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Typical Dynamics of the Digital Spiking Neuron

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Abstract—The paper studies the simple digital spiking neuron (DSN). The DSN is a discrete dynamical system and can exhibit a variety of spike-trains. For example, the DSN generates a spike-train with period 1, that with period 2, and so on. In order to consider the steady states and transient phenomena of spike-trains, we use the Field-Programmable Gate Array (FPGA) based DSN. Depending on the initial condition, the DSN can generate a variety of steady states and transient phenomena to them. Using the FPGA, such phenomena are investigated experimentally.

1. Introduction

The spiking neuron is a simple switched dynamical system. Repeating integrate-and-fire behavior between a threshold and periodic base signal, the spiking neuron can output a variety spike-trains. We consider the spike-trains (steady states and transient phenomena). In previous works of this, sinusoidal and triangular base signals have been mainly used. Applications of the spiking neurons are many and include signal processing, ultra wide band communications, and neural-prosthesis. Analysis of spiking neurons is important not only as a basic study of nonlinear dynamical system but also for engineering applications.
This paper studies the digital spiking neuron (DSN). The DSN can exhibit a variety of spike-trains. In order to consider the steady states and transient phenomena of spike-trains, we use the Field-Programmable Gate Array (FPGA) based DSN. Depending on the initial condition, the DSN can generate a variety of steady states and transient phenomena to them. Using the FPGA, such phenomena are investigated experimentally.

2. Digtal Spiking Neuron

Fig.1 shows a circuit diagram of the DSN. As shown in this figure, the DSN has M p-cells. Each p-cell has a digital state $p_i^{(k)}(t) \in \{0, 1\}$ where $i \in \{0, 1, \cdots, M-1\}$ is an index of the p-cell, and

$$t = 0, 1, 2, \cdots$$

is a discrete time. In this paper, we assume that one p-cell has a state 1 and the others have states 0s. Then we can define the following scalar state variable $P^{(k)}(t) \in$

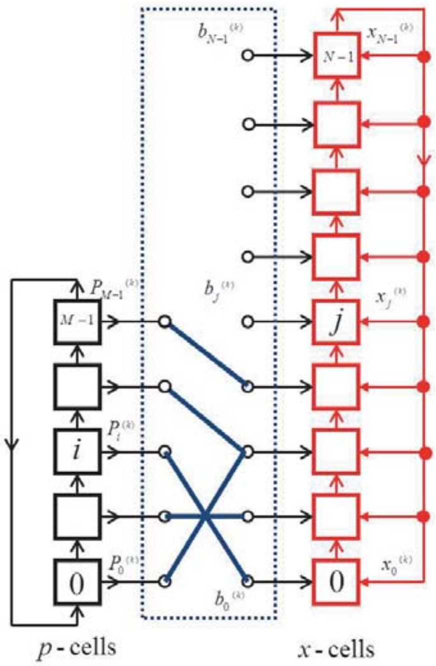

Figure 1: Circuit diagram. The numbers of the cells are $(M, N) = (5, 9)$

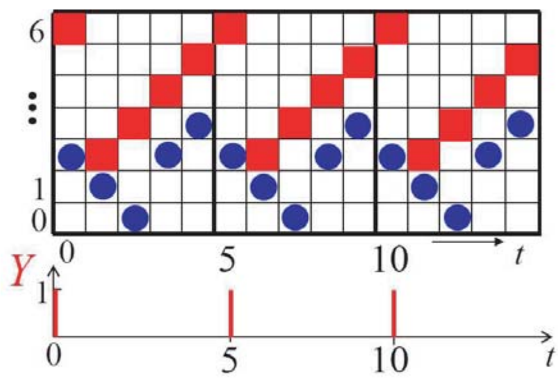

Figure 2: Time-waveforms of the base siganal $B^{(k)}(t)$, and the membrane potential $X^{(k)}(t)$

$\{0, 1, \cdots, M-1\}$ of the p-cells.

$$P^{(k)}(t) = i \quad \text{if} \quad p_i^{(k)}(t) = 1$$

As shown in Fig.1, the p-cells are ring-coupled and their dynamics is described by

$$P^{(k)}(t+1) = P^{(k)}(t) + 1 \pmod{M} \tag{1}$$

In this paper, we assume the initial states of the p-cells to be $P^{(k)}(0) = 0$ without loss of generality. Then the state $P^{(k)}(t)$ is described as a function of the time t as follows.

$$P^{(k)}(t) = t \pmod{M} \tag{2}$$

As shown in Fig. 1, the M p-cells are one-way connected to the N x-cells through the reconfigurable wires. In this paper, we assume that a single wire is connected from a p-cell and arbitrary number of wires are connected to an x-cell. Under this assumption, the pattern of the wires is represented by the following function $A^{(k)} : \{0, 1, \cdots, M-1\} \to \{0, 1, \cdots, N-1\}$.

$$A^{(k)}(i) = j$$

if the i-th p-cell is wired to the j-th x-cell.
The function $A^{(k)}$ is referred to as the wiring function and is characterized by the following parameter vector.

$$A^{(k)} \equiv (A^{(k)}(0), A^{(k)}(1), \cdots, A^{(k)}(M-1))$$

where "≡" represents the "definition" hereafter. The parameter vector $A^{(k)}$ is referred to as wiring pattern. In the case of Fig.1, the number of cells are $(M, N) = (5, 9)$ and the wiring pattern is

$$A^{(k)} = (2, 1, 0, 2, 3). \tag{3}$$

As shown in Fig.1 the reconfigurable wires output a binary signal vector $(b)_0^{(k)}(t), \cdots, b_{N-1}^{(k)}(t)$ which can be represented by the following scalar signal $B^{(k)}(t) \in \{0, 1, \cdots, N-1\}$

$$B^{(k)}(t) = j \quad \text{if} \quad b_j^{(k)}(t) = 1$$

The signal $B^{(k)}(t)$ is referred to as the base signal. Using the wiring function $A^{(k)}$, the base signal $B^{(k)}(t)$ is described by

$$B^{(k)}(t) = A^{(k)}(P^{(k)}(t)). \tag{4}$$

Fig.2 show a time-waveform of the base signal $B^{(k)}(t)$ generated by the DSN in Fig.1. Now we consider the x-cells. Each x-cell has a digital state $x_j^{(k)} \in \{0, 1\}$, where $j \in \{0, 1, \cdots, N-1\}$ of the x-cells.

$$X^{(k)}(t) = j \quad \text{if} \quad x_j^{(k)}(t) = 1$$

Since the state $X^{(k)}$ corresponds to a membrane potential of an integrate-and-fire neuron model, $X^{(k)}$ is referred to as the membrane potential. Using the membrane potential $X^{(k)}$, the dynamics of the x-cells is described by

$$X^{(k)}(t+1) = \begin{cases} X^{(k)}(t) + 1 & \text{if} \quad X^{(k)}(t) < N-1 \\ B^{(k)}(t) & \text{if} \quad X^{(k)}(t) = N-1 \end{cases} \tag{5}$$

In this paper we assume the initial states of the x-cells to be $X^{(k)}(0) = N-1$ without loss of generality. Fig.2 shows a waveform of the membrane potential $X^{(k)}(t)$. If the membrane potential $X^{(k)}$ is below $N-1$ (which corresponds to a firing threshold of an integrate-and-fire neuron model), the membrane potential $X^{(k)(t)}$ is shifted upward. If the membrane potential $X^{(k)}(t)$ reaches the firing threshold $N-1$ at $t = t_f$, the membrane potential $X^{(k)}(t_f + 1)$ is reset to the base signal $B^{(k)}(t_f)$. At the reset moment $t = t_f$, the DSN outputs a spike $Y^k = 1$. Repeating such shift-end-reset behavior (which corresponds to an integrate-and-fire behavior of a neuron model), the DSN outputs a spike-train $Y^{(k)}(t)$ as follows.

$$Y^{(k)}(t) = \begin{cases} 0 & \text{if} \quad X^{(k)}(t) < N-1 \\ 1 & \text{if} \quad X^{(k)}(t) = N-1 \end{cases} \tag{6}$$

As a result, the DSN is described by Eqs.(1), (4), (5) and (6), and is characterized by the following parameters.

$$M, N, A^{(k)}.$$

Since the DSN has the discrete time t and the discrete states $(P^{(k)}, X^{(k)})$, the output spike-train $Y^{(k)}(t)$ is to be periodic in a steady state.

3. Digital return map

We give several basic definitions of the digital return map (Dmap). The Dmap f is a mapping from a set of lattice points to itself and its iteration can generate various sequence:

$$\theta(n) = f(\theta(n)), \theta(n) \in L_N \equiv \{l_1, \cdots, l_N\} \tag{7}$$

where L_N is a set of lattice points $l_m \equiv (m-1)/N, m = 1 \sim N$, and $N$is the number of lattice points. $\theta(n)$ is a state variable on L at discrete time n. The lattice ;points are equivalent to binary vectors and hence we refer to this systems as to digital return map. Since the domain L_N consists of finite elements, the Dmap can exhibit either a periodic orbit or a transient orbit to it. We give definition of such phenomena. A point $\theta_p \in L_N$ is said to be a period-p point if $f^p(\theta_p) = \theta_p$ and $f^q(\theta_p) \neq \theta_p$ for $0 < q < p$, where f^p is the p-fold composition of f. A sequence of the period-p points $\{f(\theta_p), \cdots, f^p(\theta_p)\}$ is said to be a period-p orbit (PEO). A point θ_e is said to be an eventually periodic point (EPP). If θ_e is not a periodic point and there exists some positive integer r such that $f^r(\theta_e)$ is a periodic point. If there exist some EPPs from which an orbit falls into some PEO, the PEO is said to be stable.

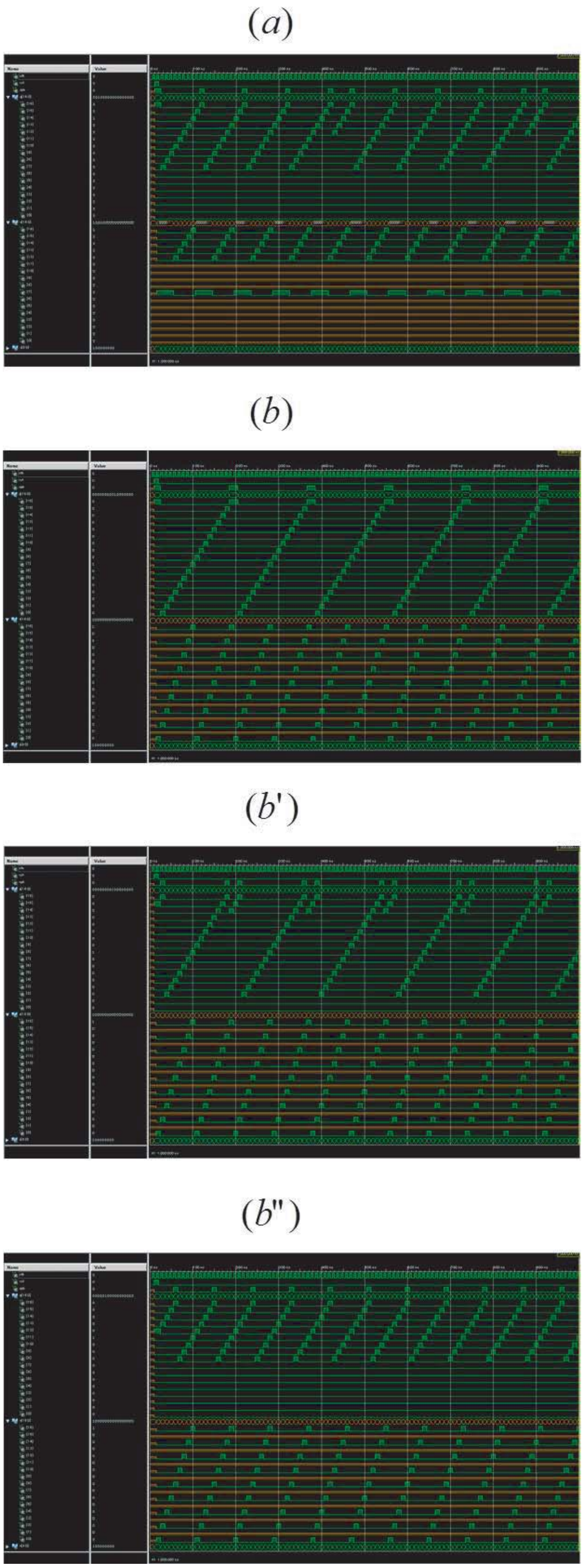

Figure 3: Waveform of the FPGA (a) $A = (7, 7, 7, 7, 12, 13, 14, 15, 16)$. (b), (b') and (b") $A = (0, 2, 4, 6, 8, 10, 12, 14, 16)$.

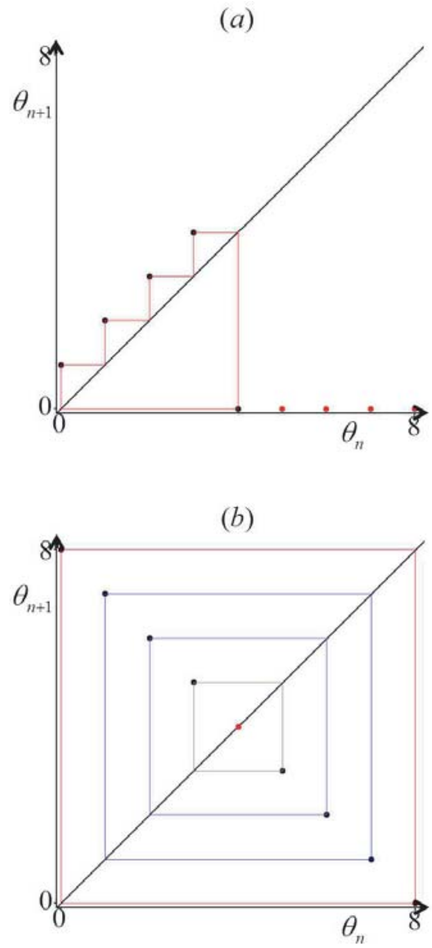

Figure 4: Digital return map (DMap) (a) $A = (7, 7, 7, 7, 12, 13, 14, 15, 16)$. (b) $A = (0, 2, 4, 6, 8, 10, 12, 14, 16)$.

4. Steady States and Transient Phenomena

We have performed experiments. The DSN exhibits various interesting phenomena and we show three examples. Let us consider three DSNs that are characterized by

$$\begin{aligned} M = 9, N = 2M - 1 \\ A^{(1)} = (7, 7, 7, 7, 12, 13, 14, 15, 16) \\ A^{(2)} = (0, 2, 4, 6, 8, 10, 12, 14, 16) \end{aligned} \tag{8}$$

Fig.3 shows the resulting spike-trains and dynamics. Fig.3 (a) exhibits spike-train that have period 5. However, Fig.3 (b) exhibits plural spike trains. Fig.3 (b) and (b') are period 2 and Fig.3 (b") is period 1. Next, we derive the digital return map and analyze the typical phenomena. Fig.4 shows Dmaps which coreespond to Fig.3. The period-5 point of the Dmap (a) corresponds to the period-5 spike-train. Stable fixed point and period-2 point of Dmap (b) corresponds to period-1 and period-2 point spike-trains, respectively. The red orbit of Dmap (b) corresponds to Fig.3 (b) and The blue orbit of Dmap (b) corresponds to Fig.3

(b’). Finally, The fixed point of the Dmap corresponds to Fig.3 (b”).

5. Conclusions

We have studied the DSN that has a various of spike-train dynamics. The dynamics can be integrated into the Dmap. The periodic/transient spike-trains correspond to the PEP and EPP of the Dmap. The DSN can exhibit a variety of steady states and transient phenomena. Presenting a simple circuit, steady states and transient phenomena are observed experimentally. Using the Dmap, the dynamics have been analyzed. Future problems include, detailed analysis of the DSN (steady states and transient phenomena) and application to engineering systems.

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