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Effects of a nonlinear packet drop probability function on RED performance

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Abstract—This paper introduces nonlinearity to the packet drop probability function of *Random Early Detection* (RED) in active queue management; that is, an algorithm to control network congestions by actively managing a router's queue. We investigate the nonlinearity contribution in terms of the fairness index, average throughput, average queue size, and average round trip time. The effects of the imposed nonlinearity on the RED performance are investigated under different traffic loads. When the network traffic is not too heavy, the RED with the nonlinearity achieves better network performance than the original RED with a linear packet drop probability function in all the statistics except for the fairness index. This work suggests the performance improvement of networks by optimizing the nonlinearity degree of the packet drop probability.

1. Introduction

In recent years, packets flowing on networks have increased more and more. The packets sent from network devices are stored in a buffer in the network's routers. When the buffer capacity is exceeded, the buffer overflows allow the network to be congested. The last technologies have realized more buffer capacity and improved the throughput, which greatly relieves the congestions caused by the buffer overflows. The cause of the congestions then moves to the queuing delay. As mentioned above, the congestion factors depend on situations, so that the congestions should be appropriately controlled in each situation to improve or solve network congestions.

The transmission control protocol (TCP) ensures the network reliability and the network congestions can be controlled by various algorithms. These algorithms are broadly categorized into end-to-end window-based algorithms [1] or router-based algorithms. Both types of the algorithms can appropriately control the congestions. Router-based congestion controlling algorithms implement two types of the queue managements: passive queue management (PQM) and active queue management (AQM) [2]. The AQM more efficiently controls the queuing delay than Tail-Drop that is a traditional PQM-based algorithm.

As the most representative AQM algorithm, the RED [3] has been extended by many researchers [4,5]. The RED algorithm computes the average queue size in a router to converge within a specific range and randomly discards packets to decrease the queuing delay. The RED's performance depends on the packet drop probability and the characteristics of the packet drop probability. The probability function includes three parameters that should be optimized for high performance. Although the optimal values of the RED parameters have been intensively searched, it is difficult to find them because they depend on the conditions, situations, and/or environments of the networks. In the original RED, the packet drop probability is a linear function of the average queue size. This linear function can be replaced with nonlinear functions. The RED algorithm with nonlinearity can potentially outperform the RED, but the best types of nonlinearity have not been clarified. Thus, finding the optimal nonlinear functions is essential for improving the throughput.

Among the nonlinearities tested to date are active ARED [4], nonlinear RED (NLRED) [5], and FWM-RED [6]. Nishiuchi et al. [7] examined quadratic, cubic, and exponential functions of the packet drop probability, and evaluated their performance in Network Simulator-2 (NS-2) [8]. All the algorithms accomplish high performance and fairness, but NLRED [5] with quadratic functions as the packet drop probability showed more robustness for the network traffic. However, to optimize the degree of the nonlinearity in the NLRED, it should be investigated how it affects the performance under various traffic conditions.

The current study aims to understand the effects of the nonlinearity in the packet drop probability function on the RED performance under various traffic loads. The current

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study chooses an exponential one [9] as a nonlinear function whose exponent is easily and precisely adjustable instead of the linear function in the original RED. The RED with the nonlinearity is implemented into a network on NS-2, and its performance is evaluated with fairness index (FI), average throughput, queue size, and round trip time (RTT) varying the nonlinearity degree and network loads. Based on the simulation results, we find the optimal strength of the nonlinearity in the packet drop probability for the RED performance (i.e., maximizes the throughput while minimizing the queuing delay).

2. Nonlinear packet drop probability function in RED

The RED [3] algorithm controls the network congestions using the average queue size $\overline{q}$ in a router and the packet drop probability $p(\overline{q})$. We employ a nonlinear function on $p(\overline{q})$. The average queue size and the packet drop probability are respectively defined as follows:

$$\overline{q} \leftarrow (1 - w_q) \cdot \overline{q} + w_q \cdot q \tag{1}$$

$$p(\overline{q}) = \begin{cases} 0 & \text{for } \overline{q} \leq q_{\min} \\ 1 & \text{for } \overline{q} \geq q_{\max} \\ p_{\max} \cdot \dfrac{\overline{q} - q_{\min}}{q_{\max} - q_{\min}} & \\ \quad \cdot \exp\left\{\gamma\left(\dfrac{\overline{q} - q_{\min}}{q_{\max} - q_{\min}} - 1\right)\right\} & \text{otherwise} \end{cases} \tag{2}$$

The queue weight w_q is a parameter in $\overline{q}$, and q is the instantaneous queue size. In Eq. (2), the maximum packet drop probability $p_{\max}$, the minimum queue threshold $q_{\min}$, and the maximum queue threshold $q_{\max}$ determine the shape of $p(\overline{q})$. The parameter γ adjusts the bending degree of the nonlinearity in the nonlinear function. Hereafter, the bending degree of the nonlinear function is called the nonlinearity strength. The RED with the nonlinear function is equivalent to the original RED when $\gamma = 0$ and larger γ strengthens the nonlinearity of $p(\overline{q})$. Therefore, we can precisely evaluate the influence of the nonlinearity strength of $p(\overline{q})$ on the network performance.

The moment the packet arrives, $\overline{q}$ is updated with the weighted summation of the current q and the $\overline{q}$ value just before the packet arrival (see Eq. (1)). As shown in Eq. (2) and Fig. 1, $p(\overline{q})$ increases exponentially between $q_{\min}$ and $q_{\max}$. The RED differently deals with packets under three conditions: when $\overline{q} \leq q_{\min}$, no packets are discarded, when $\overline{q} \geq q_{\max}$, all packets are discarded, and when $q_{\min} < \overline{q} < q_{\max}$, packets are randomly discarded based on $p(\overline{q})$.

3. Simulation setup

3.1. Network environment

The NS-2 [8] was a network simulator developed in the Visual Internet Testbed project supported by the Defense Advanced Research Projects Agency in the United States.

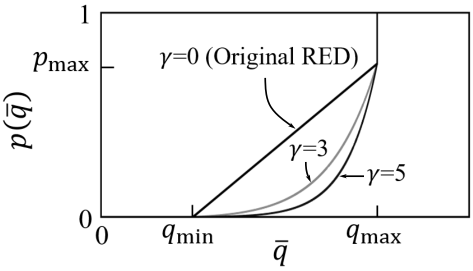

Figure 1: Nonlinear packet drop probability function $p(\overline{q})$.

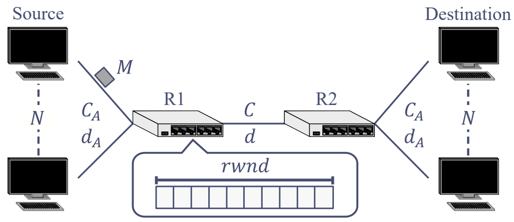

Figure 2: Network configuration.

Table 1: Network parameters.

Packet size M [bit]	8,320
Access link capacity C_A [Mbps]	1,000
Bottleneck link capacity C [Mbps]	100
Propagation delay (Access link) d_A [sec.]	0.001
Propagation delay (Bottleneck link) d [sec.]	0.010
Maximum window size *rwnd* [packet]	10,000
Buffer size B [packet]	500
Number of TCP connections N	1–10

Table 2: RED parameters.

Queue weight w_q	8.32×10^{-5}
Maximum packet drop probability $p_{\max}$	0.02
Minimum queue threshold $q_{\min}$ [packet]	30
Maximum queue threshold $q_{\max}$ [packet]	90
Nonlinearity strength γ	0–15

We constructed a RED-based network with a dumbbell topology and evaluated the nonlinearity strength contributions to the network performance (Fig. 2). The dumbbell topology was widely used for the performance evaluations in the NS-2 simulator. The RED with $p(\overline{q})$ defined by Eq. (2) was applied to the router R1 while Tail-Drop, that is the most basic control algorithm for the buffers, was implemented in the router R2. The parameter values in our simulations are same as ones in Refs. [3, 10] and are summarized in Tables 1 and 2. The network traffic was simulated for 240 s where the period from 60 to 240 s was regarded as the steady-state period of the network. Using the steady-state data, we analyzed the effect of number of TCP connections N and the nonlinearity parameter γ on the network performance.

3.2. Quantifiers for performance evaluation

The network performance was quantified in terms of the FI [11] of the throughput, the average throughput, and the average RTT.

The FI evaluates the fairness of the throughput at each source [11]. The FI range is between zero and one. The larger the FI is, the higher the fairness of the throughput is [11]. The throughput defines the amount of data transmitted per unit time. High throughput is an indicator of high network performance. The RTT is the time required for a packet to travel around between a specific source and a specific destination. The RTT quantifies how efficiently the algorithms implemented in the routers manage queuing delays. This RTT is an essential factor of network performance evaluations.

4. Simulation results

4.1. Dependence of TCP connections on bandwidth utilization

We first vary the number of the TCP connections N with the original RED ($\gamma = 0$) and confirm the effect of N on the bandwidth utilization, namely, the traffic loads on the network. The results are shown in Table 3. The bandwidth utilization exponentially increases with N and exceeded 99% for $N \geq 4$, indicating that the bandwidth utilization is sensitive to N.

Based on the bandwidth utilization results, we categorize the network traffic conditions into three categories: light (the bandwidth utilization is around 87% for $N = 1$), heavy (the bandwidth utilization is around 98% for $N = 3$), and significantly heavy (the bandwidth utilization is around 100% for $N = 10$). Then, we compared the FI, the average throughput, the average queue size, and the average RTT with different values of γ among the three conditions.

4.2. Relationship between four statistics (fairness index, average throughput, average queue size, and average RTT) and nonlinearity strength

To understand the effects of the nonlinearity strength in $p(\overline{q})$ on the performance under the three traffic conditions, we observe the FI, the average throughput, the average queue size $\overline{q}$, and the average RTT in the range $0 \leq \gamma \leq 15$.

Under the light traffic condition ($N = 1$; Table 4), the FI function of γ is flat. This indicated the FI is unaffected by the nonlinearity in $p(\overline{q})$. Under the higher traffic condition ($N = 3$ and 10; Table 4), the FI slightly fluctuates with γ but is always high. Sup et al. [12] have reported that the original RED is capable of high throughput fairness. As shown in Table 4, the fairness index is almost unity at $\gamma = 0$, which is consistent with Ref. [12]. Moreover, the FI is robust against various traffic loads and γ. More specifically, increasing γ little affects the fairness of the throughput under various traffic loads. Therefore, the high throughput of the original RED is preserved by the nonlinear packet drop probability function.

Table 5 shows the average throughput as γ. Under the light and heavy traffic conditions ($N = 1$ and 3), the average throughput monotonically increases with γ. This is because larger γ reduces the packet drop probability and prevents the excessive discarding of packets. However, the bandwidth utilization reaches around 100% with $N = 10$, as shown in Table 3, and the average throughput is constant for any values of γ under the significantly heavy traffic condition ($N = 10$; Table 5).

Table 3: Influence of the number of TCP connections N on bandwidth utilization using the original RED ($\gamma = 0$).

N	Bandwidth utilization [%]
1	87.6532
2	96.8100
3	98.1142
4	99.0375
5	99.5876
6	99.8252
7	99.8743
8	99.9052
9	99.9376
10	99.9323

A large $\overline{q}$ indicates an increase in the amount of transmission data (i.e., improved throughput). In contrast, an increase in the RTT induces larger queuing delay and degrades the performance. Tables 6 and 7 show $\overline{q}$ and the average RTT for the various values of γ. Under all the traffic conditions, $\overline{q}$ and the average RTT monotonically increase as γ increases. They increase more rapidly against the increase of γ for larger N.

Our simulation results show that larger γ allows the throughput to be higher but also the average RTT to be larger. Because the bandwidth utilization almost reaches the bottleneck link capacity, the average throughput keeps a constant. However, the average RTT tends to increase as larger γ. From our results, we believe the optimal nonlinearity strength in the RED is realized by around $\gamma = 3$ because of the maximum throughput and the minimum queuing delay.

5. Conclusions

We introduced a nonlinear packet drop probability function [9] with an adjustable nonlinearity parameter that covers the original linear function in the RED. The effects of nonlinearity on the network performance were evaluated in terms of the bandwidth utilization, the FI, the average throughput, the average queue size, and the average RTT. The statistical measures were evaluated in a network with the dumbbell topology, which was simulated on the network simulator NS-2. The traffic conditions were varied from light to significantly heavy.

Based on the analyses, we found the direct relationships between the RED performance and the nonlinearity degree in the packet drop probability under the various traffic loads. The nonlinearity in the probability function improved the performance under light and heavy traffic loads but was ineffective under significantly heavy traffic loads. The lower the traffic load was, the more influential the nonlinearity on the throughput was. In contrast, heavier traffic conditions enhanced the effects of the nonlinearity on the average queue size and the average RTT. The nonlinearity did not noticeably affect the FI. Therefore, from these facts, we conclude the optimal value of γ is around three because the value leads the RED to the saturated throughput and the minimum queuing delay under any traffic conditions.

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Table 4: Relationship between the fairness index (FI) and the nonlinearity strength γ in $p(\overline{q})$.

γ	Fairness index (FI)		
	$N = 1$	$N = 3$	$N = 10$
0	1.00000000	0.99928645	0.99424765
3	1.00000000	0.99966102	0.98934857
6	1.00000000	0.99901978	0.99711903
9	1.00000000	0.99725411	0.99760367
12	1.00000000	0.99923791	0.99624369
15	1.00000000	0.99951933	0.99687327

Table 5: Relationship between the average throughput and nonlinearity strength γ in $p(\overline{q})$.

γ	Average throughput [MBps]		
	$N = 1$	$N = 3$	$N = 10$
0	10.95664869	12.26427686	12.49153947
3	11.12064765	12.41929565	12.49700626
6	11.33184452	12.47176591	12.49714643
9	11.45642747	12.48469808	12.49749913
12	11.53638991	12.49037286	12.49436556
15	11.59476104	12.48994330	12.49153043

Table 6: Relationship between the average queue size ($\overline{q}$) and nonlinearity strength γ in $p(\overline{q})$.

γ	Average queue size [packet]		
	$N = 1$	$N = 3$	$N = 10$
0	12.14545329	28.06458115	36.51818253
3	15.65262768	36.59416766	57.73895033
6	20.70861360	50.11414592	69.40926058
9	25.00759042	58.74425322	75.02533626
12	27.64833212	64.19811188	77.96501410
15	29.35112049	67.60265929	79.72730639

Table 7: Relationship between the average RTT and nonlinearity strength γ in $p(\overline{q})$.

γ	Average round-trip time [ms]		
	$N = 1$	$N = 3$	$N = 10$
0	24.99906838	25.03974927	25.00178665
3	25.00000000	25.11137163	25.96405797
6	25.12869602	25.63629373	29.14687652
9	25.30198351	26.92937564	31.00130307
12	25.47461871	28.00153965	31.44120248
15	25.64302059	28.76889512	31.55931384

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