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An Analysis Method of a Stochastic Resonance Receiver using a Schmitt Trigger

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Abstract—Stochastic resonance (SR) enhances system responses by increasing noise. By applying SR to a receiver, a subthreshold signal not receivable by a conventional linear receiver could be received. Previously, we proposed an analysis method of the SR receiver using a comparator, and evaluated its bit error rate performance. The comparator is known as a simple non-dynamical system exhibiting SR. However, a dynamical system can have better performance since it has a memory effect. In this sense, we propose an analysis method of the SR receiver using a Schmitt trigger known as a simple dynamical system, and evaluate its bit error rate performance. A performance comparison of the comparator and the Schmitt trigger is also shown.

1. Introduction

Stochastic resonance (SR) is an interesting phenomenon in that noise enhances system response. In the past two decades, this characteristic has been discussed in the context of nonlinear physics [1, 2], and its application in signal processing has spread to various fields, such as signal detection theory [3], wireless communication [4–6], and imaging [7].

This paper discusses the SR effect in communication systems. We focus on the problem in which communication cannot be established when the received signal strength is below receiver sensitivity. Overcoming receiver sensitivity introduces new and attractive challenges in wireless communication systems. If receiver sensitivity can be lowered, we can simultaneously reduce transmission power and interference to other users. Low-power wireless systems can provide solutions for both energy-efficient green wireless communications and wireless spectrum shortage.

Previously, we proposed an analysis method of the SR receiver using a comparator, and presented an improvement of the bit error rate (BER) performance by the SR effect. The comparator is known as a simple non-dynamical system exhibiting SR. However, in general, a dynamical system can have better performance since it has a memory effect [8]. In this sense, we propose an analysis method of the SR receiver using a Schmitt trigger known as a simple dynamical system, and evaluate its BER performance. Reception sensitivity could be modeled as the threshold of a Schmitt trigger or a comparator. A performance comparison of the Schmitt trigger and the comparator is also shown.

The remainder of this paper is organized as follows. In Section 2, we present the system model of the SR receiver using a Schmitt trigger or a comparator. In Section 3, we present the method of analysis for the BER performance of the SR receiver using a Schmitt trigger or a comparator. Section 4 presents numerical results for the BER performances of the SR receiver and a performance comparison of the Schmitt trigger and the comparator. Finally, conclusions are presented in Section 5.

2. System model

In this paper, the desired signal level is assumed to be below device sensitivity. As shown in Fig. 1, when the desired signal level is denoted by A, and the device sensitivity is denoted by η, we observe that $|A| < \eta$. A conventional receiver cannot detect such a subthreshold signal, as shown in Fig. 1 (a).

Figure 2 shows the system model of the SR receiver. In Fig. 2, the SR receiver consisting of the SR system receives a desired signal $As(t)$ and channel noise $n_c(t)$, which is expressed as follows.

$$r(t) = As(t) + n_c(t). \tag{1}$$

In the desired signal, $s(t)$ is expressed as

$$s(t) = \sum_i d_i g(t - iT_s), \tag{2}$$

where d_i is a binary data sequence$\{\pm 1\}$, T_s is a symbol duration, and $g(t)$ is a rectangular pulse. The channel noise $n_c(t)$ is the zero-mean white Gaussian noise with variance σ_c^2.

The channel noise is primarily dominated by the thermal noise that occurs in the receiver; thus, its power spectral density (PSD) is assumed to be uniform and is expressed as $N_0 = k_B T_0$, where k_B is the Boltzmann constant and T_0 is noise temperature. When $T_0 = 300$ K, the noise PSD $N_0 \simeq 4.1 \times 10^{-21}$ W/Hz.

The SR receiver differs from a conventional receiver in that it adds intentional noise at the receiver front end. Figure 1 (b) illustrates the detection of a subthreshold signal by the additional intentional noise. This shows a simple SR effect. The intentional noise $n_{SR}(t)$ is assumed to be zero-mean white Gaussian noise with variance σ_{SR}^2. The intentional noise should be optimally tuned to obtain the best SR receiver performance.

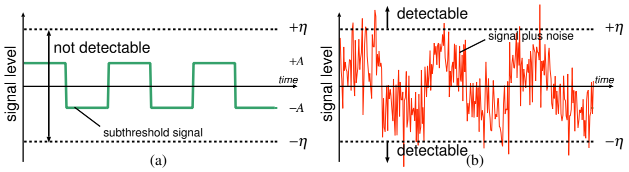

Figure 1: An example of the subthreshold signal (a) and the signal plus noise (b).

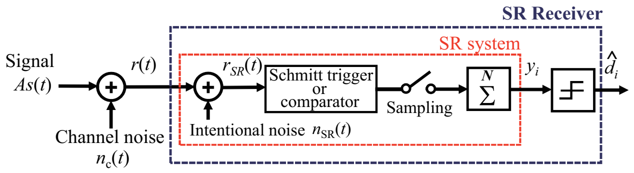

Figure 2: System model.

Reception sensitivity can be modeled as the threshold of a Schmitt trigger or a comparator which are simple nonlinear devices that exhibits the SR effect. The received signal, which is composed of $As(t)$, $n_c(t)$, and $n_{SR}(t)$, is fed into the nonlinear device. Input-output characteristics of the Schmitt trigger and the comparator are shown in Fig. 3. As can be seen in this figure, the Schmitt trigger has a hysteresis that exhibits a memory effect, and the comparator is a simple threshold system. If a subthreshold signal plus noise exceeds the thresholds, the subthreshold signal can be detected when the noise is tuned optimally. This phenomenon is known as SR.

The threshold of the Schmitt trigger is changed by the current output as

$$V_{th} = \begin{cases} -\eta & if \quad y(t) = +V \\ +\eta & if \quad y(t) = -V \end{cases}. \tag{3}$$

When the input signal $r_{SR}(t)$ is fed into the Schmitt trigger, the output of the comparator is expressed as follows.

$$y(t) = \begin{cases} -V & if \quad r_{SR}(t) < V_{th} \\ +V & if \quad r_{SR}(t) > V_{th} \end{cases}, \tag{4}$$

When the input signal $r_{SR}(t)$ is fed into the comparator, the output of the comparator is expressed as follows.

$$y(t) = \begin{cases} -V & (r_{SR}(t) < -\eta) \\ +V & (r_{SR}(t) > +\eta) \\ 0 & otherwise \end{cases}. \tag{5}$$

The η is the threshold of the Schmitt trigger or the comparator and is assumed to be equivalent to the reception sensitivity of the conventional receiver.

The output of the nonlinear device is sampled and summed to detect the subthreshold signal. When the number of samples per symbol is denoted by N, the output of the SR system is expressed as follows.

$$y_i = \sum_{k=1}^{N} y(t)\delta\{t - i(\frac{kT_s}{N})\}$$

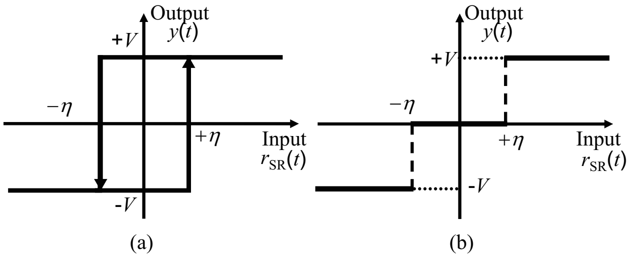

Figure 3: Input-Output characteristics of (a) a Schmitt trigger, (b) a comparator.

where

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}. \tag{6}$$

In the detector, we restore the data as $\hat{d}_i$, depending on whether the output is positive or negative, in the following manner.

$$\hat{d}_i = \begin{cases} -1 & (y_i \leq 0) \\ +1 & (y_i > 0) \end{cases}. \tag{7}$$

This means that the detector performs the major decision, and as N increases, BER performance can be improved. The BER is analytically derived in the next section.

3. BER analysis for SR

3.1. Schmitt trigger

We propose an analysis method of a Schmitt trigger using a markov model shown in Fig. 4. This model has two states shown as '0' and '1'. In the state '0', the Schmitt trigger gives $-V$ as its output, and in the state '1', it gives $+V$ as its output. In this paper, the initial state of this model is set to be '0'. The probability of transition from state i to state j, $i, j \in \{0, 1\}$, is denoted by P_{ij} as shown in Fig. 4, and the output at each transition is also shown. Their transition probabilities are given as follows.

$$P_{00} = 1 - \frac{1}{2}\mathrm{erfc}(\frac{\eta - As(t)}{\sqrt{2(\sigma_c^2 + \sigma_{SR}^2)}}) \tag{8}$$

$$P_{01} = \frac{1}{2}\mathrm{erfc}(\frac{\eta - As(t)}{\sqrt{2(\sigma_c^2 + \sigma_{SR}^2)}}) \tag{9}$$

$$P_{10} = \frac{1}{2}\mathrm{erfc}(\frac{\eta + As(t)}{\sqrt{2(\sigma_c^2 + \sigma_{SR}^2)}}) \tag{10}$$

$$P_{11} = 1 - \frac{1}{2}\mathrm{erfc}(\frac{\eta + As(t)}{\sqrt{2(\sigma_c^2 + \sigma_{SR}^2)}}) \tag{11}$$

The BER is given by depending on whether the output level of the SR system y_i is positive or negative. Thus, the BER is expressed as follows.

$$\begin{aligned} \mathrm{BER} &= P[d_i \neq \hat{d}_i] \\ &= P[y_i < 0 | d_i = +1]P[d_i = +1] \\ &\quad + P[y_i \geq 0 | d_i = -1]P[d_i = -1]. \end{aligned} \tag{12}$$

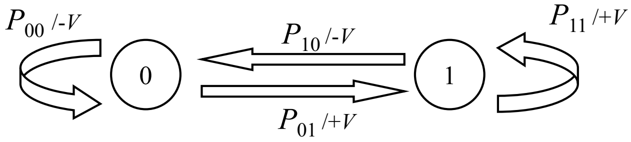

Figure 4: Analysis model for Schmitt Trigger.

Over N samples of the SR system, the all error patterns are calculated directly. For example, when N is 3, and d_i is +1, the errors occur in the transitions '0'→'0'→'0', '0'→'0'→'1', '0'→'1'→'0', and '1'→'0'→'0'. Assuming the initial state of the Schmitt trigger to be '0', this error probability is given as follows.

$$\begin{aligned} P[y_i < 0|d_i = +1] = & P_{00} \times P_{00} \times P_{00} \\ & + P_{00} \times P_{00} \times P_{01} \\ & + P_{00} \times P_{01} \times P_{10} \\ & + P_{01} \times P_{10} \times P_{00} \end{aligned} \quad (13)$$

$P[y_i \geq 0|d_i = -1]$ is also derived in the same way, and thus BER can be calculated directly.

3.2. Comparator

The output of the comparator can stochastically take the value $-V$, $+V$, or 0 depending on the input $r_{SR}(t)$, which is Gaussian with mean $As(t)$ and variance $\sigma_c^2 + \sigma_{SR}^2$. The probability that the output takes the value $\pm V$ is given by

$$P[y(t) = \pm V] = \frac{1}{2}\mathrm{erfc}\left(\frac{\eta_{comp} \mp As(t)}{\sqrt{2(\sigma_c^2 + \sigma_{SR}^2)}}\right), \quad (14)$$

where $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{+\infty} \exp(-t^2)dt$ is the complementary error function. From the above equation, the probability that the output takes the value 0 is given as follows.

$$\begin{aligned} P[y(t) = 0] = & 1 - P[y(t) = -V] \\ & - P[y(t) = +V]. \end{aligned} \quad (15)$$

The samples of the output of the comparator are statistically independent during each symbol. Thus, the BER is given by counting the sampled times of these values. Over N samples, the probability that the value $-V$ is observed n times and the value $+V$ is observed m times is given by the multinomial distribution, which is expressed as

$$P(n, m) = \frac{N!}{n!m!(N-n-m)!} P_-^n P_+^m (1 - P_- - P_+)^{N-n-m}, \quad (16)$$

where P_- is the probability that the output takes $-V$, and P_+ is the probability that the output takes $+V$. When the transmitted data d_i is +1, the probability that the output level of the SR system y_i is negative is equivalently given by the probability that the number n is smaller than the number m, which is expressed as follows.

$$P[y_i \leq 0|d_i = +1] = \sum_{n \leq m} \sum_{n}^{\lfloor N/2 \rfloor} P[n, m|d_i = +1]. \quad (17)$$

Table 1: Parameter settings for the computer simulation.

Parameter	Value
Channel and intentional noise	AWGN
PSD of Channel noise [10^{-21} W/Hz]	4.1
Symbol duration T_s [μsec]	1.0
Received signal amplitude A [μV]	1.0
Sensitivity of the conventional receiver η [μV]	1.1
Numbers of samples per symbol N	3,4,5

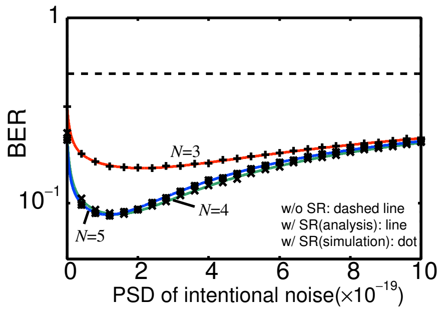

Figure 5: BER performance of the SR reiceiver using a Schmitt trigger.

Similarly, when the transmitted data d_i is -1, the probability that the output y_i is positive can be derived. Thus, the BER can be calculated by Eq. 12.

4. Numerical results

4.1. BER performance of the SR receiver using a Schmitt trigger

Figure 5 shows the BER performance versus the PSD of the intentional noise in the SR receiver using a Schmitt trigger. The analytical, simulation, and conventional results are shown by the solid line, dots, and dashed line, respectively. Parameter settings are shown in Table 1. As shown in Fig. 5, the BER performance of the SR receiver is improved with increased PSD of the intentional noise compared with that of the conventional receiver. This is a typical phenomenon exhibiting SR. This figure also shows the analysis method is appropriate to this system model since the analytical results are perfectly consistent with the simulation results.

As shown in Fig. 5, the BER is improved by increased N. However, the results of $N = 4$ and $N = 5$ are almost the same results. This depends on whether N is odd or even. When N is odd, the decision has no problem, but when N is even, the receiver has the possibility of $y_i = 0$. In this situation, the receiver must decide $d_i = +1$ or -1. In the system model, this decision depends on the initial state of the Schmitt trigger. Since the initial state '0' larger the probability that $-V$ is sampled, the receiver should decide $d_i = +1$ at $y_i = 0$ to decrease the error at the transmitted data $d_i = +1$. Thus, in Eq. 12, the error at $y_i = 0$ happens at $d_i = -1$. The difference of the BER performance between the odd N and even N depends on the initial state of the Schmitt trigger or the memory effect. This is a unique property of the SR receiver using Schmitt trigger.

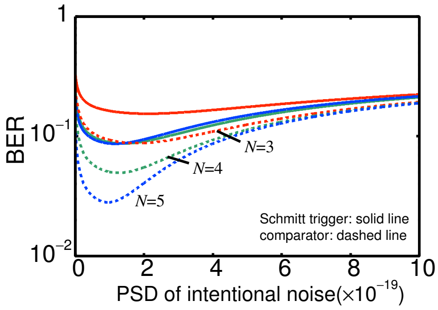

Figure 6: BER performance of the SR reiceiver using a Schmitt trigger (Fig. 5) and a comparator.

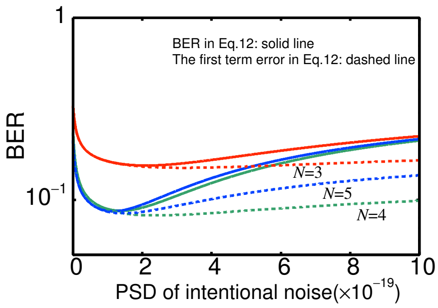

Figure 7: BER performance of the SR reiceiver using a Schmitt trigger (Fig. 5) and the first term error in Eq. 12.

4.2. A performance comparison of a Schmitt trigger and a comparator

Figure 6 shows the BER performance versus the PSD of the intentional noise in the SR receiver using a Schmitt trigger and a comparator. As shown in Fig. 6, the performance of the comparator is better than that of the Schmitt trigger. This is an unexpected result since a Schmitt trigger has a memory effect, and it should have better performance than a comparator. Why is the performance of the comparator better than that of the Schmitt trigger?

To consider this problem, we focus on the dominant error of the Schmitt trigger. Since the initial state of the Schmitt trigger is an important factor for the BER performance, we use the only first term in Eq. 12 and neglect the second term. When the initial state is '0', $P[y_i \leq 0|d_i = +1]P[d_i = +1]$ should be larger than $P[y_i > 0|d_i = -1]P[d_i = -1]$. Figure 7 shows the BER performance of the SR receiver using a Schmitt trigger (Fig. 5) and the first term error in Eq. 12. As shown in Fig, 7, the first term error is almost equal to be the BER in the relatively small PSD of the intentional noise. This shows the first term error is the dominant error of the SR receiver using Schmitt trigger. The difference between the first term error and the second term error is given by the initial state of the Schmitt trigger or the memory effect. Thus, the memory effect of the Schmitt trigger could have a bad effect on the BER performance.

In contrast, the comparator in this system have better performance. In general, a simple comparator that has only one threshold has a bad performance, but the comparator used in this paper has two symmetric thresholds, and could be good compatibility with communication systems.

5. Conclusion

We proposed an analysis method for an SR receiver using a Schmitt trigger. We also evaluated its BER performance and presented a performance comparison of the comparator and the Schmitt trigger. Numerical results show that improvement of BER performance and communication at the subthreshold level can be achieved by the SR system. The performance comparison shows the performance of the comparator is better than that of the Schmitt trigger. This is due to the memory effect of the Schmitt Trigger, and the good compatibility of the comparator in this system model with communication systems.

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