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Hopfield Neural Network with Synaptic Pruning for TSP

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Abstract—In this study, we propose two methods that incorporate the concept of synaptic pruning for an artificial neural network. The synaptic pruning is that the excess couplings between the neurons are disconnected. The performance of the two methods are investigated when these methods are applied to the Hopfield neural network for Traveling Salesman Problem (TSP).

1. Introduction

In recent years, the brain science has been widely studied in various fields. It is important to study brain science for future various kinds of applications in our society. Among these, we focus on the neural network. The neural network is calculation model which can replicate some functions of human brain. In particular, it has been researched extensively that Hopfield neural network is applied to solving method for the combinatorial problems [1]-[3]. The Hopfield neural network is applied to various fields such as associative memory [4]-[5], optimization problem [6] and so on. However, Hopfield neural network has several problems. For instance, the number of couplings between neurons is too large because Hopfield neural network is full coupling. It makes difficult to optimize the problem.

We focus on the synaptic pruning, the phenomenon which causes actually in the brain. Excess synapses are formed in the brain of the newborn creatures. Unnecessary synapses in the process of growth are cut, and normal neural circuit is completed. This process is called synaptic pruning, it has been demanded for the functional neural circuit. It follows that, the synaptic pruning is the process to increase the efficiency of the neural network [7]-[8].

In this study, we propose two methods as condition to cutting the couplings between the neurons. In order to confirm the performance of two proposed methods, we apply the proposed methods to Hopfield neural network. The first method is that all couplings of the selected neurons are disconnected. The second one is that certain couplings selected by random are cut with the disconnecting rate. In addition, we investigate Hopfield neural network with the proposed methods for Traveling Salesman Problem (TSP), which is known as representative applications of the Hopfield neural network. Furthermore, we discuss the comparison of the performance by the two proposed methods.

2. Solving for TSP by Hopfield neural network

Hopfield neural network is defined as the energy function the objective function and constraints condition to solve this problem. The objective function represents that find the shortest route, and constraints condition represents that visit each city exactly once. The state of neurons x_{ij} are renewed to decrease the energy function E. Hopfield neural network is used for solving the optimization problem. Solutions are obtained by using Hopfield neural network to minimize the energy function. For solving N-element traveling salesman problem by Hopfield neural network, $N\times N$ neurons are required.

The energy function E for the TSP is described as follows:

$$\begin{aligned} E = \ & \frac{A}{2}\sum_{i}^{N}(\sum_{j}^{N} x_{ij} - 1)^2 + \frac{B}{2}\sum_{j}^{N}(\sum_{i}^{N} x_{ij} - 1)^2 \\ & + \frac{D}{2}\sum_{i}^{N}\sum_{k}^{N}\sum_{j}^{N} d_{ik} x_{ij}(x_{k,j+1} + x_{k,j-1}). \end{aligned} \tag{1}$$

where d_{ik} is the distance from city i to city k, and the scaling parameters A, B and D are positive constants.

The first term of the above formula represents that each city is only visited once. The second term indicates that it is not possible to visit more than one city at the same time. The third term represents the energy function to find the shortest of the route.

Furthermore, the Hopfield neural network has the couplings strengths and external input given as:

$$\begin{cases} W_{ij,mn} = -A\delta_{im}(1-\delta_{jn}) - B\delta_{jn}(1-\delta_{im}) \\ \qquad\qquad - Dd_{im}(\delta_{n,j+1} + \delta_{n,j-1}). \\ \quad h_{ij} = A + B. \end{cases} \tag{2}$$

where δ_{im} is the Kronecker's delta, which is defined as:

$$\delta_{im} = \begin{cases} 1 & (i = m), \\ 0 & (i \neq m). \end{cases} \tag{3}$$

The update equation for the state of neurons is given as:

$$u_{ij} = \sum_{mn} W_{ij,mn} x_{mn} + h_{ij}. \quad (4)$$

where x_{ij} is a sigmoid function such as:

$$x_{ij} = \frac{1}{2}\left(1 - \tanh\left(\frac{u_{ij}}{0.5}\right)\right). \quad (5)$$

In next section, we discuss two proposed methods as condition to cutting couplings between the neurons.

3. Proposed Method

In this study, we investigate the influences of Hopfield neural network with synaptic pruning for TSP. In order to realize synaptic pruning, we define two methods as condition to cutting couplings between the neurons. The first method is defined neuronal death method. This neuronal death method's cutting conditions are described as follows:

- Selecting the neurons at a constant rate at random.
- Cutting all couplings of the neurons which we select.
- The internal states are retained till next cutting.

The second method is defined random cutting method. This random cutting method's cutting conditions are described as follows:

- Selecting the couplings between the neurons at a constant rate at random.
- Cutting the couplings which we select.
- The internal states are retained till next cutting.

The difference of two methods is to cut all couplings of the neurons which we select, or couplings are cut selected at random from among all of the couplings.

Here, the couplings are cut at a regular interval. The length of regular interval (update of network) is set from 1 to 20. The unnecessary neurons and couplings are decided completely random. In addition to this, we discuss the comparison of the performance by the two methods.

4. Simulation results

In this simulation, we discuss the optimization in the case of the disposition with 22 and 48 cities as shown in Figs. 1 and 2. The number of iteration is 5000 times per one trial. In addition, the cutting rate r is changed from 1 to 25 %. We compare the shortest and the average distances. We simulated ten times of trials with different initial values. The optimal solutions of the problems are known as 75.7 and 33523.7.

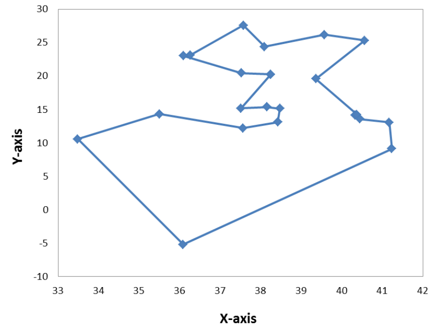

Figure 1: Disposition with 22 cities.

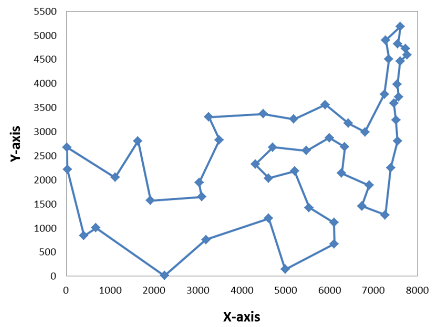

Figure 2: Disposition with 48 cities.

4.1. 22 cities

The simulation results of neuronal death and random cutting methods when the results comparison of the shortest distance are shown in Table 1, Fig. 3 and Table 2, Fig. 4. The parameters of Hopfield neural network are fixed as A = 1, B = 1 and D = 41. The cutting rate is r, the regular interval is I. These results shows that the comparison of performance of the conventional method and that of the proposed networks with 22 cities. As these simulation results, the proposed networks are improved performance than the conventional network. However, comparing the two methods, neuronal death method has a higher performance than random cutting method.

On the other hand, Tables 3 and 4 represent the results by two proposed methods when the results comparison of the average distance. By comparison with the two proposed methods, neuronal death method has a higher performance than random cutting method.

Table 1: The neuronal death method (shortest distance).

		Proposed network (neuronal death)				
	Conv.	$I = 1$	$I = 5$	$I = 10$	$I = 15$	$I = 20$
$r = 1$	99.8	88.5	81.8	84.2	82.7	83.9
$r = 5$	99.8	83.8	79.5	80.6	79.9	80.6
$r = 10$	99.8	83.3	79.3	78.4	79.7	79.4
$r = 15$	99.8	79.7	79.2	79.1	79.1	80.4
$r = 20$	99.8	80.1	79.6	79.6	79.8	79.3
$r = 25$	99.8	80.1	80.8	80.4	81.0	81.7

Table 2: The random cutting method (shortest distance).

	Conv.	Proposed network (random cutting)				
		$I = 1$	$I = 5$	$I = 10$	$I = 15$	$I = 20$
$r = 1$	108.5	83.2	92.9	100.0	98.5	100.1
$r = 5$	108.5	81.8	80.2	84.3	83.3	85.9
$r = 10$	108.5	85.8	79.9	79.8	80.4	80.6
$r = 15$	108.5	90.0	81.7	81.4	81.2	81.7
$r = 20$	108.5	92.4	82.5	81.7	82.4	82.8
$r = 25$	108.5	96.4	85.7	82.9	84.2	84.6

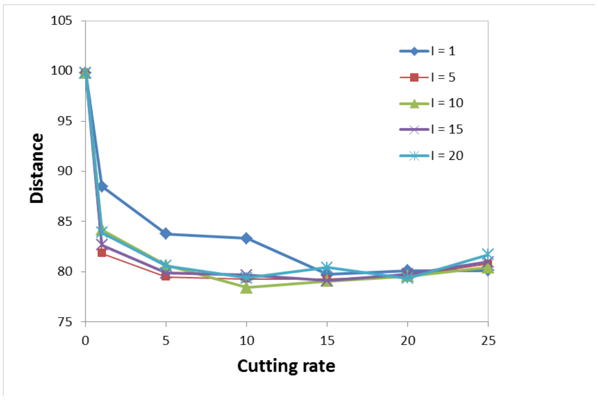

Figure 3: Comparison of performance between the neuronal death and the conventional methods.

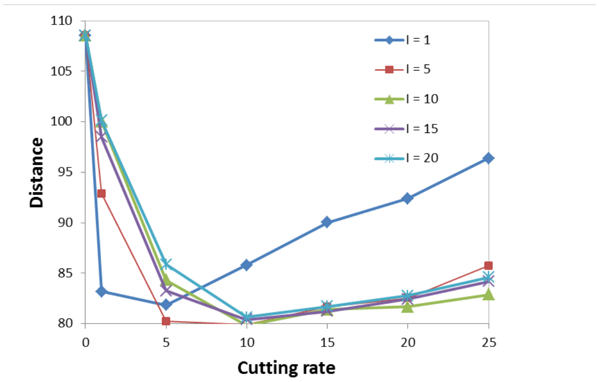

Figure 4: Comparison of performance between the random cutting and the conventional methods.

4.2. 48 cities

Next, we explain the results of 48 cities. The simulation results of neuronal death method and random cutting method with shortest distance are shown in Table 5, Fig. 5 and Table 6, Fig. 6. The parameters of Hopfield neural network are fixed as $A = 1$, $B = 1$ and $D = 100$. These results shows that the comparison of performance of the conventional method and that of the proposed networks. As the results of neuronal death method, the performance of network is improved although away from the optimal solution.

Table 3: The neuronal death method (average distance).

	Conv.	Proposed network (neuronal death)				
		$I = 1$	$I = 5$	$I = 10$	$I = 15$	$I = 20$
$r = 1$	105.1	95.1	91.2	95.2	91.1	94.0
$r = 5$	105.1	92.0	89.7	93.0	92.1	92.1
$r = 10$	105.1	93.0	94.5	93.8	95.1	94.9
$r = 15$	105.1	101.8	98.8	99.4	99.6	99.7
$r = 20$	105.1	112.9	105.6	106.9	106.9	106.1
$r = 25$	105.1	113.9	114.7	112.7	112.5	113.5

Table 4: The random cutting method (average distance).

	Conv.	Proposed network (random cutting)				
		$I = 1$	$I = 5$	$I = 10$	$I = 15$	$I = 20$
$r = 1$	112.8	95.6	103.4	108.2	109.1	112.3
$r = 5$	112.8	110.2	95.8	98.6	99.6	103.2
$r = 10$	112.8	123.9	104.0	100.7	100.3	100.2
$r = 15$	112.8	132.9	113.3	110.0	106.3	107.6
$r = 20$	112.8	141.3	121.7	116.2	113.7	111.6
$r = 25$	112.8	148.9	127.5	120.7	118.3	117.3

Table 5: The neuronal death method (shortest distance).

	Conv.	Proposed network (neuronal death)				
		$I = 1$	$I = 5$	$I = 10$	$I = 15$	$I = 20$
$r = 1$	46006.2	45357.3	43012.7	42806.0	42895.0	42464.4
$r = 5$	46006.2	44887.8	40519.0	40445.9	40200.7	41385.9
$r = 10$	46006.2	44083.7	40090.9	40210.4	40518.1	40719.2
$r = 15$	46006.2	45217.0	40844.6	40727.4	40821.3	40146.2
$r = 20$	46006.2	44850.5	41440.4	41267.0	41007.6	40552.7
$r = 25$	46006.2	45456.6	41961.0	41938.2	41156.6	41604.6

Table 6: The random cutting method (shortest distance).

	Conv.	Proposed network (random cutting)				
		$I = 1$	$I = 5$	$I = 10$	$I = 15$	$I = 20$
$r = 1$	50060.7	40718.4	44712.5	43932.3	45450.0	45413.4
$r = 5$	50060.7	51761.2	43082.1	44004.5	43235.3	43420.3
$r = 10$	50060.7	59129.9	46349.9	45832.5	45694.6	46001.2
$r = 15$	50060.7	63547.2	49307.5	47719.9	47637.4	48272.4
$r = 20$	50060.7	71495.7	51175.3	47608.1	47530.9	47942.9
$r = 25$	50060.7	77058.8	53384.5	47773.5	46852.9	47580.6

However, the results of random cutting method does not improve performance. In consequence, the performance of neuronal death method is better than random cutting method.

On the other hand, each of the Tables 7 and 8 represents the results by two proposed methods when the results comparison of the average distance. The performance of results of 48 cities, the both neuronal death and random cutting methods are not high performance when we compare with

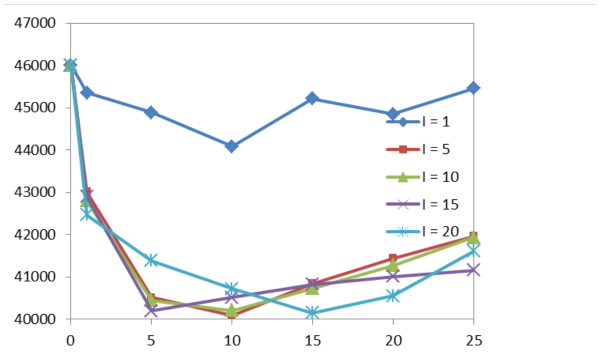

Figure 5: Comparison of performance between the neuronal death and the conventional methods.

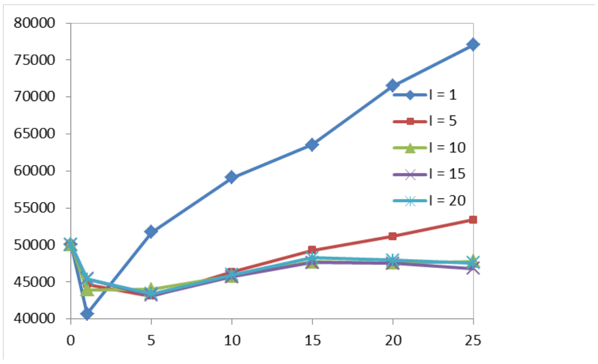

Figure 6: Comparison of performance between the random cutting and the conventional methods.

Table 7: The neuronal death method.

		Proposed network (neuronal death)				
	Conv.	$I = 1$	$I = 5$	$I = 10$	$I = 15$	$I = 20$
$r = 1$	48608.4	47904.9	46638.1	48806.2	48250.0	48981.2
$r = 5$	48608.4	49685.4	48048.4	48418.0	48983.0	49179.0
$r = 10$	48608.4	51733.6	50589.1	49940.1	49394.6	50630.4
$r = 15$	48608.4	57304.1	52281.7	51898.9	51940.6	51884.8
$r = 20$	48608.4	60390.3	54721.0	53622.4	53125.0	52473.3
$r = 25$	48608.4	63055.6	56758.4	55509.9	54817.8	54392.9

Table 8: The random cutting method.

		Proposed network (random cutting)				
	Conv.	$I = 1$	$I = 5$	$I = 10$	$I = 15$	$I = 20$
$r = 1$	53136.6	50066.0	52071.1	50224.3	50425.8	50531.4
$r = 5$	53136.6	74404.2	55519.7	54164.4	53769.8	52854.7
$r = 10$	53136.6	83887.2	65999.3	62390.1	61185.8	60568.1
$r = 15$	53136.6	92893.1	74874.1	69606.6	67991.7	66060.0
$r = 20$	53136.6	103076.8	81607.4	74152.1	71511.5	70255.7
$r = 25$	53136.6	113611.1	87703.2	78493.8	74529.0	72741.6

the results of 22 cities. However, comparing the two methods, neuronal death method has a higher performance than random cutting method.

5. Conclusion

We have studied the solution abilities of the Hopfield neural network with two methods as condition to cutting couplings between the neurons for TSP. As the simulation results, the neuronal death method shows very good performance. In addition, the random cutting method also better performance than conventional network. However, compared to neuronal death and random cutting methods the performance of the random cutting method is inferior.

In future works, we would like to devise various algorithms to cut the couplings. Furthermore, we would like to simulate Hopfield neural network for large-scale model.

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