2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

# High-frequency self-pulsations in a semiconductor laser with optical feedback in a photonic integrated circuit

Andreas Karsaklian Dal Bosco[1], Kazutaka Kanno[1,2], Atsushi Uchida[1], Marc Sciamanna[3], Takahisa Harayama[4,5] and Kazuyuki Yoshimura[4]

[1] Department of Information and Computer Sciences, Saitama University
255 Shimo-Okubo Sakura-ku, Saitama City, Saitama 338-8570, Japan
[2] Department of Electronics Engineering and Computer Science, Fukuoka University
8-19-1 Nanakuma, Johnan-ku, Fukuoka, 814-0180 Japan
[3] Centrale-Supélec, OPTEL Research Group
2 rue Édouard Belin 57070 Metz, France
[4] NTT Communication Science Laboratories, NTT Corporation
3-1 Morinosato, Wakamiya, Atsugi-Shi, Kanagawa 243-0198, Japan
[5] Department of Applied Physics, Waseda University
3-4-1 Okubo, Shinjuku-ku, Tokyo 169-8555, Japan
Emails: andreas@mail.saitama-u.ac.jp, auchida@mail.saitama-u.ac.jp

**Abstract**—We present the bifurcation cascade leading to self-pulsing dynamics in a semiconductor laser embedded in a photonic integrated circuit. The mechanism at the origin of the self-pulsations is explained by a phenomenon of beating between external cavity modes and anti-modes. This dynamics is specific to lasers subjected to time-delayed optical feedback from a short external cavity, in which the feedback delay is shorter than the laser relaxation oscillation period. On the basis of experimental observations and numerical studies, we show that increasing the feedback strength causes the laser to exhibit a succession of self-pulsing dynamics at frequencies increasing up to 17 GHz. Moreover, we demonstrate that the mechanism underlying this dynamics is a migration of limit cycles through successive modes in the phase space, which frequencies are determined by the distribution of the modes and anti-modes. Simulations using the Lang-Kobayashi model show good qualitative accordance with the experimental observations and allow to understand the frequency interval in which self-pulsations can be observed.

## 1. Introduction

The interesting properties of the rich nonlinear dynamics of semiconductor lasers with optical feedback are nowadays commonly used for optical communication and signal processing [1]. Recently, studies of nonlinear dynamics in Photonic Integrated Circuits (PICs) have been thriving [3, 4]. Due to their compactness and their high phase stability, the suitability of PICs for chaos generation or laser coupling in applications in the fields of fast random bit generation or telecommunications has been evidenced [5, 6]. Another fundamental interest of PICs is the fact that they are suitable to study lasers subjected to feedback in short external cavity configurations, i.e. when the feedback delay is shorter than the relaxation oscillation period [7].

We focus here on one kind of nonlinear dynamics specific to short cavity regimes, namely self-pulsation: state in which the emitted intensity shows harmonic oscillations versus time. Self-pulsation has been first reported theoretically [8], and has later been analyzed experimentally in lasers with ultra-short external cavity [9, 10]. In such short-cavity configurations, self-pulsation can be generated from a beating between external cavity modes and anti-modes. This beating results in a self-pulsing dynamics at a frequency determined by the frequency distribution of the modes and the anti-modes.We present an experimental observation of the bifurcation scenario showing self-pulsation originated by a beating between successive pairs of modes and anti-modes and discuss the evolution of the pulsing frequency as the feedback strength increases.

## 2. Experimental observation of self-pulsation in PIC

Our PIC consists of a distributed feedback semiconductor laser bounded by a photodiode and a 2.3-mm long active external cavity composed of two independent semiconductor optical amplifiers and a passive waveguide ended by a reflector. The external cavity frequency $f_{cav}$ is close to 17 GHz. The relaxation oscillation frequency $f_{RO}$ ranges from 2.3 to 7.4 GHz, according to the laser injection current. This fulfills the short external cavity condition, since $f_{cav} > f_{RO}$ [7].

Experimental results are presented in Fig. 1, where a transition from steady state to self-pulsation is seen as the feedback strength is increased, by adjusting the injection current in one of the optical amplifiers ($J_{SOA1}$). The laser injection current is indicated by its normalized value with respect to threshold $J/J_{th}$, where $J_{th} = 13$ mA. In Fig. 1, the value of $J/J_{th}$ is equal to 1.0. The scenario starts when

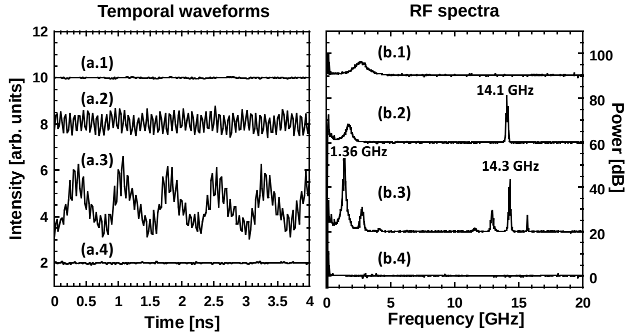


Figure 1: Experimental observation of self-pulsing dynamics when $J/J_{th}$=1.0 for increasing values of $J_{SOA1}$: (a-b.1) 9.60 mA, (a-b.2) 10.25 mA, (a-b.3) 10.53 mA and (a-b.4) 11.40 mA. The temporal waveforms (left) and the RF spectra (right) have been vertically shifted for clarity.

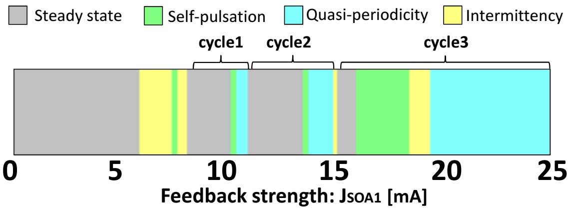


Figure 2: Experimental bifurcation diagram showing three dynamical cycles of self-pulsations seen when increasing $J_{SOA1}$ for $J/J_{th}$ fixed to 1.0. The colors correspond to different dynamics: grey is steady state, green is self-pulsation, blue is quasi-periodicity. Yellow is a peculiar intermittent regime which is not discussed here. The region labeled *cycle 1* corresponds to the scenario presented in Fig. 1.

the laser operates in steady state (Fig. 1.(a.1)). As the feedback strength ($J_{SOA1}$) is gradually increased, a bifurcation occurs and this steady state gives way to self-pulsation (Fig. 1.(a.2)) at the frequency of 14.1 GHz. If the feedback strength is further increased, a new frequency rises upon the harmonic background, inducing a transition to quasi-periodicity (Fig. 1.(a.3)). In the corresponding RF spectrum, (Fig. 1.(b.3)) the peak at 14.3 GHz is the reminiscence of the frequency of the self-pulsation and corresponds to the fast dynamics in the time trace. A second peak stands in the region of low frequencies (1.36 GHz) and corresponds to the newly-emerged slowly-varying envelope shaping the time trace in Fig. 1.(a.3). A further increase of $J_{SOA1}$ causes this quasi-periodic dynamics to collapse and the laser to recover steady state (Fig. 1.(a.4)).

As the feedback strength increases, these three successive dynamics [steady state, self-pulsation and quasi-periodicity] shape a dynamical cycle, reproducing itself several times. This observation is illustrated in Fig. 2 where an experimental bifurcation diagram obtained when varying $J_{SOA1}$ from 0 to 25 mA is represented. In this diagram, the scenario [steady state (grey), self-pulsation (green), quasi-periodicity (blue)] is reproduced three times.

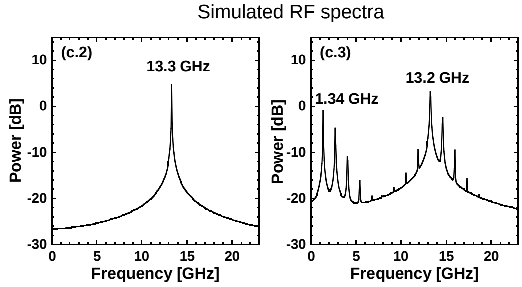


Figure 3: Simulated RF spectra. Spectra (c.2) and (c.3) illustrate the frequency contents of traces in Figs. 4.(a.2) and 4.(a.3) respectively.

## 3. Numerical analysis of bifurcation to self-pulsation

We carry out simulations making use of the Lang-Kobayashi rate equations [2] in order to give a theoretical explanation of the bifurcation cascade and mechanism underlying self-pulsation:

$$\begin{aligned}\frac{dE(t)}{dt} &= \frac{1}{2}\left[\frac{G_N\,(N(t)-N_0)}{1+\epsilon E^2(t)} - \frac{1}{\tau_p}\right]E(t) \\ &+ \kappa E(t-\tau)cos\,(\Theta(t))\end{aligned} \tag{1}$$

$$\begin{aligned}\frac{d\Phi(t)}{dt} &= \frac{\alpha}{2}\left[\frac{G_N\,(N(t)-N_0)}{1+\epsilon E^2(t)} - \frac{1}{\tau_p}\right] \\ &- \kappa\frac{E(t-\tau)}{E(t)}sin\,(\Theta(t))\end{aligned} \tag{2}$$

$$\frac{dN(t)}{dt} = J - \frac{N(t)}{\tau_s} - \frac{G_N\,(N(t)-N_0)}{1+\epsilon E^2(t)}E^2(t) \tag{3}$$

$$\Theta(t) = \omega\tau + \Phi(t) - \Phi(t-\tau) \tag{4}$$

In these equations $N$, $E$ and $\Phi$ are respectively the carrier density, the electric field amplitude and the electric field phase. $\tau$ ps and $\kappa$ represent the feedback delay and strength. $\alpha$=3 is the linewidth enhancement factor, $J$ is the laser injection current, $G_N$ is the gain coefficient, $N_0$ is the carrier density at transparency, $\tau_p$ and $\tau_s$ are the photon and carrier lifetimes. $\epsilon$ is the gain saturation coefficient. The delay $\tau$ is equal to 59 ps and corresponds to our experimental value of $f_{cav}$=17 GHz. The parameter $R$ is used to quantify the feedback strength and can be seen as a reflectivity coefficient. It is defined by $\kappa = (1-R_2^2)R/(\tau_{in}R_2)$, where $\tau_{in}$ and $R_2$ are respectively the photon round-trip time in the laser internal cavity and the reflectivity coefficient of the laser output facet. The dynamics of the photons and the carriers in the active external cavity is not taken into account in the Lang-Kobayashi model. Our aim is to understand the general mechanism at the origin of self-pulsation, which can be well rendered with this simple model. Yet it is worth to mention that analysis using the more sophisticated model of the so-called Traveling Wave Equations have been carried out, showing good quantitative compliance with experimental observations [10].

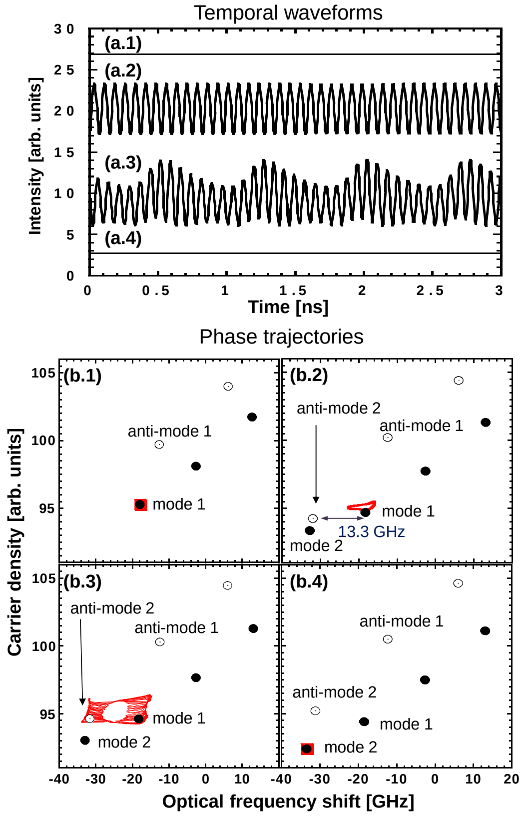


Figure 4: Simulated scenario of self-pulsing dynamics. $J/J_{th}$=1.5. Temporal waveforms (vertically shifted for clarity) and the corresponding phase trajectories in the carrier density versus optical frequency shift space are presented as the feedback strength $R$ is increased: (a-b.1) $R$=0.35, (a-b.2) $R$=0.42, (a-b.3) $R$=0.43 and (a-b.4) $R$=0.46. In the phase trajectories, the external cavity modes (anti-modes) are represented by dots (circles).

The simulated dynamical scenario is shown in Fig. 4. Modes (black dots) and anti-modes (circles) correspond respectively to stable and unstable solutions of steady intensity in the Lang-Kobayashi equations and are the the basis of the phenomenon of mode/anti-mode beating. The RF spectra corresponding to the self-pulsation and quasi-periodicity are given in Fig. 3. The sequence of qualitatively different dynamics starts with a first steady state (Fig. 4.(a-b.1)). When increasing the feedback strength, the system undergoes a bifurcation leading to a self-pulsing dynamics (Fig. 4.(a-b.2)). Self-pulsation is explained by a beating between a given mode (mode 1) and the anti-mode belonging to the next pair of mode/anti-mode (anti-mode 2). The frequency difference between the beating mode and anti-mode determines the self-pulsing frequency ($f_{SP}$). In the example of Fig. 4.(b-2), the frequency separation between mode 1 and anti-mode 2 equals 13.3 GHz. As a consequence, the peak corresponding to the self-pulsing dynamics is located at 13.3 GHz in Fig. 3.(c.2). Then if the feedback strength $R$ is increased again, this limit cycle bifurcates to a more complex attractor that involves trajectories close to both the current mode (mode 1) and the anti-mode facing it (anti-mode 2). This results in quasi-periodicity with the appearance of two incommensurate frequencies (Fig. 3.(c.3)). The fast dynamics is represented by the peak at 13.2 GHz and is still governed by the phenomenon of beating between the mode and the anti-mode. It is represented in the temporal waveform in Fig. 4.(a.3) by the fast oscillations. The slow dynamics (slowly-varying envelope) is represented by the peak at 1.34 GHz in Fig. 3.(c.3). It corresponds to the speed at which the phase trajectory switches between the two attractors located at the vicinities of mode 1 and anti-mode 2 (Fig. 4.(b.3)). Eventually, if the feedback strength is further increased, those two attractors end up by collapsing and the laser gets stabilized to a new steady state (mode 2 in Fig. 4.(b.4)). Now, if the feedback strength further increased, the same three-step-scenario will be reproduced, starting from the current mode and revealing successive self-pulsation and quasi-periodicity with attractors migrating towards the maximum gain mode as new pairs of modes and anti-modes are created. This is illustrated in Fig. 5 with a numerical bifurcation diagram showing regions of self-pulsing states and quasi-periodicity of increasing amplitude. The scenario illustrated in Fig. 4 corresponds to region between $R$=0.35 and $R$=0.46.

Since we understand that the frequency spacing between external cavity modes increases with the feedback strength, the self-pulsing frequency is expected to evolve alike.

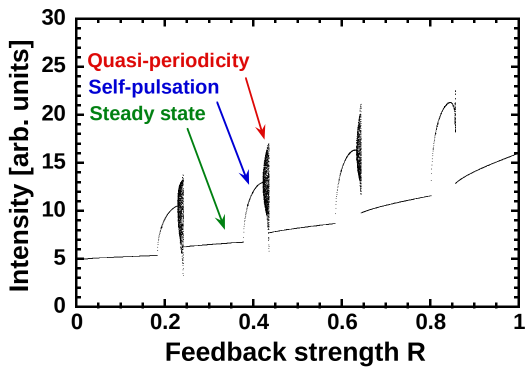


Figure 5: Simulated bifurcation diagram showing successive cycles of [steady state, self-pulsation, quasi-periodicity] for $J/J_{th}$= 1.5.

Figure 6 presents the experimental and numerical evolution of the self-pulsing frequencies ($f_{SP}$) as the feedback strength increases. The curves show a general increase of the frequency which is first rapid but gradually slows down. We understand from Fig. 6 that this increasing evolution is limited by the value of $f_{cav}$ which acts as an asymptote to the curves. As a consequence, for a given external cavity length, the self-pulsing states can be seen pulsing at frequencies up to the value of $f_{cav}$ without reaching it, though. We noticed both in the experiment and with

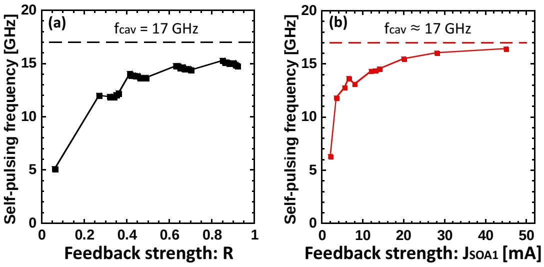


Figure 6: Simulated (a) and experimental (b) evolutions of the self-pulsing frequency as the feedback strength increases when $J/J_{th}$=2.0.

numerical analysis that for all values of the injection current, the self-pulsing frequency increases with the feedback strength. We find that, for given values of the feedback strength but for different values of the injection current, there is a discrepancy in the values of $f_{SP}$. However this discrepancy shows the following interesting tendency: it is large when the feedback is weak and gets smaller as the feedback strength increases. This implies that the feedback strength has an influence on the dependence of $f_{SP}$ on $J/J_{th}$ and $L_{cav}$. We give the interpretation that, according to the feedback strength, the relative influences of the relaxation oscillation frequency $f_{RO}$ and the external cavity frequency $f_{cav}$ on $f_{SP}$ vary. This idea is suggested by the fact that in all our observations $f_{SP}$ describes an evolution within frequency intervals bounded by $f_{RO}$ and $f_{cav}$. Therefore we see the phenomenon of self-pulsation as a pulsing dynamics dominated by the relaxation oscillation frequency at weak feedback and by the external cavity frequency at larger feedback strengths.

## 4. Conclusion

We presented an experimental and numerical analysis of self-pulsation in the case of a semiconductor laser embedded in a photonic integrated circuit. We reported on the particular dynamical scenario of cycles made of steady state, self-pulsation and quasi-periodicity undergone when the feedback strength increases. Simulations based on the Lang-Kobayashi equations gave insight into the mechanism of self-pulsation through mode/anti-mode beating. We also evidenced the fact that the pulsing frequency is a consequence of an interplay between the frequencies of both the relaxation oscillations and the external cavity.

## Acknowledgments

The authors acknowledge support from Grants-in-Aid for Scientific Research from Japan Society for the Promotion of Science and Management Expenses Grants from the Ministry of Education, Culture, Sports, Science and Technology in Japan. M.S acknowledges the financial support of Fondation Supélec, Conseil Régional de Lorraine, Agence Nationale de la Recherche (ANR) through the project TINO (grant number ANR-12-JS03-005), FEDER through the project PHOTON, and the Inter-University Attraction Pole (IAP) program of BELSPO through the project "Photonics@be", grant number IAP P7135.

## References

[1] M. C. Soriano, J. García-Ojalvo, C. R. Mirasso and I. Fischer, "Complex photonics: Dynamics and applications of delay-coupled semiconductors lasers," *Reviews of Modern Physics*, vol.85, pp.421–470, 2013.

[2] A. Uchida, "Optical Communication with Chaotic Lasers: Applications of Nonlinear Dynamics and Synchronization," *Wiley-VCH*, 2012.

[3] A. Argyris, M. Hamacher, K. E. Chlouverakis, A. Bogris and D. Syvridis, "Photonic integrated device for chaos applications in communications," *Physical Review Letters*, vol.100, pp.194101, 2008.

[4] A. Argyris, E. Grivas, M. Hamacherd A. Bogris and D. Syvridis, "Chaos-on-a-chip secures data transmission in optical fiber links," *Optics Express*, vol.18, pp.5188–5198, 2010.

[5] T. Harayama, S. Sunada, K. Yoshimura, P. Davis, K. Tsuzuki and A. Uchida, "Fast nondeterministic random-bit generation using on-chip chaos lasers," *Physical Review A*, vol.83, pp.031803, 2011.

[6] S. Sunada, T. Harayama, K. Arai, K. Yoshimura, P. Davis, K. Tsuzuki and A. Uchida, "Chaos laser chips with delayed optical feedback using a passive ring waveguide," *Optics express*, vol.19, pp.5713–5724, 2011.

[7] T. Heil, I. Fischer, W. Elsäßer and A. Gavrielides, "Dynamics of semiconductor lasers subject to delayed optical feedback: The short cavity regime," *Physical Review Letters*, vol.87, pp.243901, 2001.

[8] A. Tager and K. Petermann, "High-frequency oscillations and self-mode locking in short external-cavity laser diodes," *IEEE Journal of Quantum Electronics*, vol.30, pp.1553–1561, 1994.

[9] O. Ushakov, S. Bauer, O. Brox, H.-J. Wünsche and F. Henneberger, "Self-organization in semiconductor lasers with ultrashort optical feedback," *Physical Review Letters*, vol.92, pp.043902, 2004.

[10] O. Brox, S. Bauer, M. Radziunas, M. Wolfrum, J. Sieber, J. Kreissl, B. Sartorius and H.-J. Wünsche, "High-frequency pulsations in DFB lasers with amplified feedback," *IEEE Journal of Quantum Electronics*, vol.39, pp.1381–1387, 2003.