2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

Dependence of Rule Sets for Digital Sound Description in Cellular Automata

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Abstract—In this paper, we investigate rule sets of one dimensional cellular automata with two states and three neighbors (referred as to 1-2-3 CA hereafter), which describe digital sound data without any reproducing error. In our previous investigations with three-rules set of 1-2-3 CA, the rule set changes with target sound data, which realizes the fewest description data amount. It suggests that a strategy in finding out the three-rules set is not appropriate. Therefore, in order to improve the strategy, in this paper, we investigate description ability of two-rules sets based on 4 kinds of quantities, (i) generation ability, (ii) description ability, (iii) averaged length of resultant rule sequences and (iv) compressibility of data. From computer experiments, for data applied XOR preprocessing (referred as to xor data hereafter), the rule set (#90, #180) realize the highest compressibility even if generation ability and description ability are not so high. On the other hand, the rule sets of (#15, #240), (#60, #195), (#85, #170) and (#102, #153) take the highest generation ability and description ability but the compressibility is slightly smaller, meaning that the four two-rules sets would remain potential in improving the compressibility. Especially, we expect that the two-rules sets of (#85, #170) has possibility in improving compressibility by adding another rule.

1. Introduction

A cellular automaton is one of discrete dynamical systems which can generate complex phenomena from simple rules. Therefore, cellular automata are applied to investigate various fields, universal pattern generator, error correcting code, unsteady flow analysis, traffic stream modeling and so on [1]. In our studies, we employ 1-2-3 CA in describing digital sound data [2, 3]. We succeeded that digital sound data can be described by a two-rules set of 1-2-3 CA without reproducing error. One regards the method in describing digital sound data by a two-rules set of 1-2-3 CA as extracting a rule in a discrete dynamical system which reproduces time development of bits-pattern sequences of digital sound data. In other words, we succeeded to extract "rule dynamics" from digital sound data, which has been introduced by Aizawa and Nagai [4]. In fact, time development of bits-pattern sequences of digital sound data is represented by time development of the rule sequences consisting of the two rules in our method.

From view point of "rule dynamics", the resultant rule sequences could reflect dynamical features of the target data, complexity, chaoticity, and so on. In order to evaluate the dynamical features, it is necessary to decrease redundancy of the resultant rule sequences as much as possible. In previous investigation, for all the digital sound data, the two-rules set of (#90, #180) realizes compressive coding by introducing appropriate preprocessings [5]. For several data, however, the data amount becomes larger than the original one without the preprocessings, originating into the limitations of the two-rules set of (#90, #180). In order to overcome the limitation, we employ three-rules set in describing digital sound data by adding another rule into (#90, #180). The data amount becomes smaller but a compressible description could not be achieved. In addition, the extra rule changes with target data, suggesting that the situation is not good for a description method [6].

The results mean that a strategy in finding out the three-rules set is not appropriate. Therefore, in order to improve the strategy, the purpose of this paper is to investigate description ability of two-rules sets based on 4 kinds of quantities, (i) generation ability, (ii) description ability, (iii) averaged length of resultant rule sequences and (iv) compressibility of data. Moreover, we find out appropriate two-rules sets in realizing the description with a three-rules set.

2. Description Method with Rules of 1-2-3 CA

2.1. 1-2-3 CA

Let us explain 1-2-3 CA briefly. In 1-2-3 CA, N cells are arranged on a one dimensional chain and each cell takes 0 or 1. A state of the ith cell at time step t is denoted by a_i^t, which takes 0 or 1. The time development of a_i^t is determined by three neighboring cells of

a^t_{i-1}, a^t_i and a^{t+1}_i, then the updating rule is given by,

$$a^{t+1}_i = f(a^t_{i-1}, a^t_i, a^t_{i+1}), \tag{1}$$

where a function $f(\cdot)$ is referred as a transition function. The state of all the cells at t is denoted by $\boldsymbol{a}^t = (a^t_1, \cdots, a^t_N)$.

All the possible transition functions are given by,

$$\begin{aligned} &f(0,0,0) = f_0, \quad f(0,0,1) = f_1, \quad f(0,1,0) = f_2,\\ &f(0,1,1) = f_3, \quad f(1,0,0) = f_4, \quad f(1,0,1) = f_5,\\ &f(1,1,0) = f_6, \quad f(1,1,1) = f_7, \end{aligned} \tag{2}$$

where $f_i = 0$ or 1 $(i = 0, 1, \cdots, 7)$, indicating there are $2^8 = 256$ transition functions. In order to specify a updating rule, we employ "rule number" defined by,

$$\#r = 2^0 f_0 + 2^1 f_1 + 2^2 f_2 + 2^3 f_3 + 2^4 f_4 + 2^5 f_5 + 2^6 f_6 + 2^7 f_7, \tag{3}$$

where $\#r$ takes from #0 to #255.

2.2. Description method

Let us present our description method with 1-2-3 CA briefly [2, 3]. In general, digital sounds can be expressed as time development of bit-pattern sequences of 16 bits. Similarly, a time development of 1-2-3 CA gives bit-pattern sequences of N cells. Thus, the quantized sound signal is represented by the state of 1-2-3 CA. Time development of the sound signal ($\boldsymbol{a}^t \Rightarrow \boldsymbol{a}^{t+1}$) can be generated by applying a rule sequence $r^{(k)}$ consisting of rules of 1-2-3 CA to $\boldsymbol{a}^t$, that is, $r^{(k)} \circ \boldsymbol{a}^t = \boldsymbol{a}^{t+1}$, where $r^{(k)}$ represents a rule sequence with the length of k. If we find an appropriate rule sequence at each time step, we can regenerate the quantized sound signal from 1-2-3 CA.

The strategy in finding appropriate rule sequences is as follows:

1. First, determine a boundary condition from all the four kinds of fixed boundary conditions, L0-R0, L1-R0, L0-R1 and L1-R1, where L1 means that the boundary cell at the left of the 1st cell is fixed with 1. and R0 means that the boundary cell at the right of the 16th cell is fixed with 0. Through this paper, we apply L1-R0 boundary condition.

2. Define the number of elements in the rule set, n, and the rule set, $\mathcal{R} = \{\#r_1, \#r_2, \cdots, \#r_n\}$. $\mathcal{R}^k$ denotes k times Cartesian product of $\mathcal{R}$, that is, $\mathcal{R}^k = \prod_{i=1}^{k} \mathcal{R}$. Thus, $\mathcal{R}^k$ gives a set of all the possible rule sequences with the length of k, which is composed of elements of $\mathcal{R}$.

3. An initial state in the 1-2-3 CA, $\boldsymbol{a}^0$, is set by the initial signal of the target digital sound data.

4. Based on a technique of the breadth first search, at each time step, we search for the rule sequence with the shortest length which can reproduce the target sound signal at the next time step. Thus, starting from $k = 1$, we increase k until $\boldsymbol{a}^{t+1}$ corresponds to the target sound signal at the $t+1$ step, where $\boldsymbol{a}^{t+1} = r^{(k)} \circ \boldsymbol{a}^t$, $r^{(k)} \in \mathcal{R}^k$. If the target sound signal is not obtained within a certain maximum sequence length of $N_{\max}$, we give up description and employ the original pattern.

5. Repeat the procedure 4 until the end of the sound data.

3. Description ability

3.1. Purpose and method

Accordion to previous our results [3, 6], the best rule set would change with target data and data formats. It suggests that a strategy in finding out the three-rules set is not appropriate. Therefore, in order to improve the strategy, in this paper, we investigate description ability of two-rules sets based on 4 kinds of quantities, (i) generation ability which denotes the ratio of how many bit-patterns are generated of all the possible ones, (ii) description ability which evaluates the ratio of how many time steps the description method succeeds to describe the target data, (iii) averaged length of resultant rule sequences which can reproduce the target data without errors and (iv) compressibility of data.

The generation ability is calculated by,

$$\text{ability}_{\text{generation}} = \frac{\text{number of states reproduced within } N_{\max}}{2^{N_{\max}}}, \tag{4}$$

where $N_{\max}$ denotes the maximum application time steps of two rules.

The description ability is calculated by,

$$\text{ability}_{\text{description}} = \frac{\text{data length described by rule-set}}{\text{total data length}}. \tag{5}$$

The value of 1 means that for all the time steps, data is described by the two-rules set only. In other word, the value less than 1 means that for several time steps, the method misses to describe data within $N_{\max}$.

The the compressibility is approximately given by [3],

$$\text{compressibility} \simeq \frac{\log_2 N_{\max} + (\text{averaged length of rule sequences})}{16}. \tag{6}$$

In a calculation of the compressibility, if the description method fails to describe data within $N_{\max}$ at a certain time step, we regard its data amount as 16 bits.

In this paper, two kinds of digital sound data are applied;

- Spoken word data supplied by ATR (Advanced Telecommunication Research Institute International). In the present paper, we employ four words, "sports","knife","news" and "pocket", pronounced by two men and two women.

- 150 music data which are taken from four classic and one Japanese pops music CD. All the intervals of the data are 1 second.

The music data are digitized under the sampling frequency 44.1 kHz and the amplitude at each time step quantized into 16 bits. The data format of music is that of commercial music CD. On the other hand, in the pronounced words, the sampling frequency is 22.05 kHz and the encoding bit rate is 16 bits, respectively. But for the time

In addition, the sound data are represent by two kinds of data formats, row data and xor data. In xor data, the exclusive disjunction operation is performed between sound signals at time t and $t+1$.

In this paper, we apply eight sets of two-rules, (#15, #240), (#60, #195), (#85, #170) (#102, #153), (#150, #165), (#102, #195), (#90, #105) and (#90, #180). The first four sets can generate all the possible 2^{16} bit-patterns at 16 application time steps with the L1-R0 boundary condition, which is the least application time steps. Subsequently, the set of (#150, #165) can generate at 18, (#102, #195) at 19 and (#90, #105) at 19. The set of (#90, #105), which has been the best one in compressibility for xor data, can generate at 27. In the description, we employ L1-R0 boundary condition and $N_{\max} = 16$.

3.2. Results of description abilities

Typical examples of evaluation results of description abilities with four quantities are given in Table 1. For all the target data, the description ability of the two-rules sets of (#15, #240), (#60, #195), (#85, #170) (#102, #153) and (#150, #165) takes 100 % since the sets can generate all the possible bit-patterns within $N_{\max} = 16$, resulting in 100% generation ability. On the other hand, for raw data, the description abilities of (#90, #180) are not so high, however, for xor data, the description abilities take larger than or the almost same as (#150, #165), (#102, #195), (#90, #105) and (#90, #180). In Addition, the two-rules set realize compressive description for xor data.

Thus, the two-rules set specializes to describe xor data. On the other hand, the two-rules sets of (#15, #240), (#60, #195), (#85, #170) (#102, #153) and (#150, #165) take 100% generation and description ability even if the compressibilty slightly takes larger than1 for all the target data. Especially, the compressibilty of the two-rules set of (#85, #170) takes the nearest value to 1 for all the raw data. We expect that the two-rules set would remain possibility in improving the compressibilty by adding another rule.

4. Conclusions

In this paper, we investigate we investigate description ability of two-rules sets based on 4 kinds of quantities, (i) generation ability, (ii) description ability, (iii) averaged length of resultant rule sequences and (iv) compressibility of data, in order to find out an appropriate two-rules set which improves the compressibilty by adding another rule. Results are as follows:

- the two-rules set of (#90, #180) specializes to describe xor data, realizing the compressive description. In other word, it would be difficult to improves the compressibilty.

- The two-rules set of (#85, #170) takes 100% generation and description ability, in addition, the nearest to 1 of the the compressibilty for all the raw data. Thus, the two-rules set would remain possibility in improving the compressibilty by adding another rule.

In near future, we present the compressibilty by by adding another rule to the two-rules set of (#85, #170).

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Table 1: Evaluation results of description abilities.

	(85,170)	(15,240)	(60,195)	(102,153)	(150,165)	(102,195)	(90,105)	(90,180)
Generation	100	100	100	100	91.9	89.0	89.0	53.0
Man “sports” raw								
Description	100	100	100	100	64.77	88.64	88.04	58.98
Averaged Length	14.07	14.26	14.40	14.52	17.90	17.87	17.90	18.46
Compressibility	1.092	1.104	1.113	1.12	1.112	1.112	1.119	1.154
Man “sports” xor								
Description	100	100	100	100	85.47	82.55	88.88	84.84
Averaged Length	14.29	14.07	14.56	15.05	14.46	14.29	14.39	10.64
Compressibility	1.106	1.092	1.123	1.154	1.117	1.106	1.113	0.649
Woman “sports” raw								
Description	100	100	100	100	88.86	65.30	87.61	59.35
Averaged Length	14.09	14.27	14.35	14.47	14.48	14.49	14.53	15.04
Compressibility	1.094	1.104	1.109	1.117	1.118	1.118	1.121	1.153
Woman “sports” xor								
Description	100	100	100	100	85.93	82.79	88.88	85.39
Averaged Length	14.29	14.17	14.51	15.07	14.48	14.28	14.37	10.84
Compressibility	1.106	1.098	1.119	1.154	1.118	1.106	1.111	0.89
Classic 101 raw								
Description	100	100	100	100	87.94	88.29	88.09	55.41
Averaged Length	14.06	14.26	14.35	14.77	14.48	14.50	14.48	15.10
Compressibility	1.053	1.064	1.071	1.097	1.078	1.080	1.078	1.117
Classic 101 xor								
Description	100	100	100	100	87.43	87.04	88.60	94.98
Averaged Length	14.84	14.75	14.32	15.66	14.48	14.39	14.43	10.18
Compressibility	1.140	1.134	1.108	1.192	1.118	1.112	1.114	0.849
JPOP 101 raw								
Description	100	100	100	100	88.22	87.82	88.22	51.20
Averaged Length	14.15	14.30	14.36	14.61	14.46	14.49	14.47	15.18
Compressibility	1.058	1.067	1.071	1.086	1.077	1.079	1.078	1.123
JPOP 101 xor								
Description	100	100	100	100	88.03	89.00	88.16	90.91
Averaged Length	14.75	14.34	14.72	15.39	14.46	14.46	14.47	12.20
Compressibility	1.134	1.109	1.133	1.174	1.116	1.116	1.117	0.975
JPOP 102 raw								
Description	100	100	100	100	87.94	87.75	88.23	51.74
Averaged Length	14.17	14.32	14.35	14.57	14.48	14.51	14.46	15.18
Compressibility	1.098	1.108	1.109	1.123	1.118	1.119	1.116	1.161
JPOP 102 xor								
Description	100	100	100	100	88.26	88.96	88.45	91.94
Averaged Length	14.74	14.75	14.37	15.44	14.46	14.45	14.46	11.96
Compressibility	1.134	1.135	1.111	1.178	1.116	1.116	1.116	0.96

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