

Laboratory experiment for bifurcation phenomena of the forced Alpzur oscillator

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Abstract—This paper shows some results of laboratory experiment for bifurcation phenomena on the forced Alpzur oscillator. The oscillator denotes some special phenomena in some parameter settings and are called chaos or bifurcation. The chaos is defined as the state of disorder and occurred by the bifurcation. Therefore the bifurcation should almost always be avoided to occur. Especially in these days, an idea to solve these problems on the forced Alpzur oscillator is proposed. However laboratory experiments for the idea have never been done. Laboratory experiments are important to confirm whether numerical analyses are correct or not.

1. Introduction

In nonlinear dynamical systems, there are some special phenomena called nonlinear phenomena observed on certain parameter settings. The bifurcation and the chaos are typical phenomena of them and are well researched for a long years. The bifurcation is changing of system behavior and caused by changing of local or global properties; therefore, there are some types of bifurcations just like follows: local bifurcation: tangent bifurcation, period-doubling bifurcation and so on, and global bifurcation: grazing bifurcation, saddle-node bifurcation and so on. The bifurcations affect qualitative properties of systems and sometimes let system behavior be chaotic; here, the behavior is called the chaos. Since the chaos is a state of disorder, it should be avoided and bifurcation analysis is very important.

In these years, the systems with discontinuity are researched actively [1][2][3]. For representative example, the previous study by Tamura[2] shows bifurcation analysis for the Izhikevich neuron model, which has a discontinuous characteristic on state space. On the other hands, the study [3] shows how to control the behavior of the nonlinear systems with interruptions. Especially in those studies, we did bifurcation analysis of the systems with an external force that is constantly added on previous study[4] [5]. Above all, the study[5] shows the result of bifurcation analysis for the forced Alpzur oscillator by numerical computation. However, the study is just only by numerical analysis and there are no result from laboratory experiments. Cir-

cuit implementation is important to confirm the numerical result from the stand point of experiments. Thus, we try a laboratory experiment for the system picked up in [5] and confirm whether the numerical analyzed result is correct or not.

2. The forced Alpzur oscillator

The Alpzur oscillator, is proposed by Kawakami and Lozi[6], is one of the simple oscillators that has a switching and a nonlinear element. On previous study[5], an external force is added on the oscillator and is realized by circuit shown in Fig. 1. In addition, the circuit equations are give

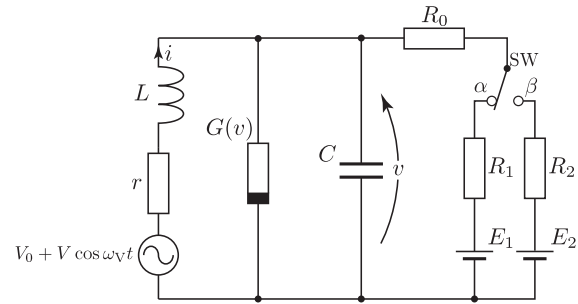


Figure 1: Circuit diagram of the forced Alpzur oscillator

as follows:

$$\begin{cases} L \frac{di}{dt} = -ri - v + V_0 + V \cos \omega vt \\ C \frac{dv}{dt} = \begin{cases} i - G(v) + \frac{E_1 - v}{R_0 + R_1} & (\text{if SW is at } \alpha) \\ i - G(v) + \frac{E_2 - v}{R_0 + R_2} & (\text{if SW is at } \beta). \end{cases} \end{cases} \quad (1)$$

Solution orbits of the model are switched at their boundaries caused by the SW

$$\begin{aligned} H &= \{(i, v) \in \mathbf{R}^2 \mid v > v_h\} \\ B &= \{(i, v) \in \mathbf{R}^2 \mid v < v_b\} \\ \partial H &= \{(i, v) \in \mathbf{R}^2 \mid v = v_h\} \\ \partial B &= \{(i, v) \in \mathbf{R}^2 \mid v = v_b\}. \end{aligned} \quad (2)$$

Now we assume that $b > h$. The behavior of the state is described as follows:

1. The flow starting from an arbitrary initial point moves within the half plane H or B , defined by Eq. (1).
2. If the flow reaches the edge ∂H or ∂B , then switching occurs.

From the above, Fig. 2 shows an example of the circuit responses.

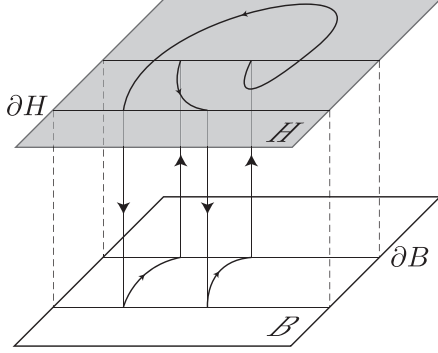


Figure 2: Conceptual figure for boundaries.

By rescaling

$$\hat{x} = \sqrt{L}i, \quad \hat{y} = \sqrt{C}v, \quad t' = \frac{1}{\sqrt{LC}}t, \quad (3)$$

assuming

$$G(v) = -a_1v + a_3v^3, \quad (4)$$

putting

$$\begin{aligned} r_1 &= \frac{1}{R_0 + R_1}, \quad r_2 = \frac{1}{R_0 + R_2}, \quad k = r\sqrt{\frac{C}{L}}, \\ \hat{V}_0 &= V_0\sqrt{\frac{C}{L}}, \quad \hat{V} = V\sqrt{\frac{C}{L}}, \quad \hat{\omega}_v = \sqrt{LC}\omega_v, \\ g_1 &= 1 - (a_1 - r_1)\sqrt{\frac{L}{C}}, \quad g_2 = 1 - (a_1 - r_2)\sqrt{\frac{L}{C}}, \\ c_3 &= \frac{3a_3}{C}\sqrt{\frac{L}{C}}, \quad \hat{B}_1 = r_1\sqrt{LE_1}, \quad \hat{B}_2 = r_2\sqrt{LE_2}, \end{aligned} \quad (5)$$

and relabeling t' as t , we have the normalized equation

$$\begin{cases} \frac{dx}{dt} = -kx - y + \hat{V}_0 + \hat{V} \cos \omega_v t \\ \frac{dy}{dt} = \begin{cases} x + (1 - g_1)y - \frac{1}{3}y^3 + B_1 & \text{if the SW is } \alpha \\ x + (1 - g_2)y - \frac{1}{3}y^3 + B_2 & \text{if the SW is } \beta \end{cases} \end{cases}, \quad (6)$$

where,

$$\hat{x} = \gamma x, \quad \hat{y} = \gamma y, \quad \gamma = \sqrt{\frac{1}{C_3}}, \quad \hat{B}_1 = \gamma B_1, \quad \hat{B}_2 = \gamma B_2. \quad (7)$$

By normalizing, the boundaries are scaled as

$$\begin{aligned} H &= \{(x, y) \in \mathbf{R}^2 \mid y > h\} \\ B &= \{(x, y) \in \mathbf{R}^2 \mid y < b\} \\ \partial H &= \{(x, y) \in \mathbf{R}^2 \mid y = h\} \\ \partial B &= \{(x, y) \in \mathbf{R}^2 \mid y = b\}, \end{aligned} \quad (8)$$

where,

$$h = \frac{\sqrt{C}}{\gamma}v_h, \quad b = \frac{\sqrt{C}}{\gamma}v_b. \quad (9)$$

3. Bifurcations and Chaotic attractor

From the result of previous study[5], analysis method for the bifurcation problem on the forced Alpaazur oscillator is suggested; and Fig. 3 is the bifurcation diagram we got by the method. Here we set the static parameters $k, \hat{V}_0, \hat{\omega}_v, g_1, g_2, B_2, h$ and b as follows:

$$\begin{aligned} k &= 0.1, \quad \hat{V}_0 = 0, \quad \hat{\omega}_v = 1.26, \quad g_1 = 0.2, \\ g_2 &= 2.0, \quad B_2 = 5.0, \quad h = -1.0, \quad b = -0.1. \end{aligned} \quad (10)$$

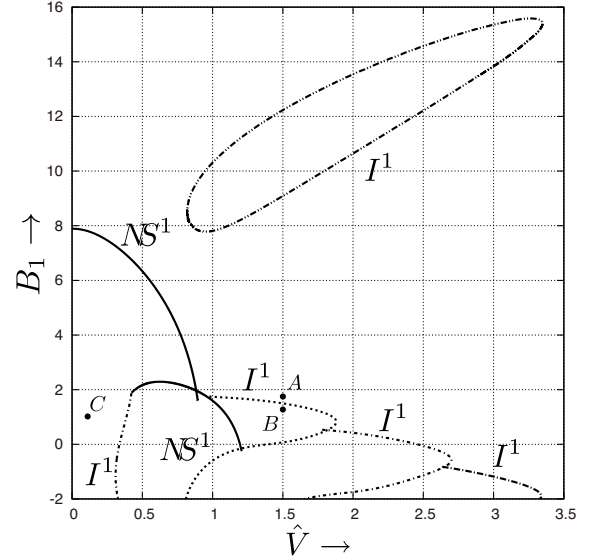


Figure 3: $\hat{V} - B_1$ plane of the bifurcation diagram.

On Fig. 3, I^i and NS^i are period-doubling bifurcation and Neimark-Sacker bifurcation from i -periodic orbit, respectively. For example at point A, the solution orbit becomes 1-periodic orbit shown on Fig. 7; and by crossing I^1 , at point B, the solution becomes 2-periodic orbit shown on Fig. 8. In addition, chaotic attractor shown on Fig. 11 is observed at point C.

4. Laboratory implementation

On this study, some parameters on Eq.(1) are fixed as $L = 50[\text{mH}]$, $C = 0.09[\mu\text{F}]$, $r = 113[\Omega]$, $R_0 = 0[\Omega]$,

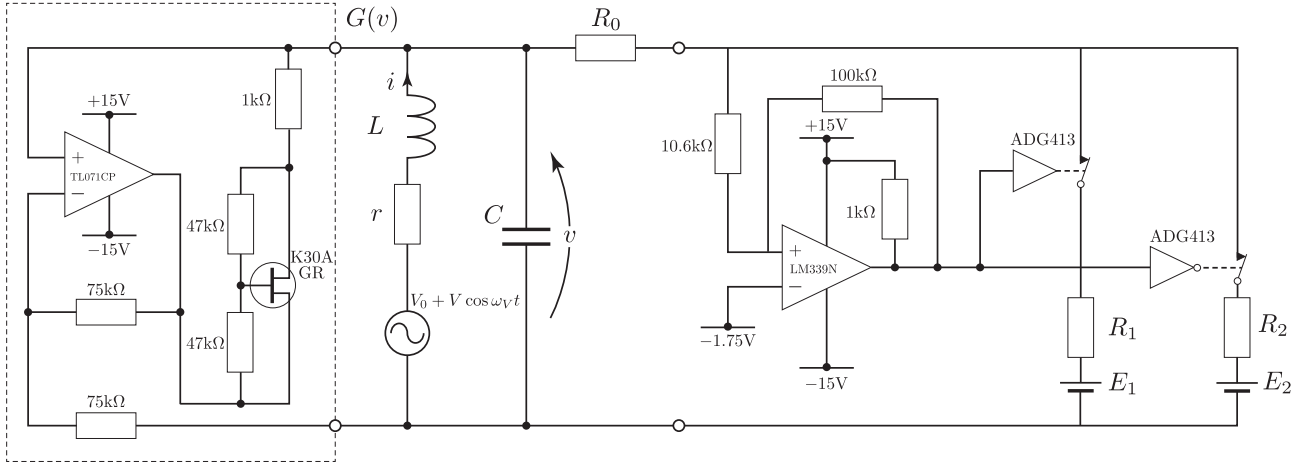


Figure 4: Rigid circuit diagram.

$R_1 = 811.7[\Omega]$, $R_2 = 212.9[\Omega]$, $V_0 = 0[\text{V}]$, $\omega_V = 1.88 \times 10^3 [\text{rad/s}]$ and $E_2 = 6[\text{V}]$ and the others V and E_1 are variable. Figure 4 shows a rigid circuit diagram we implemented. For the nonlinear resistor, the FET K30A-GR and op-amp TL071CP is used and then Fig. 5 shows the nonlinear characteristic of the resistor. Here the broken line is

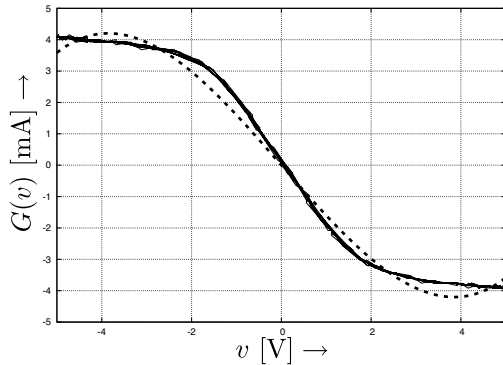


Figure 5: Characteristic of the nonlinear resistor.

$G(v)$, from fitting by GNU PLOT, and a_1 and a_3 on Eq. (4) are

$$a_1 = 1.63716 \times 10^{-3}, \quad a_3 = 3.67962 \times 10^{-6}. \quad (11)$$

For the SW, analog switch IC ADG413 is used on the right side of Fig. 4. Figure 6 shows a real circuit we implemented.

Figures 9–12 show results of numerical and laboratory experiments. From Fig. 9 to Fig. 10, solution orbit of this system occurs period-doubling bifurcation on laboratory experiment as well as on numerical experiment. As the same on Fig. 12, the chaotic attractor is observed.

5. Conclusion

We implemented the forced Alpzur oscillator on real circuit and confirmed that the bifurcation analysis by nu-

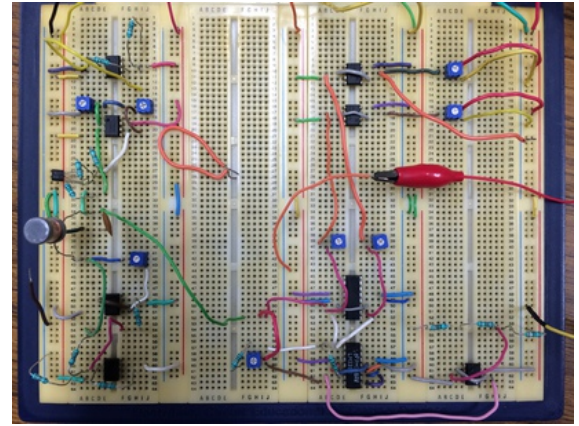


Figure 6: Real circuit.

merical method[5] is exactly correct from the stand point of laboratory experiments. Bifurcations observed from numerical experiments are observed in real circuit similarly. Also we have observed chaotic attractor on certain parameter setting either in numerical computation and in real circuit.

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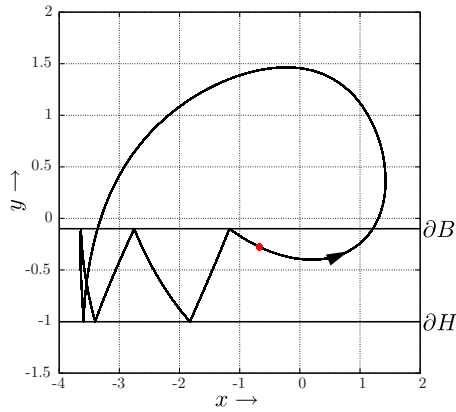


Figure 7: 1-periodic orbit at point A.

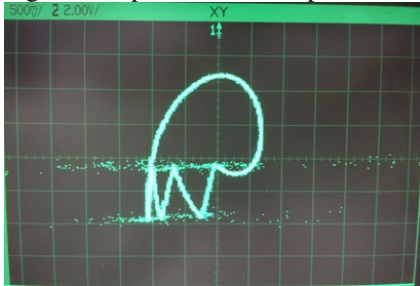


Figure 9: $V = 9[V]$, $E_1 = -3.0[V]$.

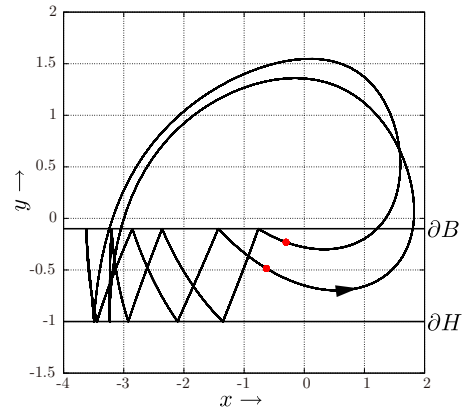


Figure 8: 1-periodic orbit at point B.

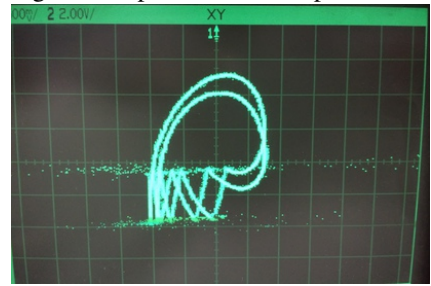


Figure 10: $V = 9[V]$, $E_1 = -4.0[V]$.

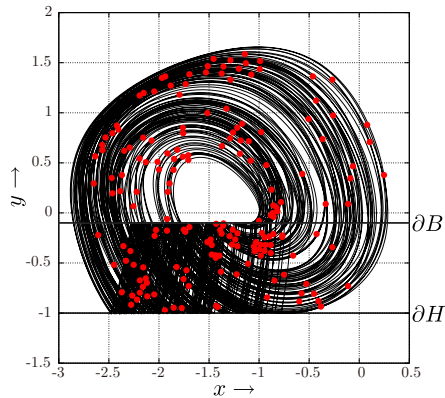


Figure 11: 1-periodic orbit at point C.

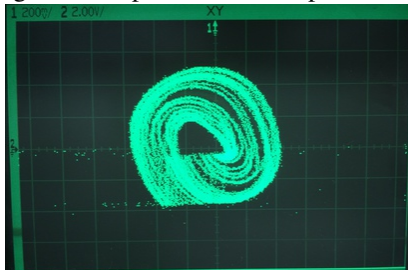


Figure 12: $V = 0.4[V]$, $E_1 = 0.1[V]$.

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