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Analysing chaotic attractors by measures of complex networks

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Abstract—Various complex phenomena in our world may have deterministic nature or stochastic one, or both. Then, it is an important issue to characterize the dynamics of these complex phenomena. Therefore, in this paper, we propose a new method to analyze deterministic chaos from a new point of view. In the proposed method, we first construct a network from an attractor of nonlinear dynamical systems. In this network, nodes correspond to points on the attractor and connections between the nodes are decided by Euclidean distance between the points on the attractor. Next, we measure the degree of the nodes in the network. As a result, we confirmed that the networks constructed from chaotic attractors show different tendency from other attractors.

1. Introduction

In the real world, various complex phenomena often occur, for example, electrical activities of neural systems such as brain waves and nerve responses, change of weather conditions, fluctuation of financial indices and so on. Although these complex phenomena may be produced from a deterministic system or a stochastic system, we suppose that these phenomena are produced from deterministic nonlinear dynamical systems.

If a complex phenomenon is produced by a deterministic nonlinear rule, it becomes hard to predict its long-term behavior, when the dynamics of the complex phenomenon exhibits chaos. Therefore, it is a fundamental and important issue to clarify a source of the complex phenomenon.

On the other hand, during the last decade a complex network theory has drastically advanced and has been widely acknowledged through several areas, such as biology, sociology, physics and so on. Several properties of complex phenomena in various real networks have been clarified by the measures of the complex network theory; for example, a small-world property discovered by D. Watts and S. Strogatz [5], a scale-free property discovered by A.-L. Barabási and R.Albert [6]. Several measures have already been proposed to analyze structures and dynamics of the complex network [7].

Recently, to quantify and analyze deterministic nonlinear dynamics, novel methods have been proposed [4, 10, 11, 12, 13]. These methods analyze nonlinear dynamics using the complex network theory. In these methods, first, a network is constructed from an attractor produced from a nonlinear dynamical system. Next, the constructed network is analyzed by the measures of complex network theory. J. Zhang and M. Small showed interesting analytical results of observed complex phenomena using the measures of the complex network theory such as the clustering coefficient, the characteristic path length, and the degree distribution[4].

Although an analysis method proposed in this paper measures the deterministic nonlinear chaotic dynamics using the measures of the complex network theory [10, 11, 12, 13], an idea in this paper is different from the conventional methods [4]: the idea is based on the fact that attractors of nonlinear dynamical systems and networks are characterized by a two-dimensional matrix. The matrix is called a recurrence plot[1, 2, 3] from the dynamical system theory and an adjacent matrix from the network theory.

To quantify the chaotic dynamics, various indices have been proposed: the Lyapunov exponent, the Kolmogorov-Sinai entropy, the fractal dimension and so on. Although these indices quantify the chaotic dynamics from different points of view, these indices also have a common feature: these indices are obtained by the interpoint distances between two points on an attractor produced from dynamical systems. The interpoint distances can be represented through a binary two-dimensional image by a recurrence plot. The recurrence plot can visualise how recurrences occur on the attractors and can also often be used to investigate nonstationarity of the dynamical systems.

The recurrence plot is represented as a two-dimensional square matrix. On the other hand, a network is also described by an adjacent matrix which is a two-dimensional square matrix that reflects connections between nodes in the network. Using this relationship of the recurrence plot and the adjacent matrix, in the previous paper [11, 12, 13], we applied the measures of the complex network theory, the clustering coefficient and the characteristic path length, to the analysis of nonlinear chaotic dynamical systems and showed that the networks constructed from chaotic attractors have a small-world property of the complex network theory. Therefore, in this paper, we propose a new method to characterize the chaotic dynamics using a new construction method of a network from an attractor of the nonlinear dynamical system.

2. Proposed method

2.1. Construct a network from an attractor

To characterize the chaotic behavior using the complex network theory, we construct a network from an attractor. Here, the attractor is obtained as an asymptotic set of a dynamical system, if we have a specific information of the dynamical system. If we only have an observed time series, the attractor is reconstructed from the time series using a time delay coordinate [8].

In our construction procedure of a network from an attractor, the network grows using the information of temporal evolution of the attractor. Therefore, nodes in the network constructed from the attractor correspond to points on the attractor. The connection between nodes in the constructed network is uniquely decided by a distance between points on the attractor.

Let $\boldsymbol{x}(i)$ $(i = 1, \ldots, N)$ be the ith point on an attractor and $v(i)$ be the ith node in a network constructed from the attractor. Then, a distance between $\boldsymbol{x}(i)$ and $\boldsymbol{x}(j)$ is defined by $d_{ij} = |\, \boldsymbol{x}(i) - \boldsymbol{x}(j)\, |$, and a strength of a connection between $v(i)$ and $v(j)$ is defined by $w_{ij} = d_{ij}$.

The algorithm for constructing a network from an attractor is described as follows:

1. Start from an initial network that consists of completely connected M_0 nodes $(v(1), \ldots, v(M_0))$.
   Set the number of nodes in the present network $N' = M_0$.
2. Add a new node $v(N'+1)$ by connecting it to different M $(M = M_0 - 1)$ nodes that have existed already in the network. The M nodes with small $w_{iN'+1}$ $(i = 1, \ldots N')$ are selected.
3. Increase N' to $N' + 1$.
4. Repeat steps 2 and 3 if $N' < N$.

In the above algorithm, we consider that the connections have no directions and no weights.

2.2. The degree and the degree distribution

To characterize the chaotic behavior, we investigate the degree of nodes and the degree distribution of the network constructed from the attractor. The degree of node $v(i)$ is k_i, and $p(k)$ is the distribution of probability that a node in the network connects to other k nodes.

3. Experiments

We applied our proposed method to three data sets: a periodic attractor, a chaotic attractor and a reconstructed attractor from a random time series. We first produced the periodic attractor and the chaotic attractor from the Rössler system[14]. The Rössler system is defined by the following three equations:

$$\begin{cases} \dot{x} &= -(y+z), \\ \dot{y} &= x + ay, \\ \dot{z} &= b + z(x-c). \end{cases} \tag{1}$$

In Eq.(1), we fixed $a = b = 0.2$, and used the following values of c: $c = 2.5$ (a period-1 attractor) and $c = 5.0$ (a chaotic attractor).

Next we generated a reconstructed attractor from a time series. The time series is an interval time series of gamma ray emission of cobalt (cobalt data) (Fig.1). This time series has erratic behavior and is considered to be a truly random one.

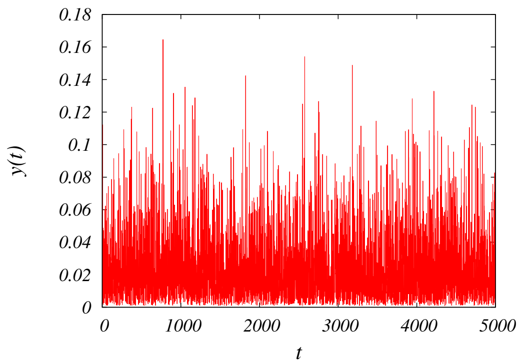


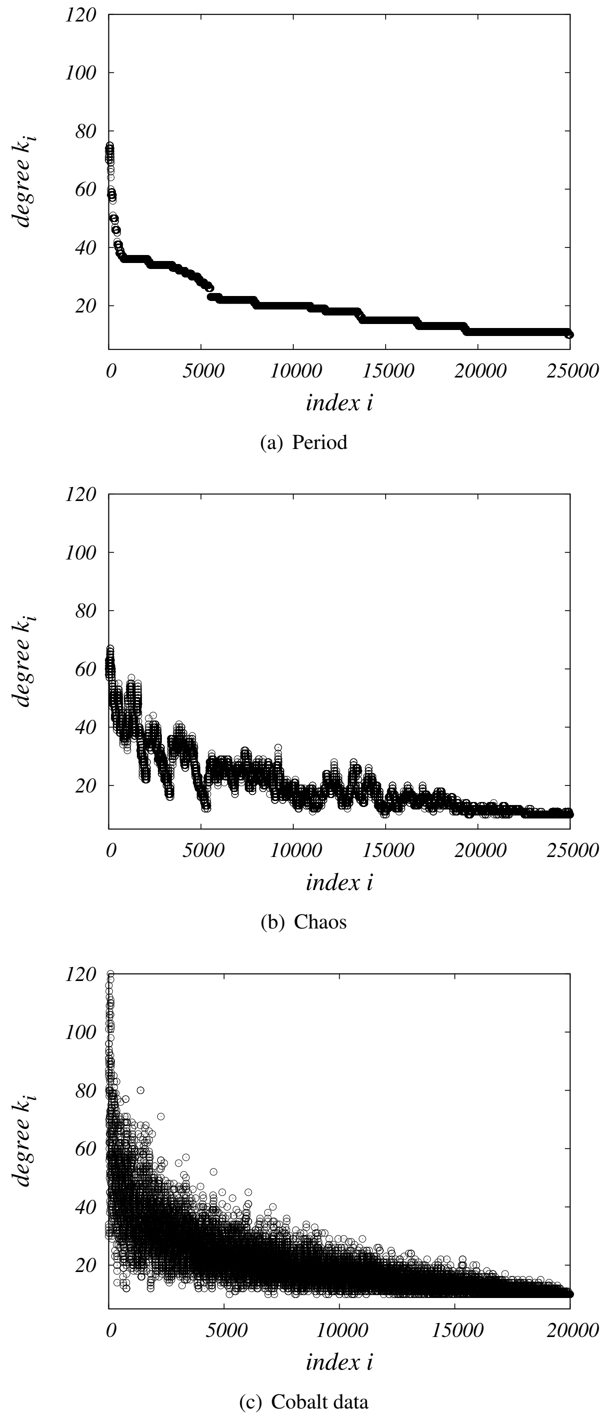
Figure 1: Time series of gamma ray emission of cobalt.

In this experiments, to construct a network from an attractor, we fixed $M = 10$. M means the number of connections of node added to the network.

4. Results and Discussion

Results of the degree of nodes in the network generated from the attractors are shown in Fig.2. Fig.2(a) shows the result of the periodic attractor, Fig.2(b) shows that of the chaotic attractor, and Fig.2(c) shows that of cobalt data. From Fig.2, the results show clearly different tendency. From Figs.2(a) and (b), the nodes $v(i)$ with small indices i have the large number of degree k_i. In particular, in the result of the periodic attractor (Fig.2(a)), this tendency strongly appears. On the other hand, the result of the chaotic attractor (Fig.2(b)) does not only show the similar tendency to Fig.2(a) but also some fluctuation. Different from Fig.2(a) and Fig.2(b), Fig.2(c) shows a wide range of values of the degree k_i.

Results of the degree distribution of the networks constructed from the attractors are shown in Fig.3. From Fig.3, the degree distributions of the chaotic attractor and the cobalt data show similar tendency, but the periodic attractor is different. From the results of the degree distribution (Fig.3), there are no clearly differences between the constructed network from the chaotic attractor and the cobalt data. However, the results of the degree (Fig.2) show the different tendency between the network of the chaotic attractor and that of the cobalt data. The reason why the clearly differences between the chaotic attractor and the cobalt data do not appear in Fig.3 is considered that the degree distribution is static information of the constructed network. On the other hand, the results of the degree (Fig.2) show the information of temporal evolution of the attractors, and we can confirm the different tendency between the chaos and the cobalt data.



(a) Period

(b) Chaos

(c) Cobalt data

Figure 2: Relations between the degree k_i and the index i of the nodes in the network constructed from (a) periodic attractor, (b) chaotic attractor (c) and cobalt data. The degree k_i means the number of connections of the node $v(i)$.

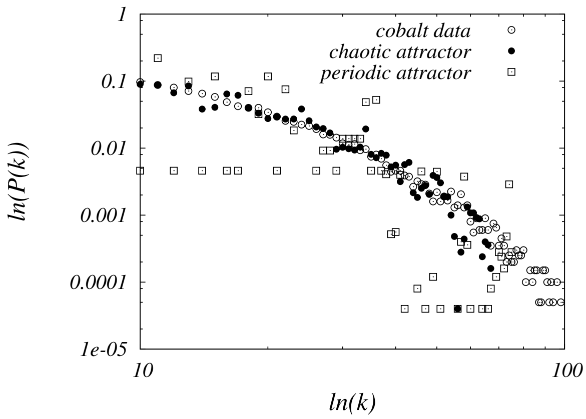


Figure 3: Results of the degree distributions of the periodic attractor, the chaotic attractor and the cobalt data.

5. Conclusion

In this paper, we proposed a construction method of a new network from an attractor to characterize the chaotic dynamics. In the proposed method, the networks grow using the information of temporal evolution of the attractors, and the connections between nodes in the network are decided by the Euclidean distance between nodes on the attractor. Using the proposed method, we first constructed the networks from three different attractors: the periodic attractor, the chaotic attractor, and the reconstructed attractor from cobalt data. We next investigated the degree of the nodes and the degree distribution of these networks. From the results with using the information of temporal evolution of attractor, we confirmed that the different tendencies exist among the constructed networks from these attractors.

In the proposed method, the connections of nodes in a constructed network have no direction and no weight. Therefore, it is an important future work to analyze the network with directional and weighted connections. In addition, it is also important to apply the various measures of the complex network theory to the constructed networks from the attractors.

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