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Analysis of Almost Periodic Oscillations in Three-Phase Circuit Using Shooting Method

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Abstract—Almost periodic oscillations are often generated in three-phase circuits. In this report, we propose a method to analyze the almost periodic oscillations in three-phase circuits by extending the shooting method, which is used for the analysis of periodic oscillations. We formulate the almost periodic oscillations using the Poincare map from closed curve to closed curve. Our approach gives a new representation of invariant tori.

1. Introduction

Symmetrical three-phase circuits in Fig.1 are basic models of non-linear electric power systems for no load or light load. Fundamental harmonic and subharmonic periodic oscillations and almost periodic oscillations have been observed in three-phase circuits for the nonlinear nature of inductors [1, 2, 3, 4]. Especially, the almost periodic oscillations are generated on a widespread region in parameter spaces.

As useful tools for the analysis of the periodic oscillations, we have the harmonic balance method [5] and the shooting method [6]. The shooting method solves the fixed-point problem of the Poincare map from point to point. Until now, many methods have been proposed also for analyzing the almost periodic oscillations [2, 7, 8, 9, 10, 11, 12]. The first approach approximates the almost periodic oscillations with the Fourier coeffcients [2, 7, 9]. The second approach represents the almost periodic oscillations which lead to a specific invariance equation by a natural parametrisation [8]. In the third approach, the almost periodic oscillations are computed as a fixed point of a generalized Poincare map [10, 11, 12]. We propose a new method in the third approach.

This paper proposes a method of obtaining the almost periodic solutions by extending the shooting method. Almost periodic oscillations are represented by invariant tori and thier Poincare sections are closed curves. Moreover we approximate the closed curve with Fourier coefficients. We formulate the determining equation to be able to use the shooting method. Our approach regards the analysis of the almost periodic oscillations as the fixed-point problem of the Poincare map from closed curve to closed curve.

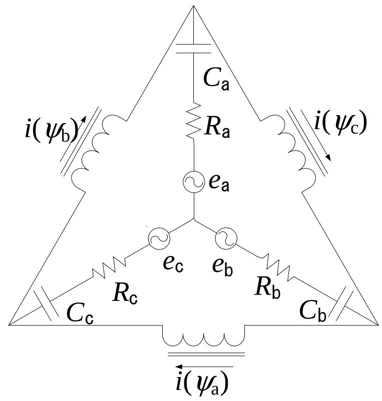

Figure 1: Three-phase circuit

2. Equation of three-phase circuit

Figure 1 illustrates the symmetric three-phase circuit, which consists of delta-connected nonlinear inductors, capacitors, resistors and balanced voltage sources. All inductors, capacitors, resistors and voltage sources are same characteristics.

The circuit equation which is normalized by the resonance frequency is written by

$$\begin{cases} \dot{\boldsymbol{\psi}}_{\rm abc} &= \boldsymbol{e}_{\rm abc} - \boldsymbol{A}_{\rm abc}\boldsymbol{u}_{\rm abc} - \boldsymbol{R}_{\rm abc}\boldsymbol{i}_{\rm abc} \\ \dot{\boldsymbol{u}}_{\rm abc} &= \boldsymbol{A}_{\rm abc}^{\rm T}\boldsymbol{i}_{\rm abc} \end{cases} \tag{1}$$

$$\boldsymbol{\psi}_{\rm abc} := (\psi_{\rm a}, \psi_{\rm b}, \psi_{\rm c})^{\rm T}, \quad \boldsymbol{u}_{\rm abc} := (u_{\rm a}, u_{\rm b}, u_{\rm c})^{\rm T},$$

$$\boldsymbol{A}_{\rm abc} := \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix},$$

$$\boldsymbol{e}_{\rm abc} := E_{\rm abc}\left(\sin(\omega t), \sin(\omega t - \frac{2\pi}{3}), \sin(\omega t + \frac{2\pi}{3})\right)^{\rm T},$$

$$\boldsymbol{i}_{\rm abc} := (i(\psi_{\rm a}), i(\psi_{\rm b}), i(\psi_{\rm c}))^{\rm T}, \quad \boldsymbol{R}_{\rm abc} := \boldsymbol{A}_{\rm abc}^{\rm T}\boldsymbol{A}_{\rm abc}R,$$

$$i(\psi) = \psi^3, \tag{2}$$

where $\boldsymbol{\psi}_{\rm abc}$ is the scaled magnetic flux vector, $\boldsymbol{u}_{\rm abc}$ is the scaled capacitor voltage vector. $E_{\rm abc}$ is the scaled voltage source, R is the scaled delta-connected resistance, ω is the scaled frequency. We approximate the magnetizing characteristic $i(\psi)$ in Eq.(2).

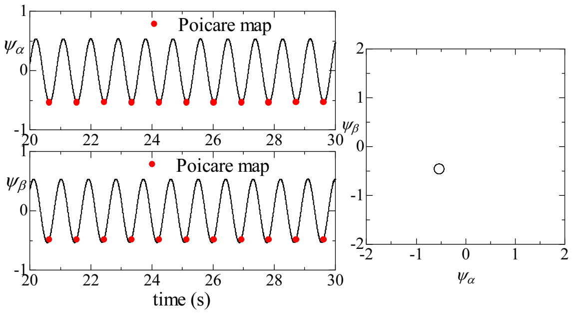

Figure 2: Waveforms of the periodic oscillation and fixed point of Poincare map

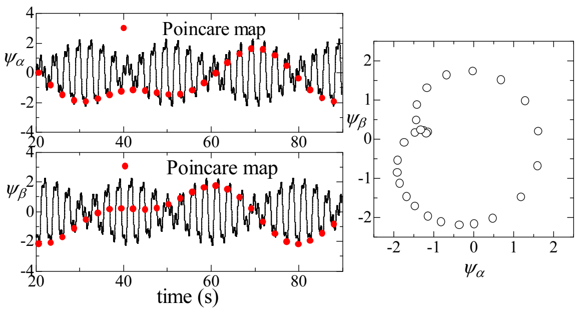

Figure 3: Waveforms of the almost periodic oscillation and approximate invariant closed curve

In order to simplify Eq.(1), we use the $0\alpha\beta$ transform defined by

$$\boldsymbol{\psi}_{0\alpha\beta} = \boldsymbol{W}_{0\alpha\beta}\boldsymbol{\psi}_{\mathrm{abc}}, \quad \boldsymbol{u}_{0\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \boldsymbol{W}_{0\alpha\beta}\boldsymbol{u}_{\mathrm{abc}},$$
$$\boldsymbol{\psi}_{0\alpha\beta} := (\psi_0, \psi_\alpha, \psi_\beta)^{\mathrm{T}}, \quad \boldsymbol{u}_{0\alpha\beta} := (u_0, u_\alpha, u_\beta)^{\mathrm{T}},$$
$$\boldsymbol{W}_{0\alpha\beta} := \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

Then, this circuit satisfies

$$\begin{cases} \sqrt{3}\psi_0 & = \psi_{\mathrm{a}} + \psi_{\mathrm{b}} + \psi_{\mathrm{c}} = const, \\ \sqrt{3}u_0 & = u_{\mathrm{a}} + u_{\mathrm{b}} + u_{\mathrm{c}} = const. \end{cases} \tag{3}$$

Equation (3) shows that the dimension of Eq.(1) decreases from 6 to 4. Thus, we can represent Eq.(1) by the following form:

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}\tau} = \boldsymbol{f}(\boldsymbol{x}, \tau), \quad \boldsymbol{x} := \begin{pmatrix} \boldsymbol{\psi}_{\alpha\beta} \\ \boldsymbol{u}_{\alpha\beta} \end{pmatrix}, \quad \tau = \omega t. \tag{4}$$
$$\boldsymbol{\psi}_{\alpha\beta} := (\psi_\alpha, \psi_\beta)^{\mathrm{T}}, \quad \boldsymbol{u}_{\alpha\beta} := (u_\alpha, u_\beta)^{\mathrm{T}},$$

3. Formulation of almost periodic oscillation

3.1. Shooting Method of periodic oscillation

The shooting method is a useful tool for obtaining the solution of the periodic oscillation. We define a periodic solution $\boldsymbol{x}(\tau)$ as integrating Eq.(4) from the initial state $\boldsymbol{x}(0) = \boldsymbol{x}_0$ over the interval $[0,T]$. Then, the Poincare map $\boldsymbol{T} : \mathbb{R}^n \mapsto \mathbb{R}^n$ for period T is defined by

$$\boldsymbol{T}[\boldsymbol{x}_0] := \boldsymbol{x}(\boldsymbol{x}_0, T) = \boldsymbol{x}_0 + \int_0^T \boldsymbol{f}(\boldsymbol{x}, \tau)\mathrm{d}\tau. \tag{5}$$

The periodic solution satisfies

$$\boldsymbol{x}_0 = \boldsymbol{T}(\boldsymbol{x}_0). \tag{6}$$

To obtain the periodic solution, we solve

$$\boldsymbol{F}(\boldsymbol{x}_0) := \boldsymbol{x}_0 - \boldsymbol{T}(\boldsymbol{x}_0) = \boldsymbol{o}. \tag{7}$$

Figure 2 illustrates an example of the periodic oscillation. The left figure shows the flux waveforms of the periodic oscillation, and as shown in the right figure we consider the periodic solution is formulated by the fixed-point problem of the Poincare map from point to point.

3.2. Almost Periodic Oscillation

Figure 3 illustrates an example of almost periodic oscillation. The flux waveforms of the almost periodic oscillation are shown in the left figure, and the Poincare map for ψ_α and ψ_β is shown in the right figure.

As shown in the right figure, the Poincare map of ψ can be expressed by the invariant closed curve in the almost periodic oscillation. In other words, almost periodic oscillations are represented by invariant tori and the cross section is a closed curve. We consider a method for obtaining the solution of the almost periodic oscillation as the fixed-point problem of the Poincare map from the closed curve to the closed curve. We define the invariant closed curve as $\bar{\boldsymbol{x}}(\theta) : \mathbb{T}^1 \mapsto \mathbb{R}^4$, where θ is a normalized phase and $\mathbb{T}^1$ denotes 1-torus.

Extending Eq. (7) for the initial value $\boldsymbol{x}_0$ of a periodic oscillation to the almost periodic oscillation, we describe the condition of $\bar{\boldsymbol{x}}(\theta)$ by

$$\bar{\boldsymbol{x}}(\theta) - \boldsymbol{T}'(\bar{\boldsymbol{x}}(\theta)) = \boldsymbol{o}. \tag{8}$$

This equation means that the maps $\boldsymbol{T}'$ preserves the closed curve $\bar{\boldsymbol{x}}(\theta)$.

To represent the closed curve efficiently, we use the Fourier series expansion. We define the determining equation as

$$\boldsymbol{F}(\bar{\boldsymbol{X}}(k)) := \bar{\boldsymbol{X}}(k) - \mathcal{F}\boldsymbol{T}'(\mathcal{F}^{-1}\bar{\boldsymbol{X}}(k)) = \boldsymbol{o}, \tag{9}$$

where $\bar{\boldsymbol{X}}(k)$ represents the Fourier coefficients of k-th higher components.

We describe a method to obtain the real vector of the Fourier series in the following section 3.3. and a method to calculate the Poincare map for the closed curve in the following sections 3.4 and 3.5.

3.3. Fourier series expansion of closed curve

In order to characterize the almost periodic oscillations, we approximate $\bar{\boldsymbol{x}}(\theta)$ by truncated Fourier series expansion with K frequency components.

In this case the closed curve $\bar{\boldsymbol{x}}(\theta) = (\bar{\psi}_\alpha, \bar{\psi}_\beta, \bar{u}_\alpha, \bar{u}_\beta)^{\mathrm{T}}$ is represented by

$$\begin{cases} \bar{\psi}_\alpha(\theta) = \Psi_{\alpha,0} + \sum_{k=1}^{K}(\Psi_{\alpha,\mathrm{c},k}\cos(k\theta) + \Psi_{\alpha,\mathrm{s},k}\sin(k\theta)) \\ \bar{\psi}_\beta(\theta) = \Psi_{\beta,0} + \sum_{k=1}^{K}(\Psi_{\beta,\mathrm{c},k}\cos(k\theta) + \Psi_{\beta,\mathrm{s},k}\sin(k\theta)) \\ \bar{u}_\alpha(\theta) = U_{\alpha,0} + \sum_{k=1}^{K}(U_{\alpha,\mathrm{c},k}\cos(k\theta) + U_{\alpha,\mathrm{s},k}\sin(k\theta)) \\ \bar{u}_\beta(\theta) = U_{\beta,0} + \sum_{k=1}^{K}(U_{\beta,\mathrm{c},k}\cos(k\theta) + U_{\beta,\mathrm{s},k}\sin(k\theta)) \end{cases} \tag{10}$$

$$\begin{aligned} \boldsymbol{\Psi}_{i,j} &:= (\Psi_{i,j,1}, \Psi_{i,j,2}, \cdots, \Psi_{i,j,K})^{\mathrm{T}}, \ \ (i = \alpha, \beta), (j = \mathrm{c}, \mathrm{s}) \\ \boldsymbol{U}_{i,j} &:= (U_{i,j,1}, U_{i,j,2}, \cdots, U_{i,j,K})^{\mathrm{T}}, \ \ (i = \alpha, \beta), (j = \mathrm{c}, \mathrm{s}) \\ \boldsymbol{\Psi}_\alpha &:= (\Psi_{\alpha,0}, \boldsymbol{\Psi}_{\alpha,\mathrm{c}}^{\mathrm{T}}, \boldsymbol{\Psi}_{\alpha,\mathrm{s}}^{\mathrm{T}})^{\mathrm{T}}, \ \ \boldsymbol{\Psi}_\beta := (\Psi_{\beta,0}, \boldsymbol{\Psi}_{\beta,\mathrm{c}}^{\mathrm{T}}, \boldsymbol{\Psi}_{\beta,\mathrm{s}}^{\mathrm{T}})^{\mathrm{T}}, \\ \boldsymbol{U}_\alpha &:= (U_{\alpha,0}, \boldsymbol{U}_{\alpha,\mathrm{c}}^{\mathrm{T}}, \boldsymbol{U}_{\alpha,\mathrm{s}}^{\mathrm{T}})^{\mathrm{T}}, \ \ \boldsymbol{U}_\beta := (U_{\beta,0}, \boldsymbol{U}_{\beta,\mathrm{c}}^{\mathrm{T}}, \boldsymbol{U}_{\beta,\mathrm{s}}^{\mathrm{T}})^{\mathrm{T}}. \end{aligned}$$

Equation (10) shows that the invariant closed curve $\bar{\boldsymbol{x}}(\theta)$ is approximated by the real vector $\boldsymbol{\Psi}_\alpha, \boldsymbol{\Psi}_\beta, \boldsymbol{U}_\alpha, \boldsymbol{U}_\beta \in \mathbb{R}^{2K+1}$.

However, the invariant closed curve $\bar{\boldsymbol{x}}(\theta)$ has arbitrary property of phase shift:

$$\bar{\boldsymbol{x}}(\theta) = \bar{\boldsymbol{x}}(\theta + \Delta\theta). \tag{11}$$

We normalize the phase shift by

$$\Psi_{\alpha,\mathrm{c},1} = 0. \tag{12}$$

Removing $\Psi_{\alpha,\mathrm{c},1}$ from $\boldsymbol{\Psi}_\alpha$, we define $\boldsymbol{X} \in \mathbb{R}^{4(2K+1)-1}$

$$\begin{aligned} \boldsymbol{X} &:= (\boldsymbol{\Psi'}_\alpha^{\mathrm{T}}, \boldsymbol{\Psi}_\beta^{\mathrm{T}}, \boldsymbol{U}_\alpha^{\mathrm{T}}, \boldsymbol{U}_\beta^{\mathrm{T}})^{\mathrm{T}}, \ \ \boldsymbol{\Psi'}_\alpha^{\mathrm{T}} := (\Psi_{\alpha,0}, \boldsymbol{\Psi'}_{\alpha,\mathrm{c}}^{\mathrm{T}}, \boldsymbol{\Psi}_{\alpha,\mathrm{s}}^{\mathrm{T}})^{\mathrm{T}}, \\ \boldsymbol{\Psi'}_{\alpha,\mathrm{c}} &:= (\Psi_{\alpha,c,2}, \Psi_{\alpha,c,3}, \cdots, \Psi_{\alpha,c,K})^{\mathrm{T}}. \end{aligned}$$

3.4. Poincare map

As the closed curve is the continuous function of θ, we discretize the invariant closed curve $\bar{\boldsymbol{x}}(\theta)$ to calculate the Poincare map. We divide the invariant closed curve $\bar{\boldsymbol{x}}(\theta)$ into $M \in \mathbb{Z}$ parts for θ:

$$\begin{aligned} \bar{\boldsymbol{x}}_D &= (\bar{\boldsymbol{x}}(\theta_0)^{\mathrm{T}}, \bar{\boldsymbol{x}}(\theta_1)^{\mathrm{T}}, \cdots, \bar{\boldsymbol{x}}(\theta_{M-1})^{\mathrm{T}})^{\mathrm{T}}, \\ \theta_m &= \frac{2\pi}{M}m, \ \ (m = 0, 1, \cdots, M-1) \end{aligned} \tag{13}$$

As we can get the discretized closed curve $\bar{\boldsymbol{x}}_D$, we can calculate the Poincare map from $\bar{\boldsymbol{x}}_D$ to $\bar{\boldsymbol{x}}'_D$ by Eq. (5):

$$\boldsymbol{T}[\bar{\boldsymbol{x}}(\theta_m)] := \boldsymbol{x}(\bar{\boldsymbol{x}}(\theta_m), T) = \bar{\boldsymbol{x}}(\theta_m) + \int_0^T \boldsymbol{f}(\boldsymbol{x}, \tau)\mathrm{d}\tau, \tag{14}$$

$$\bar{\boldsymbol{x}}'_D := (\boldsymbol{T}[\bar{\boldsymbol{x}}(\theta_0)]^{\mathrm{T}}, \boldsymbol{T}[\bar{\boldsymbol{x}}(\theta_1)]^{\mathrm{T}}, \cdots, \boldsymbol{T}[\bar{\boldsymbol{x}}(\theta_{M-1})]^{\mathrm{T}})^{\mathrm{T}}.$$

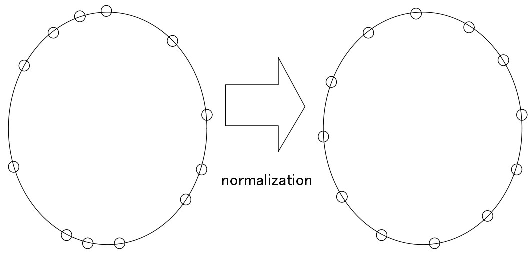

Figure 4: Example of the normalization using arc-length

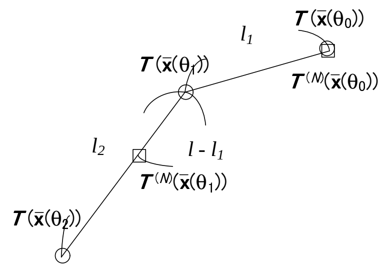

Figure 5: Linear approximation using arc-length

3.5. Normalization of θ

We assume that the θ of $\bar{\boldsymbol{x}}_D$ is normalized by arc length. After calculating the Poincare map, parameter θ has been changed. Then, the discretized closed curve $\bar{\boldsymbol{x}}_D$ contains arbitrary property in Fig. 4.

We normalize the closed curve $\bar{\boldsymbol{x}}'_D$ using the arc length. We define a part length l as

$$l = \frac{L}{M}, \tag{15}$$

$$L = \sum_{i=0}^{M-2} \|\bar{\boldsymbol{x}}'_D(\theta_i) - \bar{\boldsymbol{x}}'_D(\theta_{i+1})\|_2 + \|\bar{\boldsymbol{x}}'_D(\theta_{M-1}) - \boldsymbol{T}(\bar{\boldsymbol{x}}'_D(\theta_0))\|_2,$$

$$l = \begin{cases} \|\bar{\boldsymbol{x}}^{(N)}(\theta_i) - \bar{\boldsymbol{x}}^{(N)}(\theta_{i+1})\|_2 \ \ (i = 0, 1, \cdots, M-2) \\ \|\bar{\boldsymbol{x}}^{(N)}(\theta_{M-1})) - \bar{\boldsymbol{x}}^{(N)}(\theta_0)\|_2 \end{cases}$$

$$\bar{\boldsymbol{x}}_D^{(N)} := (\bar{\boldsymbol{x}}^{(N)}(\theta_0)^{\mathrm{T}}, \bar{\boldsymbol{x}}^{(N)}(\theta_1)^{\mathrm{T}}, \cdots, \bar{\boldsymbol{x}}^{(N)}(\theta_{M-1})^{\mathrm{T}})^{\mathrm{T}},$$

where L is the total length of the closed curve $\bar{\boldsymbol{x}}'_D$, and $\bar{\boldsymbol{x}}_D^{(N)}$ is the normalized closed curve.

We approximate the distance from point $\bar{\boldsymbol{x}}'_D(\theta_i)$ to point $\bar{\boldsymbol{x}}'_D(\theta_{i+1})$ by piece-wise line, and we use the interior division as shown in Fig. 5.

3.6. Algorithm

We show the algorithm to define the determining equation $\boldsymbol{F}(\boldsymbol{X})$ of almost periodic oscillations using the functions previously described.

1. We construct the approximated closed curve $\bar{\boldsymbol{x}}(\theta)$ by the Fourier series from the real vector $\boldsymbol{X}$.

2. We discretize the closed curve $\bar{\boldsymbol{x}}(\theta)$ by θ_m and obtain $\bar{\boldsymbol{x}}_D$ which is the series of M points.

3. We calculate $\bar{\boldsymbol{x}}'_D$ using the Poincare map.

4. We calculate $\bar{\boldsymbol{x}}'^{(N)}_D$ by the arc-length normalization.

5. We calculate $\boldsymbol{X}^{(N)}$ by Fourier series expansion from $\bar{\boldsymbol{x}}'^{(N)}_D$.

As a result, we define the determining equation of almost periodic oscillations by

$$\boldsymbol{F}(\boldsymbol{X}) = \boldsymbol{X} - \boldsymbol{X}^{(N)} \tag{16}$$

4. Example

We analyze the almost periodic oscillation using the proposed method. We set the scaled resistance $R = 0.06$, the scaled frequency $\omega = 7.0$, the scaled voltage source $E = 3.0$, the maximum order $K = 3, 6$ and $M = 64$.

Moreover, we use the Homotopy method [13] to solve Eq. (17).

Figure 6 illustrates the Poincare map obtained by Runge-Kutta method and the proposed method. The difference between the proposed method and Runge-Kutta method is the effect of approximation by Fourier series expansion. In this case, despite $K = 3$ we can approximate the almost periodic oscillation .

5. Conclusion

We proposed a method for calculating the almost periodic oscillations by extending the shooting method. We defined the determining equation of almost periodic oscillations by the Poincare map from closed curve to closed curve. We represented the closed curve by the Fourier series expansion and formulated the determining equation to be able to use the shooting method. We gives a new representation of invariant tori.

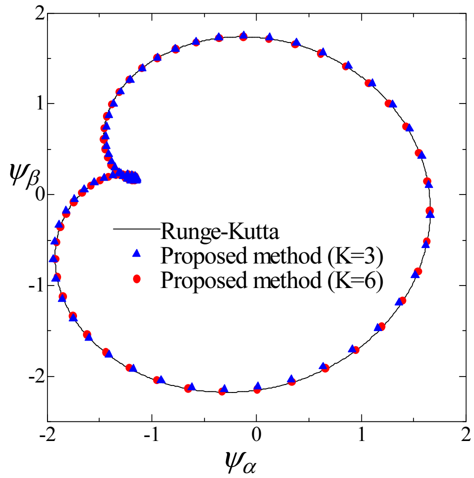

Figure 6: The invariant closed curve by the proposed method and Runge-Kutta method

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