

# Bursting of transition dynamics on series coupled two operational amplifiers

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Abstract—An operation amplifier (op-amp) is used occurred drastically with varying parameters. in various electrical circuits and one of the most famous electronic devices. In general applications, opamps behave as differential amplifiers, but in parameter miss matching or miss connections, the op-amp occurs complex high frequency oscillations. We find complex oscillation phenomena in the modified simple oscillator with the op-amp, and that these phenomena depend on the dynamics of circuits and through rates of op-amps.

# 1. Introduction

Op-amps are important electrical devices for various electric circuits [1]. For example, in an amplifier, an oscillator, and a comparator, they are essential devices for modern electrical circuits. Basically, op-amps behave as differential amplifiers, and various applications are designed using this characteristic.

A square wave generator (SWG) is the simple oscillator with the op-amp, and it consists of a capacitor and three or four resistors only [2]. One can configure various frequencies and duty ratios by adjusting them. From reason of a simple and cheep devises, it is used for not only application circuits, but also learning kits for electrical circuit beginners. Additionally, they can be considered as relaxation oscillators, and it is used for circuit experiments of the network model of several simple rhythm oscillators, i.e. creatures with a rhythm [3], [4], [5]. In our previous studies, LED firefly circuits are proposed, and a frequency response, and synchronization phenomena of them are clarified [6].

In laboratory experiments, we find a complex oscillation mode on the oscillator with two coupled opamps. This circuit is composed based on the square wave oscillator, and circuit parameters are set as that the frequency is around 70Hz. However, the complex oscillation mode shows about 100kHz-5MHz, and it is In this study, we show the simple circuit model that

causes complex oscillation modes using two op-amps, and compare laboratory experiments and behavior of the circuit. First, we demonstrate complex oscillations on electrical circuits, and clarified the relation ship between a capacitor voltage, a reference voltage and two outputs of op-amps. From these results, we explain reasons of these phenomena and its dynamical properties. Finally, we mention mechanisms of them.

### 2. Square wave generator

Figure 1 is a square wave generator, which is simple and easy implementable. This circuit can be also considered as a hybrid system because it has two modes while oscillations, and the circuit can oscillate by repeating them each other [7][8]. Circuit equations are



Figure 1: Square wave generator circuit.

shown as follows:

$$C\frac{dv}{dt} + \frac{1}{R_2}(v - v_{\text{out}}) = 0, \ v_{\text{out}} \in \{0, \ E\}, \quad (1)$$

where v is a capacitor voltage,  $v_{\rm out}$  is the output of an op-amp. Especially,  $v_{out}$  defined two modes of the circuit. When  $v_{\rm out} = 0$ , it is a discharge mode, otherwise it is a charge mode of the capacitor. They are flipped on two threshold values  $\alpha$  and  $\beta$ . Switching rules are shown in Eq. (2).

if 
$$v < \beta E$$
 then  $v_{\text{out}} = E$ ,  
if  $v > \alpha E$  then  $v_{\text{out}} = 0$ , (2)

where  $\alpha$  and  $\beta$  show threshold values of switching motions and are defined as follows:

$$\alpha = \frac{\frac{1}{R_3} + \frac{1}{R_5}}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}}, \ \beta = \frac{\frac{1}{R_5}}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}}.$$
 (3)

Figure 2 shows time waveforms from Eq. 1. Exactly



Figure 2: Time waveform from circuit equations.

solution of the capacitor voltage v(t) can be solved because each mode behaves based on a linear dynamics. Thus, characteristics of circuit including a fundamental frequency  $f_0$  and a duty ratio d can be derived as follows:

$$f_0 = \frac{1}{C_1 R_2} \left( \log \frac{\alpha(\beta - 1)}{\beta(\alpha - 1)} \right)^{-1}, \qquad (4)$$

$$d = \frac{\log \frac{\beta - 1}{\alpha - 1}}{\log \frac{\alpha(\beta - 1)}{\beta(\alpha - 1)}}.$$
(5)

Next, let us consider the modified square wave generator circuit shown in Fig. 3. Totally, Fig. 3 is similar to Fig. 1, but an inverter is attached at the output terminal of op-amp. By doing this, one can reversed a sign of  $v_{\rm out}$ . Therefor, Eq. 2 is rewrite as follows:

if 
$$v < \alpha E$$
 then  $v_{\text{out}} = E$ ,  
if  $v > \beta E$  then  $v_{\text{out}} = 0$ . (6)

#### 3. Circuit experiments

Next, let us show laboratory experiments. Electrical devices are shown in Tab. 1. Accordingly, the oscillating frequency is  $f_0 \approx 72.135$ [Hz], and the duty ratio is



Figure 3: Modified square wave generator circuit.

Op-amp	TL082
Capacitor $C_1$	$0.1 \mu F$
Resistor $R_2$	$100 \mathrm{k}\Omega$
Resistors $R_{3, 4, 5}$	$10k\Omega$
Power-supply $E$	5V

Table 1: Device list on laboratory experiments.

d = 50%. The threshold value  $\alpha$  and  $\beta$  are 2/3 and 1/3, respectively, since resistor  $R_3$ ,  $R_4$ , and  $R_5$  are the same.

Figure 4 shows circuit experiments of the square wave generator (Fig. 1). We can see that  $v_{out}$  oscillates about  $f_0 = 76.9$ Hz and d = 55%. The capacitor voltage v(t) also oscillate dependently on  $v_{out}$ . Note that, amplitudes of  $v_{out}$  are about 2Vpp, not 5Vpp, because we use the op-amp TL082, which is not a rail-to-rail, in experiments. The differences from Eq. (4) and (5) is appeared from it and a tolerance of the capacitor or resistors.



Figure 4: Circuit experiments of the square wave oscillator. Two lines mean the output of op-amp and capacitor voltage, respectively. (X-axis: 5msec/DIV)

Next, we show circuit experiments of the modified

SWG in Fig. 5. Time scales and voltage ranges are the same of previous one in Fig 4. Thus, frequencies of oscillations are changed higher values drastically. It cannot confirm shapes of waveforms. In this circuit, the capacitor  $C_1$  and resistor values  $R_2$  are not changed, so the time constant is the same of previous one. It can be considered that the circuit oscillates with another rule irrelevantly to basic oscillation modes.



Figure 5: Circuit experiment of bursting of transition. Two lines mean the output of op-amp and capacitor voltage, respectively. (X-axis: 5msec/DIV)

Figure 6 shows the magnification diagram of Fig. 5. The time scale is  $1\mu$ sec/DIV, thus frequency of bursting of transition is about 333.3kHz, and it is hundreds of times of fundamental frequency. However, it is confirmed that shapes of two waveform are simple and they are governed some kind of dynamical rules.



Figure 6: Magnified diagram of Fig. 5. (X-axis:  $1\mu$ sec/DIV) The output of the op-amp is changed independently of the capacitor voltage.

## 4. Bursting of transition

Figure 7 shows phase portraits of the SWG and the its modified circuit. Note that the y-axis is the output of the op-amp and it takes a binary value (0 or E).

The x-axis is the capacitor voltage and it is an analog value.

From circuit diagrams, behavior of the SWG is shown as Fig. 7 (a). Now, if the capacitor voltage is located as point (i), it is charged toward to v(t) = E. Finally, v(t) reaches to  $\alpha E$  and transitions are occurred to  $v_{\text{out}}(t) = 0$ . Thereafter, the capacitor voltage is discharged toward to v(t) = 0. Since these sequences are repeated, the SWG shows steady oscillations. Note that, oscillations depend on the dynamics of the capacitor.

On the modified SWG, you can see that the capacitor voltage is converged to a constant value from laboratory experiments. Thus, the capacitor voltage does not oscillate, but the output of the op-amp oscillates with very high frequencies. Figure 7 (b) shows the sketch of behavior of the modified SWG. The x and y-axis are analog dynamics of the capacitor and digital dynamics of the op-amp, respectively. It is confirmed that  $\alpha E$  and  $\beta E$  are interchanged, and there are regions (I) that the digital state which  $v_{out}$  is not determined at specific state. If the current state is set at point (ii),  $v_{\text{out}}$  is flipped to zero immediately since the capacitor voltage is larger than  $\alpha E$ . After that, the state become point (iii), but it is clear that the state is lower than the threshold value  $\beta E$ . Therefore,  $v_{\text{out}}$  soon turns back to E again. In this range, two states with any capacitor voltages are flipped each other without charge/discharge dynamics of the capacitor. Thus, transitions of the output of the op-amp are occurred one after another. Actually, the output of opamp is flipped dependently on a slew rate that TL082 has  $13[V/\mu sec]$ , and these oscillations are governed by it. As a result, a high frequency oscillation mode is appeared. These phenomena cannot be observed numerical simulation based on the mathematical models Eqs. (1) and (6) because these equations express ideal behavior without slew rates of op-amps.

Next, we show the relationship between threshold values and oscillating frequencies or bursting of transitions. The parameter map is shown in Fig. 8. The gradation (II) and white (III) regions show regular mode oscillations and bursting of transitions, respectively. In the region (II), the circuit shows square waves depended on circuit components, i.e. the capacitor and resistors. However, high frequency oscillations that are regardless of them without the slew rate of the opamp are appeared in (III). In this region, oscillating frequencies are governed by only op-amps.



Figure 7: Sketches of circuit behavior of (a) a SWG and (b) modified SWG.

#### 5. Conclusion

In this study, we have clarified complex oscillation phenomena using two op-amps.

The oscillators with op-amps are useful applications for various fields. However, it is confirmed that the unexpected oscillation mode is appeared in circuit experiments since parameter miss matching, and so on. These modes generate high gain and high frequency noise, but they are governed by the dynamical rule of oscillations depended on the circuit structure. We call this mode the bursting of transitions of op-amps.

As an example, we have shown laboratory experiments of this phenomenon. The modified SWG with op-amps (TL081), oscillator shows about 333.3kHz. In this circuit, slew rates of op-amps play an important parameter, and frequencies of oscillations depend on it. When we use other op-amps, frequencies of waves are also changed. Especially, if we use high-speed comparators (i.e. LM319), we can get mega hertz oscillation waves.

These phenomena are observed not only in peculiar electrical circuit, but also in multipurpose circuit with op-amps. For example, they can be happened in a hysteresis comparator. High frequency oscillation modes are easily observed with the set of inappropriate parameters.



Figure 8: Parameter map of oscillating frequency with varing parameters  $\alpha$  and  $\beta$  ( $C_1R_2 = 0.01$ ).

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