2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

Emergence of Cooperators with Extortioners in Homogeneous Random Networks

Zhihai Rong†

†CompleX Lab, Web Sciences Center,University of Electronic Science and Technology of China, Chengdu 610054, China
Department of Electronic and Information Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong
Email: rongzhh@gmail.com

Abstract—In this paper, we investigate the influence of time scales to the evolution of extortioners with cooperators and defectors in homogeneous random networks. When letting strategies' lifetime is related to their fitness, the extortioners are easy to invade the clusters of defectors, and form stable relationship with cooperative neighbors. Therefore, introducing the time scale factor into game dynamics promotes the stable existence of extortioners and furthermore enhances the cooperation level in homogeneous random networks. With the network becomes denser, The frequency of cooperators will decrease monotonically, which is due to the fact that cooperators are easy to meet defectors and be exploited by them.

1. Introduction

Cooperation phenomenon is ubiquitous in both the nature and human society. How to understanding the emergence of cooperation under the assumption of individual's selfishness remains a riddle and attracts scientists from many different fields, who usually employ game theory as a theoretical framework [1]. The Prisoner Dilemma(PD) is one of the most famous game models which receives the most attention as a metaphor of cooperation between unrelated individuals. In the PD game, two players interact simultaneously with each other by choosing cooperation or defection as a strategy. A Cooperator will pay a cost c to let her opponent receive a benefit b, whereas a defector will pay nothing. Therefore, two cooperators can receive the reward $R = b - c$ and two defectors will obtain the punishment $P = 0$. When a cooperator meets a defector, the former will obtain the sucker's payoff $S = -c$ and the latter will get the temptation $T = b$. Under this conditions, it is always better to defect regardless of the opponent behavior, resulting in the outcome of mutual defection, although mutual cooperation yields the highest collective payoff.

Recently, Press and Dyson discovered a class of strategies called zero-determinant(ZD) strategies, which allow a player to enforce a linear relation between her own payoff and the opponent payoff unilaterally in the *PD* game [2]. A subset of ZD strategies called extortion strategies ensure the extortioner X can receive a payoff surplus exceeding the surplus of her co-player Y by a fix percentage. However, in the realm of evolutionary game, where players are set to update their own strategies by imitating the neighbors whose strategies perform better, extortion strategies spread rapidly and, as mutual extortion resulting in zero yield, and the evolution of extortioners in well-mixed populations are deeply investigated [3, 4, 5, 6, 7].

The networked reciprocity is an efficient mechanism to support the evolution of cooperation in population [8]. Previous investigations showed that cooperators can form tight clusters to defend the invasion of defectors in regular or complex networks if strategies update in terms of imitation dynamics [9, 10, 11]. Many important factors, such as degree heterogeneity [10, 11], individual aspiration [12], etc., play key influence on the evolution and maintenance of cooperation in networks. Szolnoki and Perc [13] recently showed that if the strategy updating is guided by the myopic best response rule, the extortion strategy can stably exist with other strategies in structured population, which furthermore promotes the emergence of cooperation. Through the aspiration-driven strategy updating rule [12], Wu and Rong [14] showed that the involvement of extortioners facilitates the boom of cooperators in the square lattice.

There are two time scales in game dynamics, i.e., the interaction time scale which depicts how frequently individuals play games with each other, and the strategy-selection time scale which characterizes how frequently they update their strategies. The two processes are interdependent, and many previous investigations consider that they have the same time scale, i.e., every individual immediately updates her behavior after one round of game. However, the evolution of cooperation changes if individuals in well-mixed or structured population own nonidentical time scales [15, 16]. Especially, by investigating the evolution of extortioners in well-mixed population, Hilbe *et al.* [4] showed that the extortion strategy can also exist in two distinct well-mixed populations if the two populations evolve in different time scales, i.e., extortioners can be dominant in the population with a slow time scale and exploit the individuals in another population with a fast time scale. Rong *et al.* [15, 16] also previously studied the coevolution of time scale and cooperation in networked *PD* game, and found that the cooperation can be promoted if permitting an individual with high payoff to hold onto her successful strategy for a longer time. This motivates us to investigate how the extortion strategy evolves in networked systems where individuals can adaptively adjust their strategy-selection time scales.

2. Models

Consider an individual X uses a memory-one strategy $\mathbf{p}^X = (p_R, p_S, p_T, p_P)$, where $\mathbf{p}_i$ is the conditional probability to cooperate after receiving the outcome $i \in (R, S, T, P)$ in the previous round. If an individual X adopts the zero-determinant strategy $\tilde{\mathbf{p}} = \mathbf{p}^X - \mathbf{e}_{12} = \phi[(\mathbf{S}_X - l\mathbf{1}) - \chi(\mathbf{S}_Y - l\mathbf{1})]$, she can enforces an linear relation between her long-term payoff A_X with her opponent Y's payoff A_Y, i.e., $A_X - l = \chi(A_Y - l)$, regardless of any strategy $\mathbf{p}^Y$ that Y adopted [5, 6]. Here, $\mathbf{S}_X = (R, S, T, P)(\mathbf{S}_Y = (R, T, S, P))$ is the payoff vector of individual X (Y), and the vector $\mathbf{e_{12}} = (1, 1, 0, 0)$ and $\mathbf{1} = (1, 1, 1, 1)$. The parameter $\phi > 0$ should be sufficiently small so that there exist the feasible strategies. The extortion factor $\chi = 1$ implies X lets she owns the same long-term payoff with her opponent's, which corresponds to the kind of fairness strategy. The famous Tif-for-Tat(TFT) strategy is an example of fairness strategy. $l \in (P, R)$ is the baseline payoff that implies the benefit of the two individuals if one adopts the fairness strategy with $\chi = 1$. If $l = P$ and $\chi > 1$, this is the extortion strategy (E_χ) where the individual X with such strategy can ensure that her own surplus is the χ-times of the Ys.

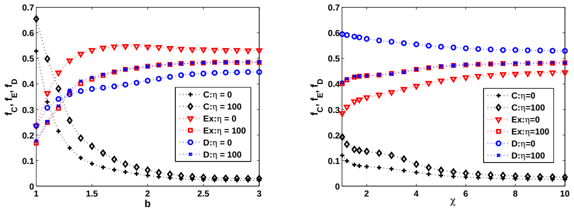
In this paper, we focus on the evolution of extortion strategy with unconditional cooperation (C) / unconditional defection (D) strategy as well as in the donation game (a kind of Prisoner's dilemma game). According to Ref. [4], the payoff matrix among extortion(E_χ), unconditional cooperation (C) and unconditional defection (D) strategies is:

$$\begin{array}{c|ccc} & E_\chi & C & D \\ \hline E_\chi & 0 & \frac{(b^2-c^2)\chi}{b\chi+c} & 0 \\ C & \frac{b^2-c^2}{b\chi+c} & b-c & -c \\ D & 0 & b & 0 \end{array}. \tag{1}$$

We consider each individual x locates on a node in a network, who plays the donation game with her immediate neighbors and obtain her accumulate payoff P_x in terms of Eq.(1). In social and biological systems, individuals tend to adopt the behavior with high fitness, which can be characterized by her payoff. For every round t, each individual i obtains the accumulated payoff P_i via playing the donation game with her neighbors. With probability $p_i(t)$, which will be defined later, an individual i will change her behavior from the current strategy s_i to another randomly selected strategy s_i' with probability q in terms of the myopic best response rule [13], i.e.,

$$q(s_i' \to s_i) = \frac{1}{1 + \exp[(f_i - f_i')/\kappa]}, \tag{2}$$

where the fitness f_i corresponding to strategy s_i is reaped by $f_i = P_i/k_i$, and k_i is the degree of individual i. And f_i' is the fitness of the same individual adopting strategy s_i' to play game within the same neighborhood. The parameter κ represents the noise of environment and is set as 0.05 following the previous paper [13].



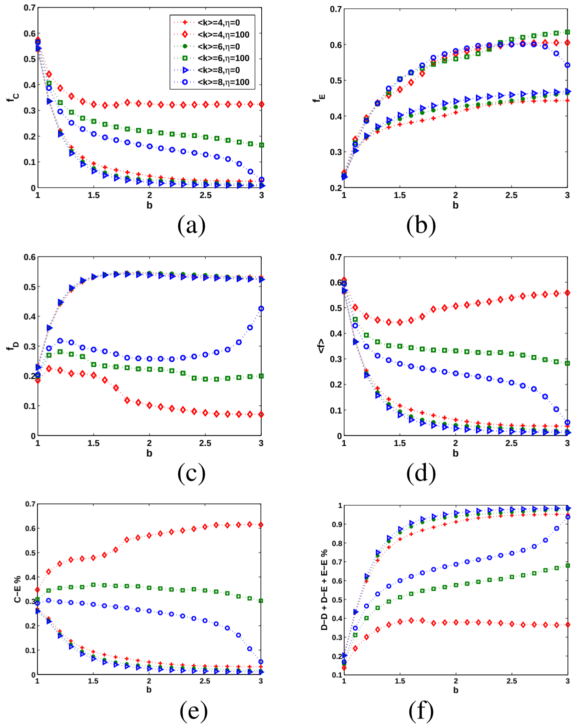
(a) (b)

Figure 1: Evolution of cooperation, extortion and defection strategies versus (a) the benefit factor b with $\chi = 5$ and (b) the extortion factor χ with b=1.2 in well-mixed population with N=10000.

In this paper, we consider the strategy-selection time scale to be longer than the interaction time scale, which indicates that individuals can hold onto their current strategies and play game with neighbors for several rounds before they modify their behaviors. This implies that the strategy has lifetime. From the social and biological points of view, the lifetime of a strategy is related to the fitness that an individual obtains through the strategy. If an individual owns positive fitness in the current generation, she tends to hold her current advantageous behavior for a longer time. Whereas, for an individual obtaining negative fitness in the current generation, she will try other possible behaviors. Therefore, in this paper we consider the case where an individual i updates her behavior with probability $p_i(t) = \frac{1}{1+\eta \max(0, f_i)}$. The time scale parameter $\eta \geq 0$ adjusts how long individuals update their behaviors. The case of $\eta = 0$ corresponds to original networked game model where individuals immediately update their strategies after one round of game. For $\eta > 0$, the behavior with higher fitness has longer lifetime. Below we will investigate how individuals evolve their strategies under the control of parameter η in different networked systems.

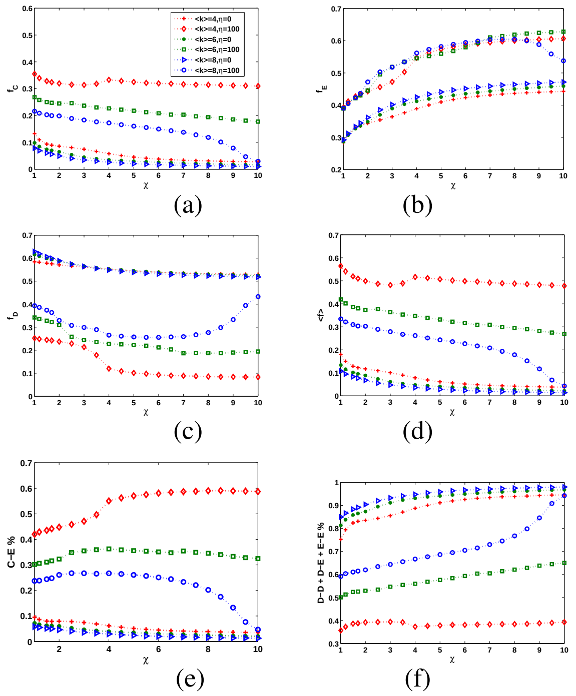
3. Results

At the beginning, let's study the evolution of extortion strategy in well-mixed population. For a well-mixed population (complete graph) when cooperators interact with defectors and extortioners together, the cooperation behavior will slightly decrease for a large value of η=100, which is replaced by extortioners and defectors (see Fig. 1). This is due to the fact that, when cooperators interact with defectors and extortioners in the well-mixed population, each individual can interact with all other individuals, and introducing the time scale factor lets defector or extortioner own long lifetime to exploit a neighboring cooperator. Hence, the time scale factor can not promote the emergence of cooperation in well-mixed population. Whereas, the situation may change when individuals interact in spatial networks.



(a) (b) (c) (d) (e) (f)

Figure 2: Evolution of (a) cooperation, (b) extortion, (c) defection, (d) fitness, (e) cooperator-extortioner pairs, (f) pairs that bring punishment versus the benefit factor b in the homogeneous random networks with $< k >$=4,6,8, respectively, and χ=5.



(a) (b) (c) (d) (e) (f)

Figure 3: Evolution of (a) cooperation, (b) extortion, (c) defection, (d) fitness, (e) cooperator-extortioner pairs, (f) pairs that bring punishment versus the extortion factor χ in the homogeneous random networks with $< k >$=4,6,8, respectively, and b=2.

Let's then consider the influence of the average degree on the evolution of extortion behavior in homogeneous random networks with different degrees. A homogeneous networks can be obtained by reshuffling edges of a nearest-neighbor network sufficient times so that the edges are randomly rewired without changing the degree of nodes in the original network [17].

Firstly we investigate the change of frequencies of strategies with a function of the benefit factor b. It is found from Fig. 2 that the evolution of three strategies is similar in three kind of homogeneous random networks for η=0, the frequencies of cooperation will decrease with the increase of b. Whereas, the increase of η promotes the frequencies of cooperators and extortioners in all the three kinds of homogeneous random networks. This is due to fact that in terms of Eq. (1), extortioners are neutral with defectors and they coexist in the network, whereas, the snowdrift-like relation between extortioners and cooperators makes the partner of extortioner more likely to turn to cooperator under the myopic best response rule, and extortioners can invade cooperative clusters. When introducing the time scale factor and increasing the parameter η, there are different results for defectors, extortioners and cooperators. A defector can obtain high payoff from her cooperative neighbors, but leave negative payoff as a return to them. As a consequence, those neighboring cooperators of defectors tend to adopt either the defection or the extortion strategy in the subsequent rounds, which in return diminishes the gains for defectors, hence leading to the short-term lifetime of the defection strategy. In contrast, those neighboring cooperators of extortioners are much more better off since they can obtain some tiny positive payoffs, irrespective of being extorted by them. Consequently, when the strategy's lifetime is related to her fitness, extortioners can form stable relationship with cooperators in a long term, which leads to the boom of both cooperators and extortioners in the network, and there exist lots of cooperator-extortioner pairs in network when η=100. The frequency of cooperators will decrease monotonically with the network becoming dense, which is due to the fact that cooperators are easy to meet defectors and be exploited by them in denser networks, which are validated by the decrease of cooperator-extortioner pairs and the increase of pairs that bring punishment in dense networks.

Then we turn to study the influence of the extortion factor χ to the evolution of extortioners and cooperators in the homogeneous random networks. The cooperators obtain less with the increase of χ, as exploited more by extortioners. It is shown from Fig. 3 that, for η=0, the frequency of cooperators monotonically decreases with the increase of χ, which is mostly replaced by extortioners. Whereas, for $\eta > 0$, the evolution of cooperation and extortion will become nontrivial, which can be understood through strategy pairs. Following the increase of η, there are more extortioners replacing defectors in the random network since extortioners can invade clusters of defectors and induce more cooperators around them. It is validated by Figs. 3(e) and 3(f) that there are more cooperator-extortioner pairs that replacing pairs that bring punishment, i.e., defector-defector pairs, defector-extortioner pairs and extortioner-extortioner pairs. Hence, the time scale factor can play nontrivial roles in the evolution of cooperation in the homogenous random networks. Following the increase of network density, the frequency of cooperators will descend since the cooperators are easy to be explored by defectors, which leads to the decrease of cooperator-extortioner pairs and the increase of pairs that bring punishment in dense networks.

4. Conclusion

In this paper, we studied the influence of time scales on the evolution of extortioners in random networks. We shows that if strategies' lifetime is related to their fitness, it is easy for the extortioners to invade the clusters of defectors, and form stable relationship with cooperative neighbors. Therefore, introducing the time scale factor into game dynamics promotes the stable existence of extortioners and furthermore enhances the cooperation level in networked systems. Particularly, different from the traditional networked game theory, where cooperators can form tight clusters to defend the invasion of defectors in *PD* game, the snowdrift-like relation between extortioner and cooperator. The cooperation level will decrease monotonically with the network becoming dense since cooperators are easy to be exploited by defectors in denser networks. The discovery of zero-determinant strategies is changing our viewpoint of game theory.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (No.61473060), the Research Foundation of UESTC and Hong Kong Scholars Program (No. XJ2013019 and G-YZ4D), and the Open Project Program of State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, China (No.Y5KF201CJ1).

References

[1] M. A. Nowak, "Five rules for the evolution of cooperation," *Science*, vol.314, pp.1560-1563, 2006.

[2] W. H. Press, F. J. Dyson, "Iterated Prisoners Dilemma contains strategies that dominate any evolutionary opponent," *Proc. Natl. Acad. Sci. USA*, vol.109, pp.10409-10413, 2012.

[3] C. Adami, A. Hintze, "Evolutionary instability of zero-determinant strategies demonstrates that winning is not everything," *Nature Communications*, vol.4, pp.1038/ncomms3193.

[4] C. Hilbe, M. A. Nowak, K. Sigmund, "Five rules for the evolution of cooperation," *Proc. Natl. Acad. Sci. USA*, vol.110, pp.6913–6918, 2013.

[5] A. J. Stewart, J. B. Plotkin, "From extortion to generosity, evolution in the Iterated Prisoners Dilemma," *Proc. Natl. Acad. Sci. USA*, vol.110, pp.15348-15353, 2013.

[6] C. Hilbe,M. A. Nowak, A. Traulsen, "Adaptive dynamics of extortion and compliance," *PLoS ONE*, vol.8, pp.e77886, 2013.

[7] A. J. Stewart, J. B. Plotkin, "Collapse of cooperation in evolving games," *Proc. Natl. Acad. Sci. USA*, vol.111, pp.17558-17563, 2014.

[8] G. Szabó, G. Fáth, "Evolutionary games on graphs," *Phys. Rep.*, vol.446, pp.97-216, 2007.

[9] M. A. Nowak, R. M. May, "Evolutionary games and spatial chaos," *Nature*, vol.359, pp.826-829, 1992.

[10] F. C. Santos, J. M. Pacheco, T. Lenaerts, "Evolutionary dynamics of social dilemmas in structured heterogeneous populations," *Proc. Natl. Acad. Sci. USA*, vol.103, pp.3490-3494, 2006.

[11] Z. Rong, X. Li, X. F. Wang, "Roles of mixing patterns in cooperation on a scale-free networked game," *Phys. Rev. E*, vol.76, pp.027101, 2007.

[12] X. J. Chen, L. Wang, "Promotion of cooperation induced by appropriate payoff aspirations in a small-world networked game," *Phys. Rev. E*, vol.77, pp.017103, 2008.

[13] A. Szolnoki, M. Perc, "Evolution of extortion in structured populations," *Phys. Rev. E*, vol.89, pp.022804, 2014.

[14] Z. Wu, Z. Rong, "Boosting cooperation by involving extortion in spatial prisoners dilemma games," *Phys. Rev. E*, vol.90, pp.062102, 2014.

[15] Z. Wu, Z. Rong, P. Holme, "Diversity of reproduction time scale promotes cooperation in spatial prisoner's dilemma games," *Phys. Rev. E*, vol.80, pp.036106, 2009.

[16] Z. Rong, Z. Wu, G. Chen,"Coevolution of strategy-selection time scale and cooperation in spatial prisoner's dilemma game," *EPL*, vol.102, pp.68005, 2013.

[17] S. Maslov,K. Sneppen,"Specificity and stability in topology of protein networks," *Science*, vol.296, pp.910-913, 2002.