

# A Hardware-Efficient Neural System Model based on Asynchronous Bifurcation Processor

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**Abstract**—A novel network of asynchronous cellular automaton neuron models with spike timing dependent plasticity and a dopamine modification is presented. It is shown that the presented model can reproduce typical synaptic changes observed in a reward learning of a network of computationally efficient ODE neuron models.

## 1. Introduction

Many mathematical and electronic hardware models of neurons and networks of neurons have been presented (see the references in [2][3]). These neural system models can be classified into the following four classes based on the continuousness and discontinuousness of state variables and times.

- Class CTCS (continuous time and continuous state): A nonlinear differential equation model of a membrane potential, which has a continuous time and continuous states. Such a model can be implemented by an analog nonlinear circuit.
- Class DTCS (discrete time and continuous state): A nonlinear difference equation model of a membrane potential, which has a discrete time and continuous states. Such a model can be implemented by a switched capacitor circuit or by a Poincare map.
- Class DTDC (discrete time and discrete state): A numerical integration model of a membrane potential, which has a discrete time and discrete states. Such a model can be implemented by a digital processor. A cellular automaton model of a membrane potential also belongs to this class, which has a discrete time and discrete states. Such a model can be implemented by a traditional synchronous sequential logic circuit.
- Class CTDS (continuous time and discrete state): An asynchronous cellular automaton model of a membrane potential, which has a continuous (state transition) time and discrete states. Such a model can be implemented by an asynchronous sequential logic circuit.

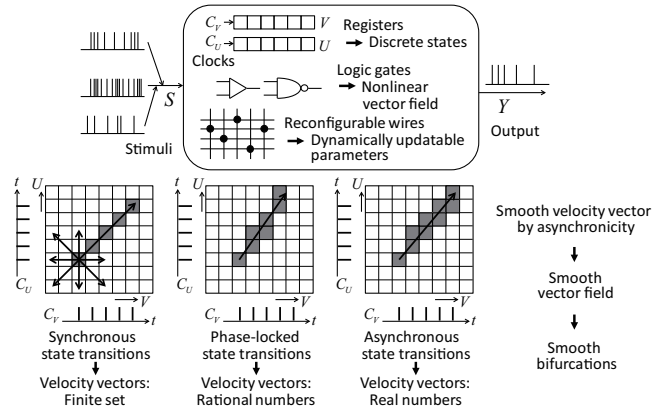


Figure 1: Concepts of neural system modeling by asynchronous cellular automaton. The velocity vectors triggered by synchronous state transitions are characterized by a finite set. The velocity vectors triggered by phase-locked state transitions are characterized by rational numbers. The velocity vectors triggered by asynchronous state transitions are characterized by real numbers. The asynchronous transitions of the discrete states realize smooth velocity vectors, smooth vector fields, and thus smooth bifurcations.

It goes without saying that most conventional neural system models are belonging to the classes CTCS, DTCS, and DTDC. On the other hand, our group has been developing neural system models belonging to the class CTDS (see [2][3] and references therein) and is referring to class CTDS systems designed to exhibit nonlinear phenomena (especially, bifurcation phenomena) as *asynchronous bifurcation processors*. Concepts of such a neural system modeling method based on the asynchronous bifurcation processor are illustrated in Fig. 1, where one of the key concept is that the asynchronous bifurcation processor neural system model wisely utilizes the asynchronous state transition to realize smooth bifurcations in a low resolution discrete state space. Advantages of such a neural system modeling method have been demonstrated in both electronic circuit literatures and neural system literatures, e.g., low hardware cost and on-chip dynamic reconfiguration (on-chip plasticity and learning) capabilities (see [2][3] and references therein). In this paper, a novel network of neuron models based on the asynchronous bifurcation proces-

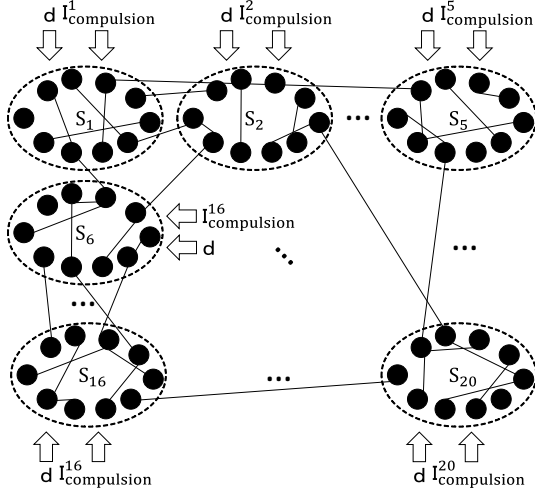


Figure 2: Sketch of the presented network model based on the asynchronous bifurcation processor.

processor with spike timing dependent plasticity and a dopamine modification is presented. It is shown that the presented model can reproduce typical synaptic changes observed in a reward learning of a network of computationally efficient ODE neuron models.

## 2. Neural network model with STDP and Dopamine Modification based on ABP

### 2.1. Internal States, Internal Periodic Clocks, and External Random Stimulations

Fig. 2 shows a sketch of the presented network model based on the asynchronous bifurcation processor. The network consists of 1000 neurons, where 20% of neurons are inhibitory with fixed synaptic weights and 80% of neurons are excitatory with STDP-modulated synaptic weights. The structure of the presented network model is inspired by a network of ODE neuron models with STDP learning in [1] and thus the readers are encouraged to refer it in order for better understanding of the structure of the presented network in this paper. Referring to [2][3], in this paper we introduce a single neuron model based on the asynchronous bifurcation processor whose parameter values are specialized for this paper. The  $i$ -th neuron model has the following internal discrete states.

#### Internal discrete states of each neuron model

Membrane potential:	$V_i \in \{0, 1, \dots, N-1\}$ ,
Recovery variable:	$U_i \in \{0, 1, \dots, M-1\}$ ,
Membrane velocity counter:	$P_i \in \{0, 1, \dots, K-1\}$ ,
Recovery velocity counter:	$Q_i \in \{0, 1, \dots, J-1\}$ .

where  $N = M = K = J = 64$  in this paper. The network has the following internal periodic clocks.

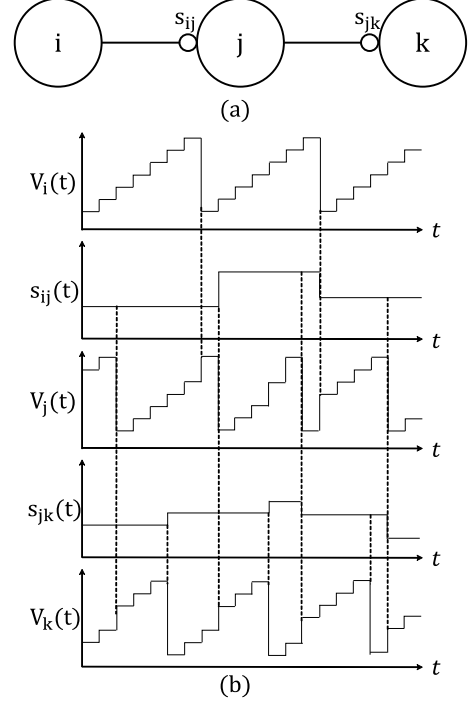


Figure 3: Basic dynamics of the  $j$ -th neuron affected by the  $i$ -th neuron via the synaptic strength  $s_{ij}$  and affecting the  $k$ -th neuron via the synaptic strength  $s_{jk}$ .

#### Internal periodic clocks

$$C_{main}(t) = \begin{cases} 1 & \text{if } t = 0, T_{main}, 2T_{main}, \dots, \\ 0 & \text{otherwise,} \end{cases}$$

$$C_{ACAN}(t) = \begin{cases} 1 & \text{if } t = 0, T_{ACAN}, 2T_{ACAN}, \dots, \\ 0 & \text{otherwise,} \end{cases}$$

$$C_c(t) = \begin{cases} 1 & \text{if } t = 0, T_c, 2T_c, \dots, \\ 0 & \text{otherwise,} \end{cases}$$

where  $T_{main} = 1$ ,  $T_{ACAN} = 50$ , and  $T_c = 10$  in this paper. Also, the network has the following external random stimulation inputs.

#### External random stimulation inputs

$$I_{rand}(t) = \begin{cases} 1 & \text{if } t = \tau_{rand}, \\ 0 & \text{otherwise,} \end{cases}$$

$$I_{comp}(t) = \begin{cases} 1 & \text{if } t = \tau_{comp}, \\ 0 & \text{otherwise,} \end{cases}$$

### 2.2. Asynchronous state transitions

The discrete states of each neuron are updated asynchronously according to the internal periodic clocks and the external stimulation inputs. Each neuron has the following discrete excitatory synaptic weight.

### Discrete excitatory synaptic weight

$$s_{ij} \in \{0, 1, \dots, L\}.$$

Fig. 3 shows basic dynamics of the  $j$ -th neuron affected by the  $i$ -th neuron via the synaptic strength  $s_{ij}$  and affecting the  $k$ -th neuron via the synaptic strength  $s_{jk}$ . The dynamics are described by the following equations.

### Asynchronous transitions of internal discrete states triggered by internal periodic clocks

If  $C_{ACAN}(t) = 1$ , then

$$P(t^+) := \begin{cases} P(t) + 1 & \text{if } P(t) > F(V(t), U(t)), \\ 0 & \text{otherwise,} \end{cases}$$

$$Q(t^+) := \begin{cases} Q(t) + 1 & \text{if } Q(t) > G(V(t), U(t)), \\ 0 & \text{otherwise,} \end{cases}$$

If  $C_{ACAN}(t) = 1$  and  $P(t) \geq F(V(t), U(t))$ , then

$$V(t^+) := \begin{cases} A & \text{if } V(t) = N - 1, \\ V(t) + 1 & \text{if } V(t) \neq N - 1 \text{ and} \\ & F(V(t), U(t)) + 1 > 0, \\ V(t) - 1 & \text{if } V(t) \neq 0 \text{ and} \\ & F(V(t), U(t)) + 1 < 0, \\ V(t) & \text{otherwise,} \end{cases}$$

If  $C_{ACAN}(t) = 1$  and  $Q(t) \geq G(V(t), U(t))$ , then

$$U(t^+) := \begin{cases} B(U(t)) & \text{if } U(t) = M - 1, \\ U(t) + 1 & \text{if } U(t) \neq M - 1 \text{ and} \\ & G(V(t), U(t)) + 1 > 0, \\ U(t) - 1 & \text{if } U(t) \neq 0 \text{ and} \\ & G(V(t), U(t)) + 1 < 0, \\ U(t) & \text{otherwise,} \end{cases}$$

where  $F(V, U) = \lfloor [N(\gamma_1(V/N - \gamma_2)^2 + \gamma_3 - U/M)/\lambda] \rfloor - 1$  and  $G(V, U) = \lfloor [\mu M(\gamma_4(V/N - \gamma_2) + (\gamma_3 + \gamma_5) - U/M)/\lambda] \rfloor - 1$ ,  $A = \lfloor \rho_1 N \rfloor$ ,  $B(U) = U + \lfloor \rho_2 M \rfloor$ , and  $\lfloor \cdot \rfloor$  is the floor function. In this paper we propose to use the following parameter values to build the network model.

$$p = (\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \lambda, \mu, \rho_1, \rho_2),$$

Excitatory neurons :

$$p = p_{excitatory} = (7, 0.3, 0.2, 0.5, 0.05, 64, 4, 0.25, 0.4),$$

Inhibitory neurons :

$$p = p_{inhibitory} = (7, 0.3, 0.2, 0.5, 0.1, 64, 4, 0.37, 0.35).$$

### Asynchronous transitions of internal discrete states triggered by random stimulation inputs

If  $I_{rand}(t) = 1$ , then

$$V(t^+) := \begin{cases} V(t) + 1 & \text{if } V(t) \neq N - 1, \\ V(t) & \text{otherwise,} \end{cases}$$

$$P(t^+) := 0,$$

If  $I_{comp}(t) = 1$ , then

$$V(t^+) := N - 1.$$

### Asynchronous state transitions of discrete states related to synaptic weights

If  $C_{main}(t) = 1$ , then

$$c(t^+) := \begin{cases} c(t) + STDP(t) & \text{if } t = t_{post}, \\ c(t) - [\alpha_c STDP(t)] & \text{if } t = t_{pre}, \end{cases}$$

$$c_{cnt}(t^+) := 0,$$

If  $C_c(t) = 1$ , then

If  $c_{cnt}(t) = \lfloor \beta_c c_{cnt}(t)^{\gamma_c} \rfloor$ , then

$$c(t^+) := \begin{cases} c(t) + 1 & \text{if } c(t) < 0, \\ c(t) - 1 & \text{if } c(t) > 0, \end{cases}$$

$$c_{cnt}(t^+) := 0,$$

else

$$c_{cnt}(t^+) := c_{cnt} + 1,$$

where  $\alpha_c = 1.5$ ,  $\beta_c = 300$ , and  $\gamma_c = 1.5$  in this paper.

If  $C_{main}(t) = 1$  and  $t = t_{pre}$ , then

$$STDP(t^+) := \alpha_{STDP}.$$

$$STDP_{cnt}(t^+) := 0,$$

If  $C_{main}(t) = 1$ , then

If  $STDP_{cnt}(t) = \lfloor \alpha_{STDP} STDP_{cnt}(t)^{\beta_{STDP}} \rfloor$ , then

$$STDP(t^+) := STDP(t) - 1 \text{ if } STDP(t) > 0,$$

$$STDP_{cnt}(t^+) := 0,$$

else

$$STDP_{cnt}(t^+) := STDP_{cnt} + 1.$$

### Asynchronous state transitions of discrete synaptic weights

If  $C_{main}(t) = 1$  and  $d(t) \uparrow$ , then

$$s(t^+) := s(t) + c(t)d(t)/\alpha_s$$

where  $\alpha_s = 10$  in this paper.

### Asynchronous state transitions of discrete dopamine

If  $C_{main}(t) = 1$  and  $t = t_{rew}$ , then

$$d(t^+) := d(t) + \alpha_d$$

$$d_{cnt}(t^+) := 0$$

If  $C_{main}(t) = 1$ , then

If  $d_{cnt}(t) = \lfloor \beta_d d_{cnt}(t)^{\gamma_d} \rfloor$ , then

$$d(t^+) := d(t) - 1 \text{ if } d(t) > 0,$$

$$d_{cnt}(t^+) := 0,$$

else

$$d_{cnt}(t^+) := d_{cnt} + 1$$

where  $\alpha_d = 50$ ,  $\beta_d = 300$ , and  $\gamma_d = -1.2$  in this paper.

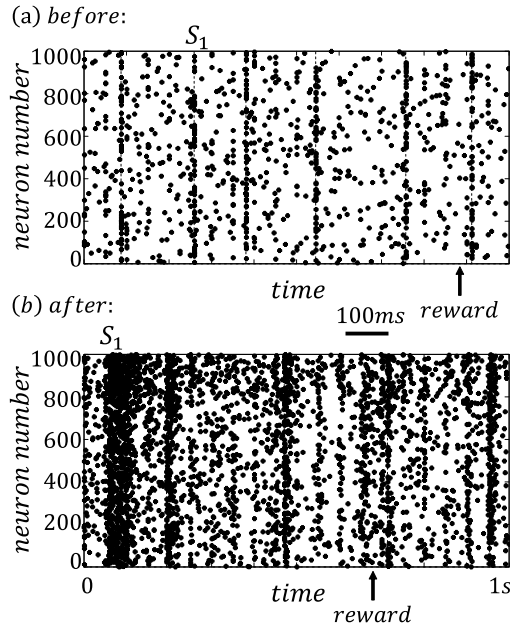


Figure 4: Spike timings of the neurons in the network. (a) Before learning. The neurons rarely respond to every stimulation. (b) After learning. The neurons strongly respond only to the stimulation to the neuron group  $S_1$ .

### Asynchronous state transitions of EPSP

If  $C_{main}(t) = 1$   $t = t_{pre}$ , then

$$EPSP(t^+) := \begin{cases} EPSP(t) + (s(t)/\alpha_{EPSP})^{\beta_{EPSP}} & \text{if } V \in \{\text{excitatory neuron}\}, \\ EPSP(t) - \gamma_{EPSP} & \text{if } V \in \{\text{inhibitory neuron}\}, \end{cases}$$

If  $C_{main}(t) = 1$  and  $EPSP(t) \geq \delta_{EPSP}$ , then

$$\begin{aligned} V(t^+) &:= V(t^+) + 1 \text{ if } V \neq N - 1, \\ P(t^+) &:= 0, \\ EPSP(t^+) &:= EPSP(t) - \delta_{EPSP} \end{aligned}$$

where  $\alpha_{EPSP} = 100$ ,  $\beta_{EPSP} = 2$ ,  $\gamma_{EPSP} = 100$ ,  $\delta_{EPSP} = 400$  in this paper.

### 3. Reward learning with STDP

Fig. 4(a) shows the spike timings of the neurons in the network before learnings. It can be seen that the neurons rarely respond to every stimulation. Fig. 4(b) shows the spike timings of the neurons in the network after learnings, where, during the learnings, each external stimulation to the neuron group  $S_i$  (see Fig. 2) leads to an emission of dopamine signal after a short random delay (see [1]). It can be seen that the neurons strongly respond only to the stimulation to the neuron group  $S_1$ . Fig. 5 shows the values of the excitatory synaptic weights from the neuron group  $S_1$  after the learning. It can be seen that the synaptic

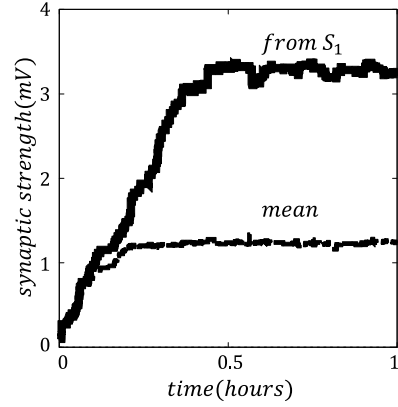


Figure 5: Excitatory synaptic weights (normalized) from the neuron group  $S_1$  after the learning.

weight distribute between zero and the normalized maximum value 4 without strong saturations. We note that the spike timings of the neurons before and after learnings and the distribution of the excitatory synaptic weights after the learnings are qualitatively similar to those from a network of ODE neuron models [1]. Detailed analysis will be presented in a future journal paper.

### 4. Conclusions

In this paper, the novel network of asynchronous cellular automaton neuron models with spike timing dependent plasticity and the dopamine modification was presented. It was shown that the presented model can reproduce typical synaptic changes observed in the reward learning of the network of computationally efficient ODE neuron models citeiz. Future problems include (a) detailed analysis of behaviors of the presented model, (b) hardware implementation of the presented model, and (c) detailed comparisons with existing models.

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### References

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