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AC-DC Converters with Hysteresis Switching and Stabilization

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Abstract—This paper studies nonlinear dynamics of a switched circuit based on ac-dc converters in current mode control. The switching has hysteresis characteristics that cause rich phenomena. In order to analyze the dynamics, we derive a 1D return map of switching phase. The map has simple shape and we can analyze the chaotic/periodic phenomena precisely. We also consider stabilization by adding periodic compulsive switching to the system. It is confirmed that our method can realize stable operation in wider parameter range.

1. Introduction

Switching power converters are important not only as efficient energy supply systems but also as nonlinear dynamical systems having interesting phenomena [1] [2]. The converters include nonlinear switches that can cause a variety of periodic/chaotic phenomena. Analysis of the phenomena is basic to develop novel bifurcation theory and to design efficient practical circuits.

This paper studies a switched circuit based on ac-dc converters in current mode control (CMC) [3]-[5]. The CMC is used for achieving faster transient response in boost converters and for lower voltages with higher current capabilities by current sharing in parallel converters [6]. In order to control the circuit, we present hysteresis switching that can cause rich phenomena and consideration. For simplicity, we assume that voltage regulation is achieved in high frequency modulation, so that the much slower dynamics of the outer voltage loop can be ignored, and the output side can be represented by a constant voltage source [6] [7]. Applying this simplifying assumption, we can derive 1D return map of the switching phase. The map can be described exactly based on exact piecewise solution. Using the map we can investigate rich periodic/chaotic phenomena and related bifurcation phenomena. It should be noted that the 1D return map is available not only for ac-dc boost converters but also for a variety of switching circuits including delta modulator for PWM control [8].

Next, we consider stabilization of the circuit. Adding periodic compulsive switching to the circuit, stabilization is possible for wider parameter range. The results of this paper is important not only as basic study but also for practical applications. For example, the results provide basic information to realize stable operation, distortion removal [4] and EMI improvement [9].

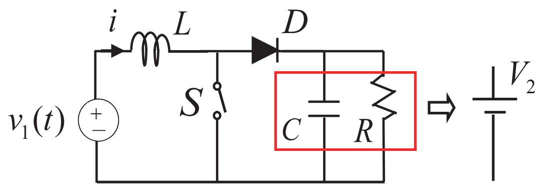

Figure 1: The circuit model and simplification

2. The Circuit Model

Fig. 1 shows the objective circuit based on a ac-dc boost converter with CMC control. The input $v_1(t) = V_1|\sin\frac{\pi}{T}t|$ is an output of rectifier where T is period of $v_1(t)$. The rectifier is omitted in the figure. In order to define the switching, we introduce two reference values which are proportional to the input: $I_{\rm ref}(t) = kv_1(t) + I_2$, $I'_{\rm ref}(t) = kv_1(t)$. For simplicity, voltage regulation is assumed to be achieved in high frequency modulation. It enables us to analyze. Under the CMC, when the switch is on, the inductor current i rises, and when it reaches a reference value $I_{\rm ref}(t)$, the switch S is turned off. When S is off D turns on, and the inductor current decays. It is turned on by i reaches a reference value $I'_{\rm ref}(t)$. Thus we have two possible states:

State 1: S conducting, D blocking and $0 < i$
State 2: S blocking, D conducting and $0 < i$

The switching rules are:

State 1 $\longrightarrow$ State 2: when $i = I_{\rm ref}(t)$
State 2 $\longrightarrow$ State 1: when $i = I'_{\rm ref}(t)$

As stated earlier, we make the simplifying assumption $T << RC$ and voltage regulation is achieved. In this case, we can replace the RC with the constant voltage source V_2. Thus the circuit equation becomes

$$L\frac{d}{dt}i = \begin{cases} v_1(t) & \text{for State 1} \\ v_1(t) - V_2 & \text{for State 2} \end{cases} \quad (1)$$

where $0 < V_1(t) < V_2$.

We introduce the following dimensionless variables and parameters

$$x = \frac{i}{kV_1},\ \tau = \frac{t}{T},\ \alpha = \frac{T}{Lk},\ \beta = \frac{TV_2}{LkV_1},\ \gamma = \frac{I_2}{kV_1} \quad (2)$$

through which Eq. (1) is transformed into

$$\frac{d}{d\tau}x = \begin{cases} \alpha s(\tau) & \text{for State 1} \\ \alpha s(\tau) - \beta & \text{for State 2} \end{cases} \quad (3)$$

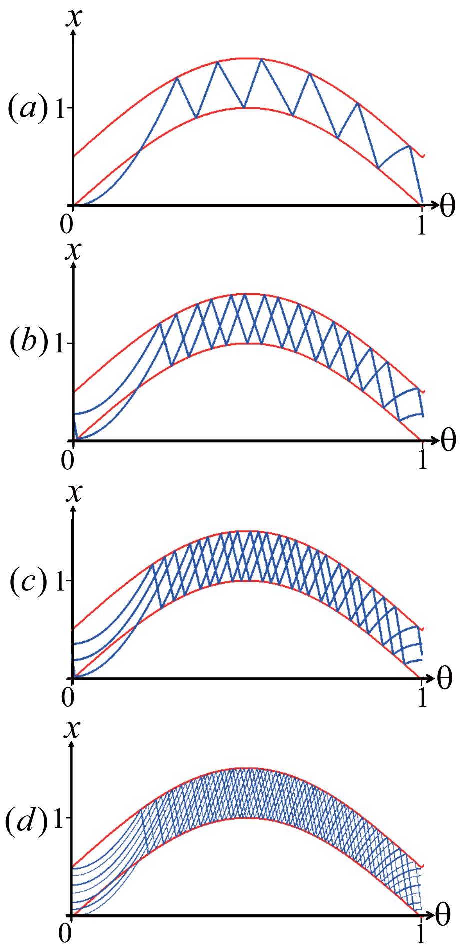

Figure 2: Typical waveforms for $\alpha = 10.0$ and $\gamma = 0.5$. (a) periodic trajectory for $\beta = 16.0$, (b) periodic trajectory with period 2 for $\beta = 20.5$, (c) periodic trajectory with period 3 for $\beta = 24.5$, (d) quasi-periodic trajectory for $\beta = 27.0$

where $s(\tau) = |\sin \pi\tau|$, $\alpha s(\tau) > 0$ and $\alpha s(\tau) - \beta < 0$. The switching rules become:

$$\begin{array}{l} \text{State 1} \rightarrow \text{State 2 if } x = s(\tau) + \gamma \\ \text{State 2} \rightarrow \text{State 1 if } x = s(\tau) \end{array}$$

Note that the original six parameters (T, L, k, I_2, V_1, V_2) are integrated into the dimensionless three parameters α, β and γ of Eq.(2). Fig. 2 (a) shows typical periodic waveform. As the parameter β increases, this periodic trajectory is changed into periodic trajectory with period 2 and 3 as shown in Fig. 2 (b) and (c), respectively. The periodic trajectory is changed into quasi-periodic trajectory as shown in Fig. 2 (d).

3. Analysis

We now derive 1D return map of switching phase. Let τ_n denote n-th switching moment at which x reaches the upper threshold $s(\tau) + \gamma$ and State 1 is changed into State2.

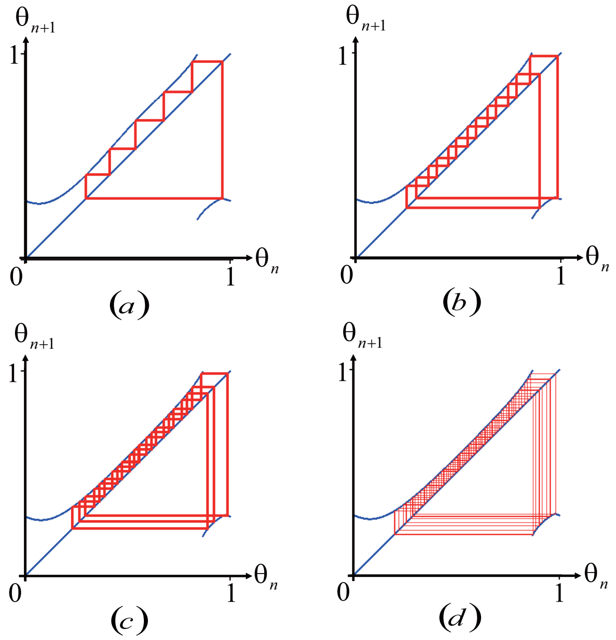

Figure 3: Return maps for $\alpha = 10.0$ and $\gamma = 0.5$. (a) periodic orbit for $\beta = 16.0$, (b) periodic orbit with period 2 for $\beta = 20.5$, (c) periodic orbit with period 3 for $\beta = 24.5$, (d) quasi-periodic orbit for $\beta = 27.0$

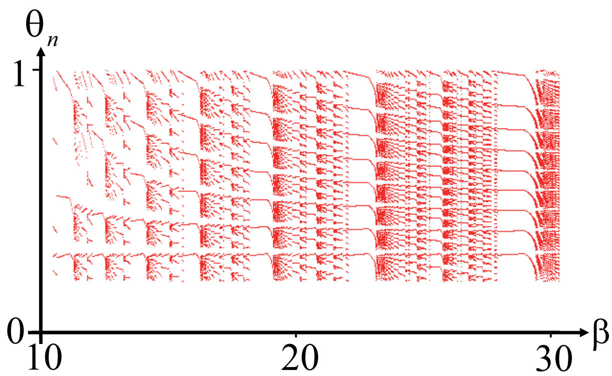

Figure 4: Typical bifurcation phenomena for $\alpha = 10.0$ and $\gamma = 0.5$.

Since τ_{n+1} is determined by τ_n we can define 1D map of the form $\tau_{n+1} = f(\tau_n)$. Since the system is period 1, we introduce phase variable $\theta_n = \tau_n \bmod 1$ and the map can be reduced into the return map from $I_1 \equiv [0, 1)$ to itself: $\theta_{n+1} = F(\theta_n) = f(\theta_n) \bmod 1$.

Fig. 3 (a) shows a return map of periodic orbit corresponding to periodic trajectory in Fig. 2 (a). As β increases, the periodic orbit with period 1 is changed into periodic orbit with period 2 and 3 as shown in Fig. 3 (b) and (c). They correspond to periodic trajectories in Fig. 2 (c) and (d), respectively. As β increases further, periodic orbits are changed into quasi-periodic orbit as shown in Fig. 3 (d) corresponding to quasi-periodic trajectory in Fig. 2 (d).

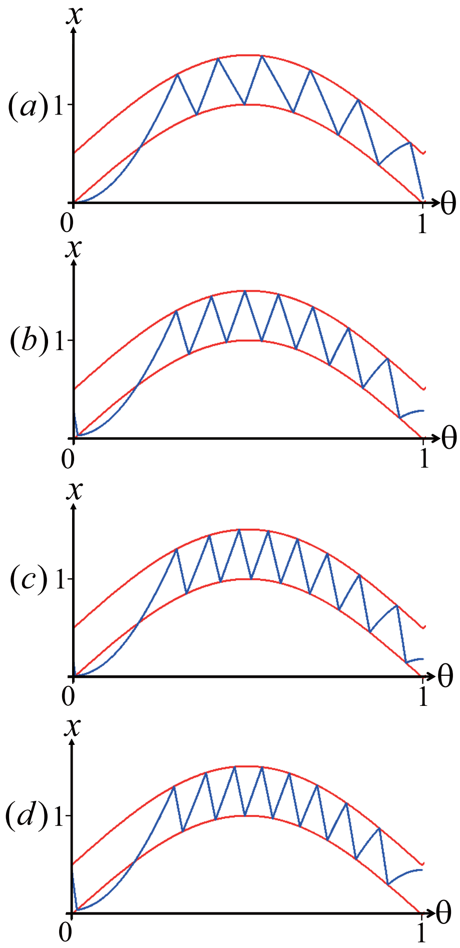

Figure 5: Typical waveforms of stabilization for $\alpha = 10.0$ and $\gamma = 0.5$. (a) periodic trajectory for $\beta = 16.0$, (b) periodic trajectory for $\beta = 20.5$, (c) periodic trajectory for $\beta = 24.5$, (d) periodic trajectory for $\beta = 27.0$

These phenomena are summarized in the bifurcation diagram in Fig. 4. Such complex behavior relate deeply to undesired operation such as current distortion. Note that the map does not have discontinuous points. The map of this paper is simpler than that of [7] having many discontinuity points.

4. Stabilization

In order to stabilize the unstable behavior, we consider the stabilization in this switching rule:

$$\begin{aligned} &\text{State 1} \;\rightarrow\; \text{State 2 if } x = s(\tau) + \gamma \textit{ or } \tau = n \\ &\text{State 2} \;\rightarrow\; \text{State 1 if } x = s(\tau) \end{aligned}$$

That is, the compulsory and periodic switching to State 2 is applied at every period end. We refer to this rule as rule S.

Fig. 7 shows return maps switching rule S. Fig. 7 (a) to (d) correspond to Fig. 3 (a) to (d). All the maps has a part

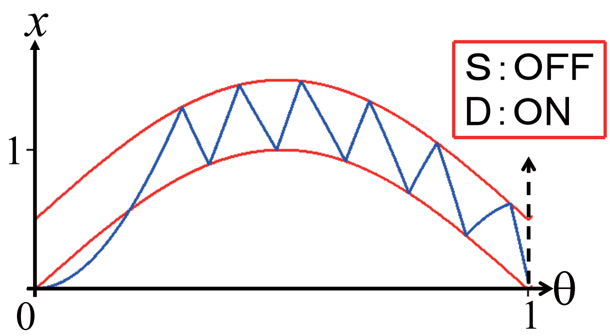

Figure 6: Stabilization method

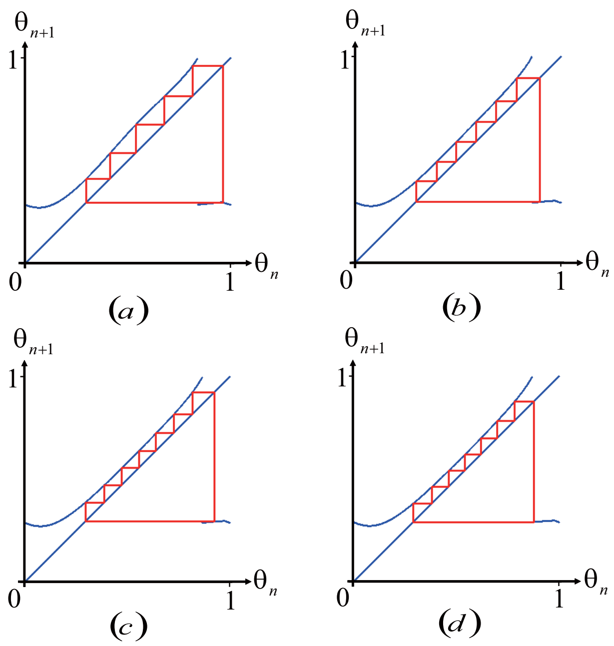

Figure 7: Return maps of stabilization for $\alpha = 10.0$ and $\gamma = 0.5$. (a) periodic orbit for $\beta = 16.0$, (b) periodic orbit for $\beta = 20.5$, (c) periodic orbit for $\beta = 24.5$, (d) periodic orbit for $\beta = 27.0$

with small slope and exhibit stable periodic orbits. These phenomena are summarized in the bifurcation diagram in Fig. 8 where we can confirm stable periodic behavior only: stabilization is achieved in wide parameters range.

5. Conclusions

This paper studies nonlinear dynamics of a switched circuit based on ac-dc converters in CMC. The hysteresis switching rule is used in the circuit. In order to analyze the rich phenomena, we derive the 1D return map that describes switching phase of the circuit. An effective stabilization method with compulsory periodic switching is also presented.

Future problems are many, including the following: de-

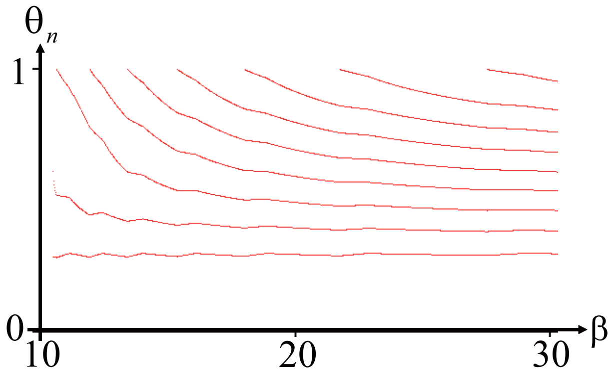

Figure 8: Typical bifurcation phenomena of stabilization for $\alpha = 10.0$ and $\gamma = 0.5$.

tailed analysis of bifurcation phenomena, generalization of 1D return map of switching phase and experiments of practical circuits.

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