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Evolutionary games of Cooperative Optimization in Unreliable MANETs

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Abstract—In this paper, we study the cooperative optimization in self-organized mobile ad hoc networks (MANETs) for the scenario where the number of interactions between any pair of players are finite. We propose an indirect reciprocity framework based on evolutionary game theory, and analyze the evolutionary dynamics of cooperative strategies to guarantee the convergence of cooperation. The numerical simulations illustrate the evolutionary stability against the perturbation effect.

1. Introduction

Self-organized mobile ad hoc networks (MANETs) is a network composed of elements that can dynamically adapt to varying network conditions to optimize end-to-end performance through learning and reasoning [1]. As a basic example, the nodes must make a mutual contribution to packet forwarding to ensure an operable network. However, since the nodes are usually constrained by limited computation resources, such as battery, memory and processing capacity, selfish nodes may refuse to be cooperative. As shown in the literature, such selfish behavior can dramatically degrade the performance of an entire system [2]. Therefore, a key problem in MANETs is how to encourage cooperative packet forwarding among selfish nodes.

In the literature, many approaches have been proposed to stimulate nodes cooperation towards common network services, which can be classified into two main categories. One approach is to use payment-based schemes to enforce cooperation [3, 4]. Another approach is to use reputation-based schemes to enforce cooperation [5, 6]. Recently, a considerable amount of efforts have been devoted with game theory to analyzing how cooperation can be enforced [7, 8, 9]. For example, in [10], Félegyházi et al. proposed a packet forwarding model in ad hoc networks based on game theory, and derived the conditions under which cooperation yields the Nash Equilibrium. Besides, in [11, 12], the authors applied game theory to analyze cooperation among selfish nodes, and focused on the updating of individual' interaction strategies based on the behaviors of others in order to maximize their benefits. Comprehensive review on this topic refer to Ref. [13, 14].

However, most of the existing game theoretical frameworks rely on the assumption that the game between a pair of players is directly played for infinite times [15, 16, 17]. In reality, due to mobility or changes of environment, nodes will periodically update their partners to achieve better performance, which means that any pair of players are supposed to play for only finite times with the termination time are either known or can be estimated by both players. Motivated by the aforementioned points, we propose an indirect reciprocity framework to enforce cooperation for the scenario in unreliable MANETs where the number of interactions between any pair of players are finite.

2. Indirect reciprocity game model

2.1. The basic model

Consider a self-organized MANET with sufficiently large population of nodes where each nodes belong to different authorities, see Figure 1. Due to the constraint of communication range, the source of service provider cannot reach the destination directly. At each time slot, a fraction of players are chosen from the population to form pairs to forward packets. Within each pair, one player acts as a provider, and the other player acts as a relay. During the process of packet forwarding, the relay will chooses his strategy, X, from the strategy set $\mathbb{A} = \{F, D\}$, where F and D are packet forwarding and dropping, respectively.

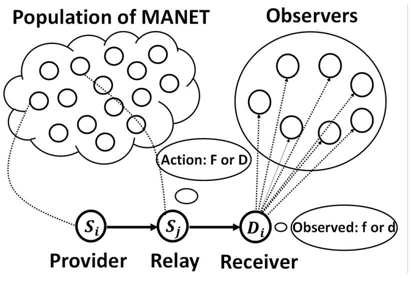

Figure 1: The illustration of system model.

If channels are reliable (loss free), the well-known Prisoner's Dilemma characterizes this scenario of packet forwarding [16, 17]. For each node, when the packets are successfully delivered to the receiver(the destination of the packet forwarding), the provider will get a payoff, denoted as b. Meanwhile, the forwarding effort of relay nodes will give rise to certain cost, denoted as c. Thus, the payoff matrix between F and D is expressed as:

$$\begin{array}{c} \\ \mathrm{F} \\ \mathrm{D} \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{F} & \mathrm{D} \end{array} \\ \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix} \end{array}. \tag{1}$$

However, imperfect observation usually exists in such MANETs due to channel noise. Although the nodes' strategies are hidden due to the channel, some traffic monitoring mechanisms are launched by each node to keep tracking of its neighbors' strategies [2, 5]. Consider that the receiver of each node observes a private signal of the opponent's strategy from the set $\Theta = \{f, d\}$, where f and d are the observations of packet forwarding and dropping, respectively. Since the node's observation is imperfect, the forwarding strategy F of one node may be observed as d by the other node due to link breakage or transmission errors. Denote such channel loss probability as p_e. For example, node S_j forwards the packet for S_i, but the forwarding strategy might fail due to the channel noise, thus the receiver D_i of S_i observes the signal of node S_j is f with probability $1 - p_e$, or d with probability p_e. If S_j drops the packet, the observed signal of node S_j from D_i is d, see Figure 1. In this case, the gain of a provider is b when the packets are successfully delivered to the destination with the probability $(1 - p_e)$, and the cost of a relay with forwarding strategy is c,

In this paper, we consider a binary reputation system, where each node is endowed with a binary reputation: good(G) and bad(B). At each time slot, the relay will forward or drop the packets of the provider to the receiver according to the provider's reputation. After the interaction, the relay's reputation will be updated based on the observed signal of receiver, while the reputation of the provider remains the same. In some cases, the traffic monitoring mechanism of reputation collection can be unreliable, leading occasionally to false reports [12]. Thus, the reputation system must be fault tolerant. In our model, this uncertainty is captured by parameter $\mu(0 \leq \mu \leq 1/2)$, i.e., with probability $1 - \mu$, an incorrect reputation is assigned; with probability μ, a correct reputation is assigned. Finally, the relay's reputation is propagated to the whole population from the receiver and the observers through a noisy gossip channel.

After the interactions, each participant goes back to the population with probability ω, or leaves the population with probability $1 - \omega$ never to return. Here, the parameter ω plays a role of a discounting factor of the future. In exchange for each player who leaves the population, a new individual enters with either a good or bad reputation according to the proportion of good and bad players in the current population. Therefore, the total population size remains unchanged.

2.2. Action rules

An action rule, $\tilde{\mathbb{A}}$, is an action table of the relay, which depends on the provider's reputation. Specifically, a player with $\tilde{X}$ ($\tilde{X} \in \tilde{\mathbb{A}}$) takes strategy $s(G)$ for a good provider, and strategy $s(B)$ for a bad one. Each of $s(G)$ and $s(B)$ can be either F or D. Thus, the action rule, $\tilde{\mathbb{A}}$, has $2^2 = 4$ possible elements: $\tilde{\mathbb{A}} = \{s(G)s(B)|FF, FD, DF, DD\}$. For example, FD means that taking strategy F towards a good provider and strategy D towards a bad one. In this paper, we only consider three of these strategies, i.e., FF, FD and DD, since strategy DF is illogical in practice.

2.3. Social norms

A social norm, Q, is a matrix used for updating the reputation of players. Each element Q_{ij} in the social norm stands for the reputation assigned to a relay who has taken the strategy i toward a provider whose reputation is j. Without loss of generality, we assume that all players in the population share the same norm. To simplify the analysis, we only consider the special case when there are two strategies of relay ($i = \{F, D\}$) and binary reputation of provider ($j = \{G, B\}$). However, the results can be extended to the case of multi-strategies and multi-reputation.

Based on the intuition that forwarding packets for the provider with good reputation or denying forwarding packets for the provider with bad reputation establishes a good reputation, and will be rewarded by others. Generally, a relay who takes the choice $X(X \in \{F, D\})$ towards a provider with reputation R ($R \in \{G, B\}$) will be assigned a new reputation $R^{'}(R, X)$ ($R^{'} \in \{G, B\}$). In this paper, we adopt the second-order social norms in the binary reputation model [18], i.e., the reputation of relay is updated according to the notation as "*GGBG*".

3. Evolutionary dynamics in unreliable MANETs

3.1. Replicator dynamic equation

During the forwarding process, players may take non-optimal action rule due to uncertainty of the system and/or the noisy parameters. Therefore, it is necessary to take the perturbation effect into account, which motivate us to evaluate the evolutionarily stability of cooperative strategies.

Denote x_1, x_2, and x_3 as the frequencies of strategy FF, FD, and DD, respectively. Then, we have $x_1 + x_2 + x_3 = 1$. Under the stationary reputation distribution $x_g^* = 1 - \mu$, the expected payoff of a strategy can be calculated. For a FF player, he has $\frac{1}{2}$ chance to be a relay, and cooperate with cost c. With $\frac{1}{2}$ chance being a provider, he meets a FF, FD and DD player with probability x_1, x_2 and x_3, and is expected to get the gain of $b(1 - p_e)$, $b(1 - p_e)(1 - \mu)$ and 0, respectively.

Similarly, we can obtain the gain and cost of FD and DD player, which results in the expected payoffs of actions FF, FD and DD as

$$\begin{cases} P_1 = \frac{1}{2}(-c) + \frac{1}{2}[b(1 - p_e)x_1 + b(1 - p_e)(1 - \mu)x_2] \\ P_2 = \frac{1}{2}(1 - \mu)(-c) + \frac{1}{2}[b(1 - p_e)x_1 + b(1 - p_e)(1 - \mu)x_2] \\ P_3 = \frac{1}{2}(0) + \frac{1}{2}[b(1 - p_e)x_1 + b(1 - p_e)(1 - \mu)x_2], \end{cases} \tag{2}$$

where P_1, P_2, and P_3 are the expected payoffs of strategy FF, FD, and DD, respectively.

As we known, replicator dynamic equation is widely used to characterize the population evolution in evolutionary game theory [19, 20, 21]. The innate character of the replicator dynamic equation is: if the expected payoff of action i is higher than the average level, then the growth of the population share using action i is proportional to the difference between the expected payoff of the action i and the entire population. Thus, the payoff of a action can be interpreted as its fitness, and actions with higher fitness have more chance to reproduce.

In the following, we use the replicator dynamic equation to model the evolution of frequency at Δt time interval, which means that the evolution of x_i $(i = 1, 2, 3)$ is given by the following equation

$$\begin{aligned} \Delta x_i &= [\omega a(x) + (1-\omega)b(x)]\Delta t - x_i \Delta t \\ &= \omega[\eta x_i(P_i - \bar{P}) - x_i]\Delta t, \end{aligned} \tag{3}$$

where $x = (x_1, x_2, x_3)^T$, η is a scale factor controlling the speed of the evolution, P_i is the expected payoff of player i, and $\bar{P} = \Sigma_{i=1}^{3} x_i P_i$ is the average payoff of three actions. Here, the first term $a(x) = \eta x_i(P_i - \bar{P})$ in Eq. (3) denotes the frequency variation which is caused by internal competition, which occurs with probability ω. And the second term $b(x) = x_i$ in Eq. (3) denotes the frequency variation which is caused by the external mobility, which happens with probability $(1-\omega)$.

Define $\hat{P}_i = P_i - P_3$, and $\tilde{P} = \Sigma_{i=1}^{3} x_i \hat{P}_i$. Then, we get the transformed deterministic dynamical evolution of frequency as

$$\begin{cases} \dot{x}_1 &= \omega[\eta x_1(\hat{P}_1 - \tilde{P}) - x_1] \\ &= \omega[(-c\eta - 1)x_1 + c\eta x_1^2] \\ &+\omega\eta \frac{[(1-2\mu)b(1-p_e)+c]x_1 x_2 - (1-2\mu)b(1-p_e)x_1 x_2(x_1+x_2)}{2-\frac{1-2\mu}{1-\mu}(x_1+x_2)} \\ \dot{x}_2 &= \omega[\eta x_2(\hat{P}_2 - \tilde{P}) - x_2] \\ &= \omega(c\eta x_1 x_2 - x_2) \\ &+\omega\eta \frac{-cx_2+[(1-2\mu)b(1-p_e)+c]x_2^2 - (1-2\mu)b(1-p_e)x_2^2(x_1+x_2)}{2-\frac{1-2\mu}{1-\mu}(x_1+x_2)} \end{cases} \tag{4}$$

Note that Eq. (4) is defined on simplex $S_3 = \{(x_1, x_2, x_3) | x_1 + x_2 + x_3 = 1, x_i \geq 0\}$, each corner of the simplex is an equilibrium of the dynamics corresponding to a monomorphic state. Therefore, we can investigate the stability of Eq. (4) to characterize the evolutionarily stability of actions.

3.2. Numerical Results

In this subsection, we illustrate the phase portrait of Eq. (4) with different parameters, where strategies FD and DD are evolutionary stable, while strategy FF is unstable. As shown in Fig.2, we know that decreasing the probability of transmission error p_e and reputation updating error μ (or increasing benefit b) will enlarge the attraction basin of FD-type CESS, i.e., it is easier for cooperation thrives when p_e, μ are small and b is large. Here, the attraction basin of a strategy are the sets of all initial strategy distributions in a feasible domain that converge to the CESS. Therefore, given appropriate system parameters to satisfy the conditions of Theorem 1 and the strategy distributions in the attraction basin of FD-type CESS, cooperation of packet forwarding in MANETs can be enforced with the indirect reciprocity mechanism.

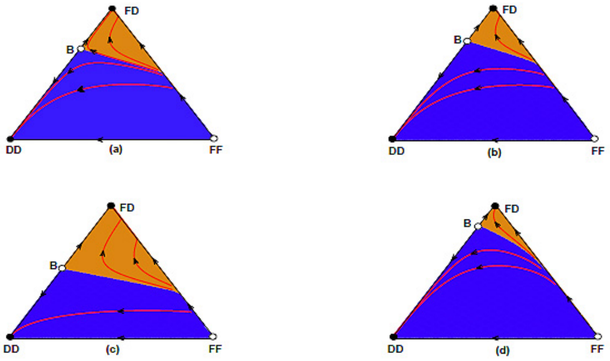

Figure 2: Phase portrait of Eq. (4). Each vertex represents a state with players taking the same strategy. The upper yellow part is the attraction basin of FD-type CESS, and the lower light blue part is the attraction basin of DD-type CESS. We set the system parameters as $\beta = 10$, $\omega = 0.8$, $\eta = 0.5$ and (a) $b = 3$, $c = 2$, $\mu = 0.01$, $p_e = 0.01$; (b) $b = 3$, $c = 2$, $\mu = 0.01$, $p_e = 0.08$; (c) $b = 4$, $c = 2$, $\mu = 0.01$, $p_e = 0.01$; (d) $b = 3$, $c = 2$, $\mu = 0.1$, $p_e = 0.01$.

4. Conclusion and Discussion

Game theory has been applied to analyze an integrated model of transmission losses, buffer overflows, packet forwarding and routing information dissemination in self-organized MANETs. In this paper, we start the analysis of the packet forwarding problem by considering a simpler game between two nodes that take turns to send their packets, in such a way that each node requires the retransmission services of the other, as shown in Fig. 1. Although this two-node scenario is a simplified model, we build an analytically tractable, non-cooperative game with incomplete information, the Forwarding Dilemma (FD). The analysis method we devised show its superiority over the classical prisoner dilemma of reputation model of MANETs, due to the evolutionarily stable strategies based on indirect reciprocity is effective and robust against packet loss and imperfect estimation of reputation. Besides, our analysis method shed light on the study of the multi-hop packet forwarding model in MANETs.

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