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Analysis of Various Super-Stable Periodic Spike-Trains in Bifurcating Neuron with Two Triangular Inputs

Risa Takahashi†, Yusaku Yanase† and Toshimichi Saito†

†Dept. Electrical and Electronic Eng., Hosei University, Tokyo, 184-8584 Japan
Email: risa.takahashi.3j@stu.hosei.ac.jp, tsaito@hosei.ac.jp

Abstract—This paper studies dynamics of the bifurcating neuron with two triangular inputs: the first input is period T and the second input is period $T/3$. Repeating integrate-and-fire behavior between a threshold and the base signal, the neuron can output various spike-trains. Deriving one-dimensional map of spike phases, the dynamics can be analyzed precisely. Especially, this paper analyzes a variety of super-stable periodic spike-trains and related bifurcation phenomena.

1. Introduction

The bifurcating neuron (BN, [1]) is a switched dynamical system inspired by spiking neurons. Repeating integrate-and-fire dynamics between a periodic base signal and a constant threshold, the BN outputs a spike-train. The BN can be analyzed precisely by a one-dimensional map of spike phases (Pmap). The analysis of BN is basic to consider spike-based nonlinear dynamics and engineering applications: image processing, digital communications, analog-to-digital converters, neural prosthesis [2]-[5], etc. Analysis of the BN is important not only as a fundamental study nonlinear dynamical system, but also for engineering applications [6].

This paper studies dynamics of the bifurcating neuron with two periodic base signal inputs. The first input is a triangular waveforms with period T and the second input is a triangular waveforms with period $T/3$. For simplicity, we consider a parameter range where the BN exhibits chaotic spike-train if either the first or the second input is applied. In this parameter range, if both the first and second inputs are applied, the BN can have a variety of super-stable periodic spike-trains. We then derive the Pmap that is piecewise linear and includes segments with zero-slope. Using the Pmap, we analyze the super-stale spike-trains and related bifurcation phenomena precisely.

For novelty of this paper, we note the following two points. First, we have presented the BN with two triangular base signal inputs and the BN exhibits chaos and super-stable spike-trains. Second, we have derived the piecewise linear Pmap including zero-slope segments. The Pmap enables us to analyze the phenomena exactly.

2. Circuit model of bifurcating neuron

Figures 1 and 2 show a circuit model and dynamics of the BN, respectively. Below a threshold V_T, the capacitor voltage v increases by integrating a constant current $I > 0$. If v reaches V_T, the BN outputs a spike $Y(t) = E$. The spike closes a switch SW and v is reset to the periodic base signal ($B(t)$) with period T. Repeating this behavior the BN outputs spike-train $Y(t)$. For simplicity, the inner resistors are ignored ($r_1 \rightarrow \infty, r_2 \rightarrow 0$) and the switching is assumed to be ideal: v_1 is reset instantaneously without delay. The dynamics is described by

$$\begin{cases} C\dfrac{dv}{dt} = I, & Y(t) = -E \quad \text{for } v(t) < V_T \\ x(t_+) = B(t_+), & Y(t_+) = E \quad \text{if } v(t) = V_T \end{cases} \qquad (1)$$

$$B(t) = K_1B_1(t) + K_3B_1(3t) + E_0,\ B_1(t+T) = B_1(t) \qquad (2)$$

$$B_1(t) = \begin{cases} -(A-2)t/T & \text{for } -d < t/T < d \\ A(t/T - 2d) + 2d & \text{for } d < t/T < 1-d \end{cases} \qquad (3)$$

where $B(t) < V_T$.

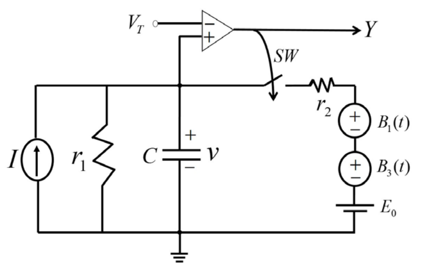

Figure 1: Bifurcating neuron circuit model.

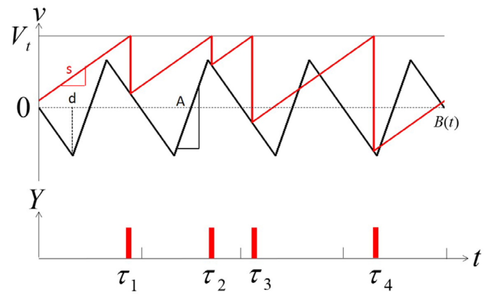

Figure 2: Bifurcating neuron dynamics.

Using dimensionless variables and parameters:

$$\tau = \frac{t}{T}, x = \frac{v}{V_T}, \dot{x} = \frac{dx}{d\tau}, y = \frac{Y+E}{2}, \quad k_1 = \frac{K_1}{V_T}, k_3 = \frac{K_3}{V_T}, a_0 = \frac{E_0}{V_T}, s = \frac{IT}{CV_T} \tag{4}$$

Eqs. (1)-(4) are transformed into

$$\begin{cases} \dot{x} = s & \text{for } x < 1 \\ x(\tau_+) = b(\tau_+) & \text{if } x(\tau) = 1 \end{cases} \tag{5}$$

$$b(\tau) = k_1 b_1(\tau) + k_3 b_1(3\tau) + a_0, \ b_1(\tau+1) = b_1(\tau) \tag{6}$$

$$b_1(\tau) = \begin{cases} -(A-2)\tau & \text{for } -d < \tau < d \\ A(\tau - 2d) + 2d & \text{for } d < \tau < 1-d \end{cases} \tag{7}$$

where $\dot{x} \equiv dx/d\tau$. The base signal is characterized by a parameters A and d. For simplicity, we assume

$$2 < A < 4, \ \ 0 < d < 0.5$$

In this paper, we consider three cases of the $b(\tau)$.

Case 1: The first component only ($k_1 = 1$, $k_3 = 0$, $a_0 = 0$)

Case 2: The second component only ($k_1 = 0$, $k_3 = \frac{1}{3}$, $a_0 \neq 0$)

Case 3: Two inputs ($k_1 = 1$, $k_3 = \frac{1}{3}$, $a_0 \neq 0$)

It goes without saying that the theorem of superposition is not valid in this nonlinear system.

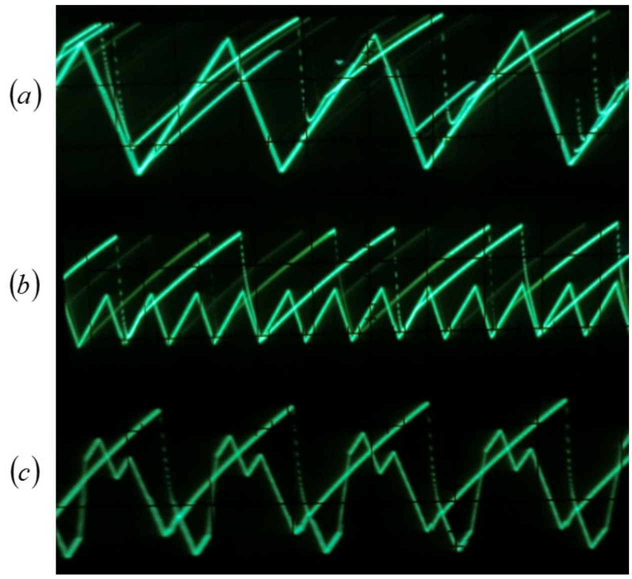

Figure 3: Typical waveforms of BN. ($T = 1[ms]$, $C = 0.022[\mu F]$, $r_1 = 120.2[k\Omega]$, $r_2 = 1.11[k\Omega]$, $V_T = 1[V]$, $A = 4.0$, $d = 0.33$, $E_0 = 0$) (a)first component only ($k_1 = 1$, $k_3 = 0$), (b)second component only ($k_1 = 0$, $k_3 = \frac{1}{3}$), (c)two inputs ($k_1 = 1$, $k_3 = \frac{1}{3}$).

3. Experiments

In order to confirm typical phenomena, we have fabricated a breadboard prototype of the BN. Figure 3 shows typical phenomena. The BN exhibits chaos for $B(t) = B_1(t)$ (first component only) or $B(t) = B_3(t)$ (second component only). However, if $B(t) = B_1(t) + B_3(t)$ then the BN exhibits periodic waveform as shown in Fig. 3(c). That is, chaotic behavior of each BN can be changed into periodic behavior by the two inputs.

4. Spike-phase map

In order to analyzed the dynamics, we derive the Pmap of the BN. Let τ_n denote the n-th spike position. The spike-train is characterized by the spike positions. Since τ_{n+1} is determined by τ_n, we can define the spike-position map.

$$\tau_{n+1} = \tau_n + (1 - b(\tau_n))/s \equiv F(\tau_n) \tag{8}$$

Since $F_1(\tau + 1) = F_1(\tau) + 1$ is satisfied, we introduce the phase variable $\theta_1(n) = \tau_1 \bmod 1$. Using this, we can define the Pmap as shown in Fig. 4:

$$\theta_{n+1} = f(\theta_n) \equiv F(\theta_n) \bmod 1 \tag{9}$$

Substituting $k_3 = 0$ and $a_0 = 0$ into Eq. (6), we obtain the Pmap for the first component. Substituting $k_1 = 0$ into Eq. (6), we obtain the Pmap for the second component.

$$f(\theta_n) = \begin{cases} a\theta & \text{for } -d < \theta < d \\ -a\theta + (1+a)/2 & \text{for } d < \theta < 1-d \end{cases} \tag{10}$$

The shape of the Pmap depends on the shape of $b(\tau)$. As the parameter varies, the shape of Pmap varies and BN can exhibit various spike-trains. Using Eqs. (10)-(**??**), the Pmap can be calculated precisely. Since the base signal is piecewise linear, the maps are also piecewise linear and precise numerical analysis is possible.

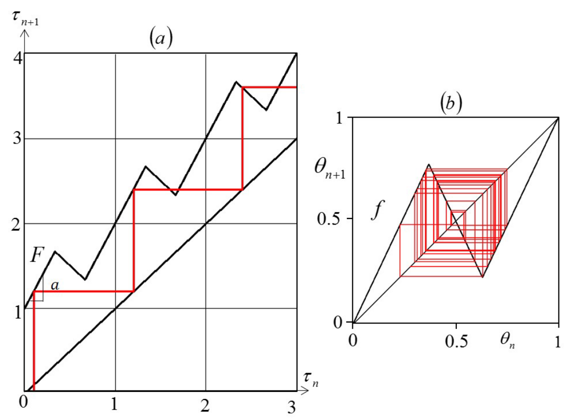

Figure 4: (a) Spike-position map (Smap), (b) Spike-phase map(Pmap).

5. Bifurcation phenomena

Using the Pmap, we consider basic bifurcation phenomena. Figure 5 shows typical examples of Pmaps and orbits in the Cases 1 and 2. We can see that the BN exhibits chaotic spike-trains in both Cases 1 and 2. Figure 6 shows several Pmaps in Case 3 (two inputs). The BN exhibits periodic spike-trains, e.g., the fixed points in Fig. 6(a) means periodic spike-train with period 1. These Pmaps show that chaotic spike-trains in Cases 1 and 2 are changed into the periodic spike-train in Case 3 (addition of two base signals in Cases 1 and 2). Note that the piecewise linear Pmap in Fig. 6 consists of several segments and slope of each segment is either 0, $2a$, or $-2a$. If a periodic orbit hits the branch with slope 0 then the periodic orbit is super-stable and the BN generates a super-stable periodic spike-train (SSPT).

Next, we consider bifurcation of SSPTs in Case 3. First we fix $a = 3, d = 0.33$ and select the dc component a_0 as a control parameter. As a_0 varies, the BN exhibits various SSPTs as shown in Fig. 6. The BN exhibits SSPTs with period 2, 3 and 4 as shown in Figs. 6(b), (c), and (d), respectively. Figure 7 shows a bifurcation diagram of SSPTs for a_0. Note that the orbits of Pmap must hit a segment with slope 0 and all the spike-trains are SSPTs. Figure 8 shows parameter regions for SSPTs with period 1: a basic results of the bifurcation analysis in the a_0-a plane.

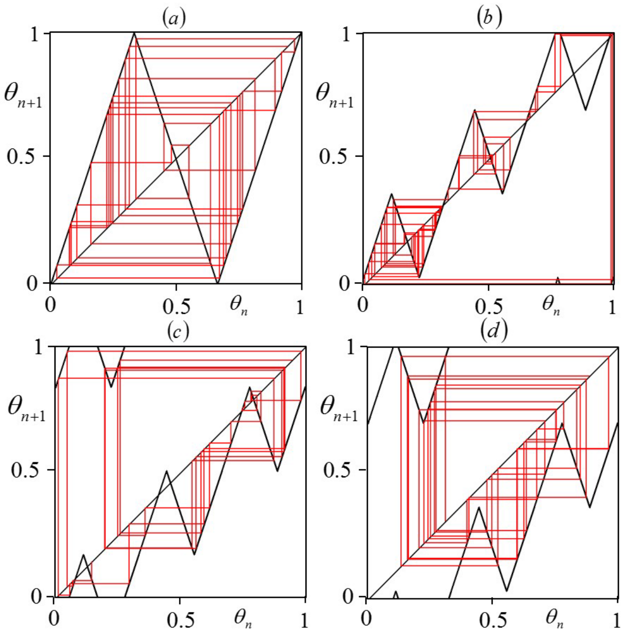

Figure 5: Pmap for $a = 3$, $d = 0.33$. (a) The first component only ($k_1 = 1, k_3 = 0$), (b) The second component only ($k_1 = 0, k_3 = 1/3, a_0 = 0.03$), (c) ($k_1 = 0, k_3 = 1/3, a_0 = 0.84$), (d) ($k_1 = 0, k_3 = 1/3, a_0 = 0.7$).

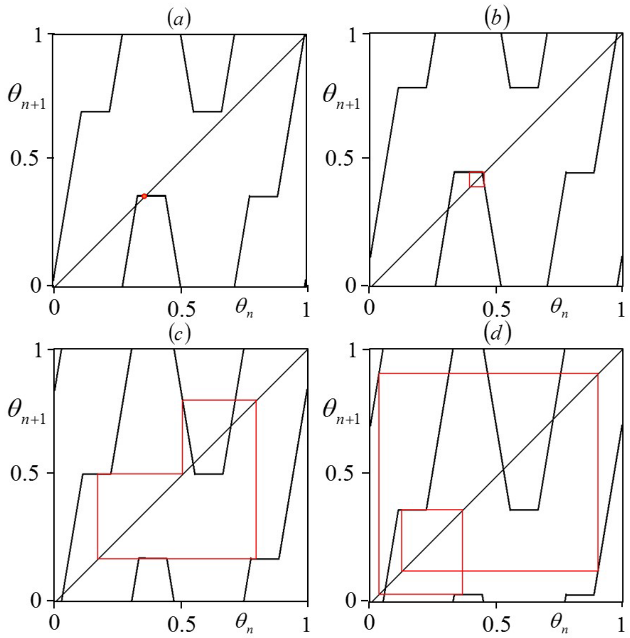

Figure 6: Pmap for ($k_1 = 1, k_3 = 1/3, a = 3, d = 0.33$). (a) $a_0 = 0.03$, (b) $a_0 = 0.12$, (c) $a_0 = 0.84$, (d) $a_0 = 0.7$.

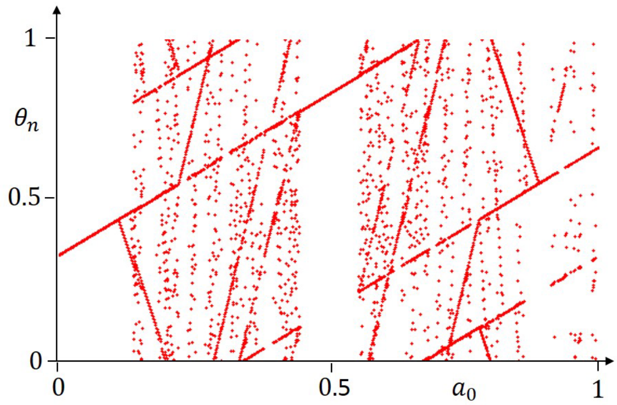

Figure 7: Bifurcation diagram for ($k_1 = 1, k_3 = 1/3, a = 3$, $d = 0.33$).

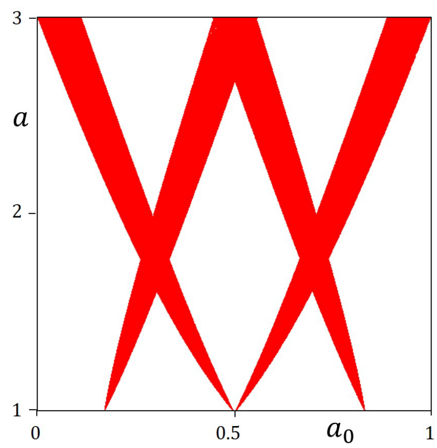

Figure 8: Parameter regions for super-stable periodic spike-train with period 1. $(k_1, k_3) = (1, 1/3)$, $d = \frac{1+a}{4a}$

6. Conclusions

We have studied dynamics of BN with two triangular base signal inputs. If either input is applied, the BN exhibits chaotic spike-train. If two inputs are applied, the BN exhibits a variety of SSPTs. Using the piecewise linear Pmap including braches with slope 0, the SSPTs have been analyzed precisely. Performing basic numerical experiments, we have demonstrated typical SSPTs and related bifurcation phenomena. Future problems are many, including analysis of rich bifurcation phenomena and application to engineering problems.

Acknowledgments

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