These results form a striking contrast with ECA rule 110 in which both the computation process emulating cyclic tag system and the evolution with a random initial configuration present $1/f$ noise. The biggest difference in computation process between LIFE and rule 110 is the frequency of 'burst' caused by the collision of propagating patterns during the evolution. Sporadic bursts occur during the computation process in LIFE since signals are designed to be detoured to avoid collisions. In rule 110, however, it is inevitable for propagating patterns to avoid collisions because of its low dimensionality of the array.

We should notice that the results obtained in this research does not invalidate the hypothetical relationship between computational universality and $1/f$ noise in CAs. The dynamics that creates $1/f$ noise in most every case is compatible with non$1/f$-type behaviour accompanying a particular initial configuration. The evolution starting from random configuration can clearly unveil the essential property intrinsic to LIFE because it is affected solely by its transition rule and the power spectrum varies little in shape according to the details of subtle difference in initial configuration. On the other hand, the power spectrum of computation process is affected not only by the transition rule but also by the 'regularity' contained in the elaborately designed initial configuration capable of supporting computation. We can explain the deviation of power spectrum of computation process from $1/f$ noise as a consequence of the 'regularity' contained in initial configuration and therefore in subsequent evolution.

Here arise some questions out of the results of this research. First one is a question of the dependency of power spectra upon the details of computing mechanism. We might be able to elucidate it by investigating another computing machine such as register machine on LIFE. And the second one is a question on the power spectrum of the evolution with moderate randomness. The evolution from random configuration and the computation process are both extreme cases among all kinds of possible evolution. The former completely lacks in orderliness and the latter has no randomness. What kind of power spectrum is observed in the evolution starting from moderately orderly initial configuration? These questions are issues in future.

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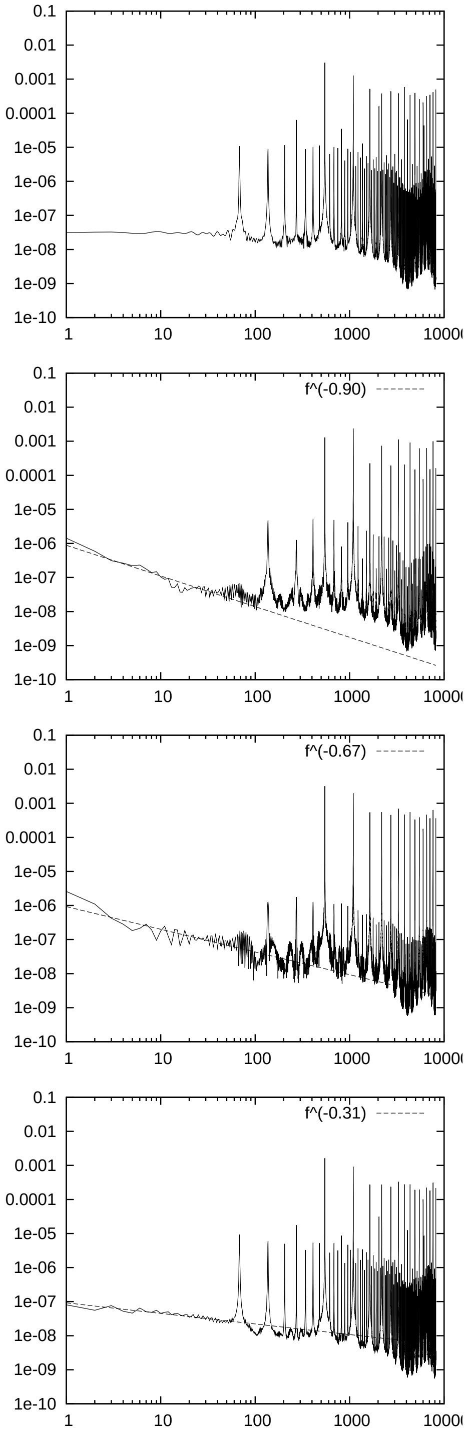

Figure 4: Power spectrum with typical shape (top) and with the least value of exponent (second from the top), the second least value (second from the bottom) and averaged over all sections during the computation process (bottom). The x-axis is the frequency f, y-axis is power S. The exponent β of the power spectrum is estimated to -0.90 (second from the top), -0.67 (second from the bottom) and -0.31 (bottom).

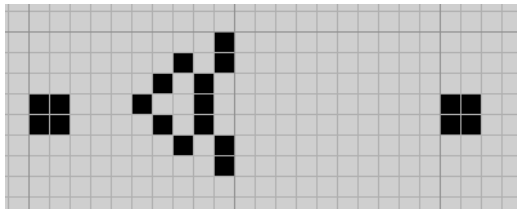

Figure 5: Periodic pattern 'queen bee shuttle' with period 30 that is most commonly used in the realization of Turing machine.

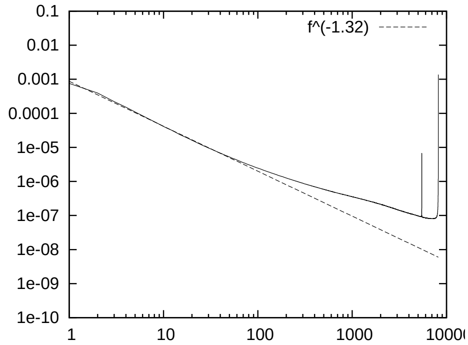

Figure 6: Power spectrum of the evolution starting from random configuration. The x-axis is the frequency f, y-axis is power S. The exponent β of the power spectrum is estimated to -1.32.

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