# Effect of heterogeneity for the self-organization of a neural network

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**Abstract**—In this paper, a self-organized neural network consisting of neurons with heterogeneous dynamics is proposed. The heterogeneity is introduced into the network by choosing the key parameter from a uniform distribution covering a wide variety of neuronal behavior. In particular, the synaptic matrix evolves according to the spike-timing dependent plasticity (STDP) mechanism and finally leads to synchronous spiking and a sparse connection. We argue that the self-emergent topology with active individuals having strong out-degree synapses essentially reflects the competition of different neurons and encodes the heterogeneity. And in order to test the efficiency of this self-organized network in signal processing, we have made comparisons to three other networks of different topologies in terms of coherence resonance (CR) and stochastic resonance (SR), which have been analyzed in various neural networks recently. It is shown that the network obtained from STDP learning can enhance the CR and SR of the entire network, indicating its high efficiency in information processing.

## 1. Introduction

In computational neuroscience, neural networks of various topologies, such as globally coupled networks [1], small-world networks [2] and scale-free networks [3], have been investigated for the performance in associative memory, stochastic resonance and so on. Specifically, instead of a prior imposition of a specific topology, it is more reasonable to consider self-organized neural networks, which have been broadly studied in [4, 5, 6, 7]. The self-organization is usually managed through spike-timing dependent plasticity (STDP), which is a form of long-term synaptic plasticity both experimentally observed [8] and theoretically studied [9, 10]. According to the Hebbian Rule [11] synapses increase their efficacy if the synapse persistently causes the postsynaptic target neuron to generate action potentials. While recent experiments shows that synapses increase their efficacy if the presynaptic neuron is activated momentarily before the postsynaptic neuron is activated. Or synapses in which the pre-synaptic input fired before the postsynaptic cell get stronger; in the inverse situation, the synapse gets weaker. Recent findings of STDP have triggered the interest in the potential roles of spike timing in processing and storage of information in neural circuits.

However, most network models in previous work did not take into account the heterogeneity of neurons that ubiquitously exist in real neural networks. For example, neurons located near the *canard region* exhibit complex behaviors in the presence of noise [12, 13, 14], where they are more sensitive to external signals and thus enhance information transfer in biological systems. Neurons having different dynamical activities will lead to the network heterogeneity, which can trigger competitions between individuals and play an important role in the coherence resonance [15] and phase synchronization [16]. In fact, the evolution of the synaptic connectivity or the network structure is closely related to the intrinsic heterogeneous dynamics of neurons.

In this paper we derive the connection of our networks through STDP over a set of heterogenous neurons. The heterogeneity is introduced into the network by choosing the key parameter from a uniform distribution covering a wide variety of neuronal behavior. After the reorganization, the active cells tend to have high out-degree synapses and low in-degree synapses, while the inactive ones are just the opposite. This self-emergent topology essentially reflects the relationships of influence and dependence among the heterogeneous neurons and thus achieves energy consumption. In order to test the efficiency of this self-organized network in signal processing, we have made comparisons to three other networks of different topologies in terms of coherence resonance (CR) and stochastic resonance (SR), which have been analyzed in various neural networks recently [14, 15, 17]. We show that the network obtained from STDP learning achieves a higher efficiency in information transfer.

## 2. Description of the Network Model

The network used in this paper is composed of $N$ FitzHugh- Nagumo (FHN) neuron models [18] described by

$$\begin{cases} \varepsilon \dot{V}_i = V_i - V_i^3/3 - W_i + I_{ex} + I_i^{syn} \\ \dot{W}_i = V_i + a - b_i W_i + D\xi_i \\ I_i^{syn} = - \sum_{1(j \neq i)}^{N} g_{ij} s_j (V_i - V_{syn}) \end{cases} \quad (1)$$

where $i = 1, 2, ..., N$. $a, b_i$, and $\varepsilon$ are dimensionless parameters with $\varepsilon$ small enough ($\varepsilon \ll 1$) to make membrane potential $V_i$ a fast variable, compared to the slow recovery variable $W_i$. $\xi_i$ is the independent Gaussian noise with zero mean and intensity $D$ that represents the noisy background,

and $I_{ex}$ stands for the externally applied current. $I_i^{syn}$ is the total synaptic current through neuron $i$. The synaptic variable $s_j$ is governed by $\dot{s}_j = \alpha(V_j)(1 - s_j) - \beta s_j$, where $\alpha(V_j) = \alpha_0/(1 + e^{-V_j/V_{shp}})$. Here the synaptic recovery function $\alpha(V_j)$ can be taken as the Heaviside function. When the presynaptic cell is in the silent state $V_j < 0$, $s_j$ can be reduced to $\dot{s}_j = -\beta s_j$; otherwise $s_j$ jumps quickly to 1 and acts on the postsynaptic cells. The synaptic conductance $g_{ij}$ from the $j_{th}$ neuron to the $i_{th}$ neuron will be updated through STDP shown later. Note that in this paper both the excitatory and inhibitory synapses are considered. The type of synapse is determined by the synaptic reversal potential $V_{syn}$, which we set to be 0 and $-2$ for excitatory and inhibitory synapse, respectively.

In this model, $b$ is a critical parameter that can significantly influence the dynamics of the system. For a single neuron free from noise, Andronov-Hopf bifurcation occurs at $b_0 = 0.45$. For $b > b_0$, the neuron is at the rest state and is excitable; for $b < b_0$, the system has a stable periodic solution generating periodic spikes. Between these two states, there exists an intermediate behavior, known as canard solution. In a small vicinity of $b = b_0$, there are small oscillations near the fixed point before the sudden elevation of the oscillatory amplitude. In our system, $b_i$ is uniformly distributed in $[0.45, 0.75]$. Hence each neuron when uncoupled has different activity when subject to external input and noisy background, and neurons with $b$ located near the bifurcation point are prone to fire in a much higher frequency than the others.

According to the experimental report on STDP [8], there is no obvious modifications of excitatory synapses onto inhibitory postsynaptic cells after their repetitive and relative activities. Hence, we set inhibitory synaptic conductance and excitatory-to-inhibitory synaptic conductance to be a constant. The remaining excitatory synapses are updated by the STDP modification function $F$, which selectively strengthen the pre-to-post synapses with relatively shorter latencies or stronger mutual correlations, while weakening the remaining synapses [4]. The synaptic conductance is updated by

$$\Delta g_{ij} = g_{ij}F(\Delta t) \qquad (2)$$

$$F(\Delta t) = \begin{cases} A_+ \exp(-\Delta t/\tau_+) & \text{if } \Delta t > 0 \\ -A_- \exp(\Delta t/\tau_-) & \text{if } \Delta t < 0 \end{cases} \qquad (3)$$

where $\Delta t = t_i - t_j$, $F(\Delta t) = 0$ if $\Delta t = 0$. $\tau_+$ and $\tau_-$ determine the temporal window for synaptic modification. $A_+$ and $A_-$ determine the maximum amounts of synaptic modification. Experimental results suggest that $A_- \tau_- > A_+ \tau_+$ which ensures the overall weakening of synapses. Here, we set $\tau_- = \tau_+ = 2$, $A_+ = 0.05$ and $A_-/A_+ = 1.05$ as used in [4]. Only the excitatory-to-excitatory synapses are modified by this learning rule and are restricted to the range $[0, g_{max}]$, where $g_{max}$ is the limiting value. Other parameters used in this paper are $a = 0.7, \varepsilon = 0.08, \alpha_0 = 2, \beta = 1, V_{shp} = 0.05, g_{max} = 0.1$. The other parameters are given


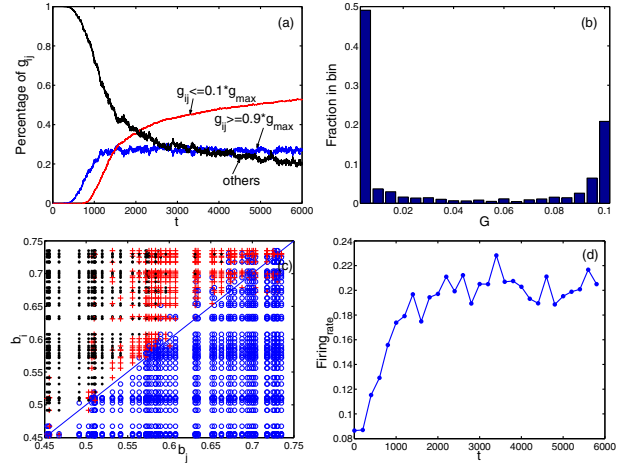
Figure 1: Evolution of the network structure.(a) Percentage of three levels of synapses: $g_{ij} \geq 0.9g_{max}$, $g_{ij} \leq 0.1g_{max}$ and the others; (b) Histogram of the synaptic matrix $G$; (c) Distribution of the synaptic matrix. Synapses $g_{ij}$ from cell $j$ to cell $i$ with $b_j$ and $b_i$ respectively are plotted. The black dots are the strong synapses satisfying $g_{ij} \geq 0.9g_{max}$, the blue circles are the weak synapses satisfying $g_{ij} \leq 0.1g_{max}$ and the red plus signs are intermediate values of synapses. (d) The average firing rate of all cells.

in each case. Numerical integrations of the system is done by the explicit Euler-Maruyama algorithm [19], with a time step 0.005.

## 3. Main Results

We consider a network of $N = 60$, which consists of 50 excitatory and 10 inhibitory neurons. All the neurons are bidirectionally and globally coupled at the beginning, and we assign $g_{max}/2$ and $3g_{max}/2$ to excitatory and inhibitory synapses, respectively. The whole network is subject to an external current ($I_{ex} = 0.1$) and noisy background ($D = 0.06$) as a learning environment.

We now check how the network structure evolves during the learning process. As is shown in Fig. 1, after competition, most of the synaptic connections converge to either 0 or the maximum $g_{max}$ from the initial value $g_{max}/2$ (see Fig. 1(a)). This structure becomes stable after about $6000s$. The synaptic connection finally becomes sparse with about 50% being 0 and 20% being $g_{max}$ (Fig. 1 (b)). Fig. 1 (c) gives a clear picture of the active-neuron-dominant synaptic connections in this network, where strong connections are mainly distributed to the synapses from active neurons (those with small values of $b_i$) to inactive ones (those with large values of $b_i$). Actually, the firing rate of the whole network plateaus after about $1500s$ when the number of synapses with $g_{ij} \geq 0.9g_{max}$ equals to that of the synapses with $g_{ij} \leq 0.1g_{max}$ (Fig. 1(d)). So the following update of the synapses is in fact an optimizing procedure that further

weakens those unnecessary connections.

The reason for generating such a special structure is that, under the same learning environment, active neurons can fire with a high frequency and thus are more likely to act as the pre-cells whose out-degree and in-degree synapses are then strengthened and weakened by STDP respectively. Such synapse distribution renders the active cells a powerful drive to the inactive ones. Hence, we can see that through the STDP learning process, the sensitive dynamics of those active neurons are fully exploited to trigger the whole network to fire synchronously, which becomes more active and sensitive than the original network. It should be noted that when the driving of external applied signal is removed, the sustained synchronous firing after learning will terminate and the whole network returns the normal rest state.

Here if the initial excitatory synapses are set to be $g_{max}$ or randomly distributed in $[0, g_{max}]$, similar results can be obtained but need longer convergence time. And to ensure that our results do not depend on the specific realization of the uniform distribution of parameter $b_i$ among neurons, we have performed the learning process over several different realizations, and find no significant changes of the final network topology.

In the following part, we investigate the efficiency of the self-organized network (SON) obtained via STDP in signal processing by comparing its performance on CR and SR with three other networks, i.e., the network with the same synaptic distribution as SON but shuffled (RNS), the random network with synapses uniformly distributed in $[0, g_{max}]$ (RNG), and the globally coupled network with constant synapses $g_{max}/2$ (CN). All these four types of network are composed of heterogeneous cells that are bidirectionally coupled and have the same mean value of synapses being about $g_{max}/2$. Ten trials are conducted for each of them.

Coherence Resonance (CR) is a noise-induced effect which describes the occurrence and optimization of periodic oscillatory behavior due to noise perturbations [12]. With an intermediate noise intensity, system can behave the most regular periodic oscillations. We take $S$ and $T_{mean}$ as the coherence factors of the firing events. They are defined as $S = \frac{1}{N} \sum_{i=1}^{N} S_i$, where $S_i = \langle T_k^i \rangle_t / \sqrt{Var(T_k^i)}$. $T_{mean} = \frac{1}{N} \sum_{i=1}^{N} \langle T_k^i \rangle_t$. $T_k^i = \tau_{k+1}^i - \tau_k^i$ is the pulse internal, where $\tau_k^i$ is the time of the $k_{th}$ firing of the $i_{th}$ cell. $\langle \cdot \rangle_t$ denotes average over time. $S$ describes the degree of spiking regularity in neural systems. $T_{mean}$ is the average inter-spike interval (ISI). Here, $I_{ex} = 0$ and all the cells are in subthreshold region in the absence of noise. Fig. 2 (a) shows that the optimal regularity occurs when noise intensity $D$ equals about 0.08. The corresponding $S$ in SON is much larger than the other networks, indicating the high coherent output of SON. The flat curve of $T_{mean}$ near the optimal case (see Fig. 2 (b)) reflects that the regular inter-spike intervals in SON can exist in a relatively wide range
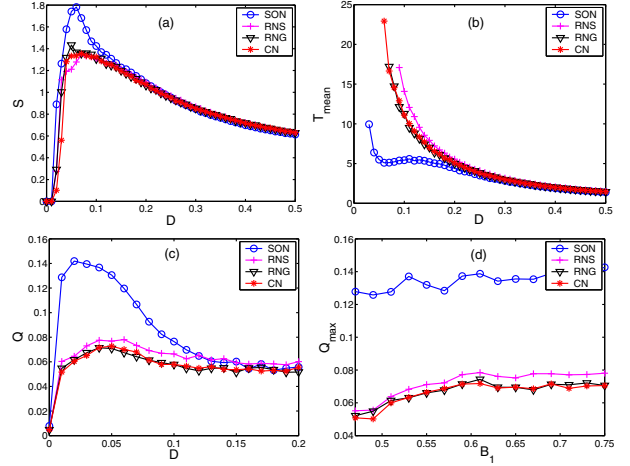


Figure 2: Comparisons of four types of neural networks on CR (a) (b) and SR (c) (d). (a) (b) $S$ and $T_{mean}$ versus noise intensity $D$ respectively; (c) $Q$ versus noise intensity $D$, where $B_1 = 0.75$; (d) The influence of inactive cells on SR. $Q_{max}$ is the maximum of $Q$. Only cells with parameter $b_i \in [0.45, B_1]$ are subject to external signal.

of noise intensity. While due to the inefficient connectivity, the other networks display unsynchronized and inactive activities, causing the small $S$ and large $T_{mean}$ (ISI). This is because, under the driving of the same noise intensity, neurons with different levels of excitability show diverse firing patterns. Only the self-organized network which has a reasonably selected synapse distribution can couple the neurons efficiently and generate regular spiking.

Stochastic resonance (SR) describes the cooperative effect between a weak signal and noise in a nonlinear system, leading to an enhanced response to the periodic force. The neuron model is an excitable system which can potentially exhibit SR [20]. To evaluate SR, we set the periodic input to be $I_{ex} = B \sin(\omega t)$, with $B = 0.1$ and $\omega = 0.3$. The amplitude of the input signal is small enough to ensure that there are no spiking for all the neurons in the absence of noise. Also, the frequency $\omega$ is much slower than that of neuron's inherent periodic spiking.

The Fourier coefficient $Q$ is used to evaluate the response of output frequency to the input frequency. It is defined as [21]: $Q = \sqrt{Q_{sin}^2 + Q_{cos}^2}$, where $Q_{sin} = \frac{\omega}{2\pi n} \int_0^{2\pi n/\omega} 2V_i(t) \sin(\omega t) dt$, and $Q_{cos} = \frac{\omega}{2\pi n} \int_0^{2\pi n/\omega} 2V_i(t) \cos(\omega t) dt$. $n$ is the number of periods $2\pi/\omega$ covered by the integration time. The quantity $Q$ measures the component from the Fourier spectrum at the signal frequency $\omega$. The maximum of $Q$ shows the best phase synchronization between input signal and output firing. Again, SON exhibits greater SR than the other cases (Fig. 2 (c)(d)). In the three other networks which have inefficient connections, active cells fire much more frequently than the periodic driven signal

while the inactive ones may be even at the rest state. The active-cell-dominant connection in SON regulates well the network activity and eventually achieves a balanced energy distribution among neurons. Moreover, in order to investigate the importance of active cells, only cells with $b_i \in [0.45, B_1]$, where $0.47 \le B_1 \le 0.75$, are subject to the periodic input. From Fig. 2 (d) we can see that whether the inactive cells are subject to external signal or not has little effect on SR. This indicates that the contributions of inactive cells to SR are negligible. While the active cells are critical and play a vital role to trigger the whole network response with external signal.

## 4. Conclusion

In this paper, a new type of self-organized neural network with heterogeneous neurons is obtained via STDP learning. Different neurons' internal dynamics are clearly encoded in the network structure after learning. In the STDP learning process, the synaptic strengths of the network are renewed by increasing the influence of active cells over the others and the dependence of inactive cells on the active cells. It makes the most of the internal dynamical properties of different neurons to synchronize and renders the whole network more sensitive to weak input. This effect is strongly reflected by its good performance on CR and SR. Therefore, we believe that this self-organized heterogeneous neural network is much efficient for signal processing.

## References

[1] DH Zanette, AS Mikhailov, "Condensation in globally coupled populations of chaotic dynamical systems," *Phys. Rev. E*, 58, 872, 1998.

[2] LG Morelli, *et al.*, "Associative memory on a small-world neural network ," *Eur. Phys. J. B*, 38, 2004.

[3] D Stauffer,*et al.*, "Efficient Hopfield pattern recognition on a scale-free neural network," *Eur. Phys. J. B*, 2003.

[4] S Song, KD Miller, LF Abbott, "Competitive Hebbian learning through spike-timing-dependent synaptic plasticity," *Nature neurosci.*, Vol. 3, No. 9, 2000.

[5] CW Shin, S Kim, "Self-organized criticality and scale-free properties in emergent functional neural networks," *Phys. Rev. E*, 74, 045101(R), 2006.

[6] EM Izhikevich, JA Gally, GM Edelman, "Spike-timing dynamics of neuronal groups," *Cerebral Cortex*, Oxford Univ Press, 2004.

[7] S Kang, K Kitano, T Fukai, "Self-organized two-state membrane potential transitions in a network of realistically modeled cortical neurons," *Neural Networks*, 17, 2004.

[8] Guo-qiang Bi, Mu-ming Poo, "Synaptic modifications in cultured hippocampal neurons: dependence on spike timing, synaptic strength, and postsynaptic cell type," *J. Neurosci.*, 18 (24), 1998.

[9] Y Dan, M Poo, "Spike timing-dependent plasticity: from synapse to perception," *Physiol. Rev.*, 86, 2006.

[10] P Roberts, C Bell, "Spike timing dependent synaptic plasticity in biological systems," *Biol. Cybern.*, 87, 2002.

[11] Hebb, R., "The organization of behavior", *Wiley, New York*, 1949.

[12] E Ullner, "Noise-induced phenomena of signal transmission in excitable neural models," *Dissertation*, 2004.

[13] VA. Makarov, VI Nekorkin, MG Velarde, "Spiking behavior in a noise-driven system combining oscillatory and wxcitatory properties," *Phys. Rev. Lett.*, 86, 3431, 2001.

[14] X Li, J Wang, W Hu, "Effects of chemical synapses on the enhancement of signal propagation in coupled neurons near the canard regime," *Phys. Rev. E*, 76, 041902, 2007.

[15] C Zhou, J Kurths, B Hu, "Array-Enhanced Coherence Resonance: Nontrivial Effects of Heterogeneity and Spatial Independence of Noise," *Phys. Rev. Lett.* 87, 098101, 2001.

[16] Y Tsubo, J Teramae, T Fukai, "Synchronization of excitatory neurons with strongly heterogeneous phase responses," *Phys. Rev. Lett.*, 99, 228101, 2007.

[17] Z Gao, B Hu, G Hu, "Stochastic resonance of small-world networks," *Phys. Rev. E 65*, 016209, 2001.

[18] R FitzHugh, "Impulses and physiological states in theoretical models of nerve membrane," *Biophys. J.*, 1, 445, 1961.

[19] DJ Higham, "An algorithmic introduction to numerical simulation of stochastic differential equations," *SIAM Rev.*, 43, 525, 2001.

[20] T Wellens, V Shatokhin, A Buchleitner, "Stochastic resonance," *Rep. Prog. Phys.*, 67, 45, 2004.

[21] L Gammaitoni, P Hänggi, P Jung, "Stochastic resonance," *F Marchesoni, Rev. Mod. Phys.*, 70, 1998.