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Causality Detection in Complex Time Dependent Systems Examplified in Financial Time Series

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Abstract— This paper examines causal relationships between the three factors in the Fama-French model and investigates, whether the efficient-market hypothesis is a suitable assumption in describing excess stock returns. Three methods were used to detect causal dependencies, namely the crosscorrelation method, the Granger causality approach and Transfer Entropy. The excess market return was found to be the leading model factor. Adding its lagged values improved the model fit.

1. Introduction

In finance a standard problem consists of predicting the future value of a financial time series. In an autoregressive model the future value is a linear function of its preceding values. More sophisticated methods may include the history of other variables. Methods detecting causalities are useful tools to investigate the impact of certain variables on others. In this paper the quantities of interest are the variables involved in the Fama-French model.

The Fama-French model describes the expected return on a stock as a linear function of three macroeconomic variables. By denoting R_i the return on stock i, its excess return over the riskfree rate R_F is given by

$$\mathbb{E}(R_i) - R_F = \beta_i(\mathbb{E}(R_M) - R_F) + s_i\mathbb{E}(SMB) + h_i\mathbb{E}(HML), \quad (1)$$

where R_M is the return on the market portfolio [4]. The first model factor is therefore the excess market return with weight β_i. The second model factor, SMB, is defined by the difference between the return on stocks of firms with small and large market capitalization, while the third variable, HML, describes the difference in the returns between stocks with high and low book-to-market ratio. The respective weights, s_i and h_i, as well as β_i, are real numbers.

In the following we assume expected returns to be accurately described by realized returns. This is justified by the rationale expectation theory, claiming realized returns to be realizations from the ex ante distributions of returns [6]. By denoting the realized excess return on the market and stock i at time t by $RMRF(t)$ and $ER_i(t)$ respectively, equation (1) in its realized form is given by

$$ER_i(t) = \beta_i RMRF(t) + s_i SMB(t) + h_i HML(t) + \xi_i(t), \quad (2)$$

where $\xi_i(t)$ is an error term which has zero expectation in time and is uncorrelated to $RMRF$, SMB and HML.

This paper examines, whether there is a driving force among the three variables describing excess stock returns by the Fama-French model. Also, we investigate, whether it may be meaningful to extend the model by adding lagged values of the driving factor. For this purpose the aim is to detect causalities among the involved financial quantities.

2. Materials and Methods

The time series, which the experiments were based on consist of monthly data from February 1973 to August 2013. Realizations of the risk-free rate as well as the Fama-French factors were taken from Kenneth French's data library (see [1]). These quantities are built upon continuously compounded returns on stocks traded in the US-stock exchanges NYSE, AMEX, and NASDAQ and the risk-free rate is given by the 1-month Treasury bill return. The return on stocks is represented by the realized, continuously compounded monthly return of the US-firm Coca Cola (KO). The underlying price was obtained from the data library "Datastream".

In order to detect causal relationships, we considered three methods. The first two, namely the crosscorrelation and Granger causality method, are well established, while the third, Transfer Entropy, is a more recent approach.

2.1. Crosscorrelation

The crosscorrelation method is based on the definition of the correlation function, which computes the correlation between two random variables. Applying the sample crosscorrelation function to realizations across two different covariance stationary time series, where one of them is shifted by a fixed time lag, one obtains a measure for the linear dependency of one time series on the other by the specified time lag.

2.2. Granger causality

In its general form the Granger causality method compares two conditional distributions of the future random variable of the target process, where one of the conditions is weaker than the other. The conditions are thereby described by sigma-algebras. The smaller is generated by the present and all previous random variables of the target process $(Y_s)_{s\leq t}$, while the larger is generated additionally by the present and all past random variables of the causing process $(X_s)_{s\leq t}$. The target process is said to be Granger caused by the other, if the conditional distributions differ [3].

2.2.1. Linear Least Squares Granger Causality

In order to make the Granger causality method operational, we consider a point forecast and therefore assume the conditional distribution to be well described by the conditional expectation. Then we approximate the later by a linear function in the random variables generating the corresponding sigma-algebra. These simplifications are justified by the facts, that, for strictly stationary processes, the conditional expectation minimizes the prediction error in a least squares sense and that the approximation of the conditional expectation by a linear function is exact in the case of Gaussian processes [2].

By comparing the two resulting linear regression fits with the classical F-test we obtain the linear least squares Granger causality, short LLSGC, method. In statistical terms the two fits are given by

$$y_{s+1} \sim y_s + y_{s-1} + \cdots + y_{s-order+1} \qquad (3)$$

$$\begin{aligned} y_{s+1} &\sim y_s + y_{s-1} + \cdots + y_{s-order+1} \qquad (4) \\ &\quad + x_s + x_{s-1} + \cdots + x_{s-order+1}, \end{aligned}$$

where *order* specifies the number of maximal lagged values incorporated in the regressions. The corresponding p-value being smaller than 0.05 indicates a statistically significant causal relationship between causing and target process.

2.3. Transfer Entropy

Transfer Entropy, a method based on information theory, is a measure for the reduction of uncertainty of the future value y_{t+1} of the target process by additionally knowing the history of the causing process $(X_s)_{s\leq t}$ to its own past $(Y_s)_{s\leq t}$. The uncertainty of the outcome of a random variable can be measured by entropy. For a discrete random variable Y taking values on a set A its entropy is given by

$$H(Y) = \sum_{y\in A} p(y)\log_2\left(\frac{1}{p(y)}\right) \quad \text{bits}, \qquad (5)$$

where $p(\cdot)$ is the probability mass function of Y. Hence, entropy only depends on the probability distribution of a random variable. Transfer entropy is defined upon conditional entropies. The conditional entropy of Y given X is given by

$$H(Y|X) = \sum_{x\in A} p(x) \sum_{y\in A} p(y|x)\log_2\left(\frac{1}{p(y|x)}\right) \quad \text{bits} \qquad (6)$$

and describes the uncertainty of Y given X. Let us now consider two stochastic processes $(X_t)_{t\in\mathbb{Z}}$ and $(Y_t)_{t\in\mathbb{Z}}$ taking the values x_t and y_t in a countable set A and denote by $X_s^{(k)} = (X_s, \cdots, X_{s-k+1})$ the vector containing the last k variables of $(X_t)_t$ starting from X_s, where s is a natural number. Then, Transfer Entropy, $T(Y_{t+1}|Y_t^{(l)}, X_s^{(k)})$, describes the reduction in the uncertainty of Y_{t+1} due to the additional knowledge of $X_s^{(k)}$ given the knowledge of $Y_t^{(l)}$. It is described by the quantity

$$T(Y_{t+1}|Y_t^{(l)}, X_s^{(k)}) = H(Y_{t+1}|Y_t^{(l)}) - H(Y_{t+1}|Y_t^{(l)}, X_s^{(k)}). \qquad (7)$$

By applying the definition of conditional entropy, Transfer Entropy can be expressed by joint and conditional probabilities,

$$T(Y_{t+1}|Y_t^{(l)}, X_s^{(k)}) = \sum_{y_{t+1}\in A, y_t^{(l)}\in A^l, x_s^{(k)}\in A^k} p(y_{t+1}, y_t^{(l)}, x_s^{(k)}) \log_2\left(\frac{p(y_{t+1}|y_t^{(l)}, x_s^{(k)})}{p(y_{t+1}|y_t^{(l)})}\right), \qquad (8)$$

where the measure is in bits and $x_s^{(k)}$ and $y_t^{(l)}$ are vectors of samples of $X_s^{(k)}$ and $Y_t^{(l)}$, respectively [5].

The quantity given in equation (8) can be estimated from time series, which should be realizations from strictly stationary processes. In order to estimate the probabilities reliably, a large amount of data points needs to be considered. Nevertheless, small probability estimations are inevitable, which can lead to an explosion of the estimated Transfer Entropy. Hence, there are several numerical issues to be considered when implementing Transfer Entropy. Note, that unlike the crosscorrelation and Granger causality method, this approach does not rely on linearities.

3. Results and Discussion

The estimated crosscorrelation between the Fama-French factors are significant for several lags. Most of them are found in the crosscorrelation plot between *RMRF* and *SMB* at negative lags (Figure 1), indicating a possible causal influence of *RMRF* on *SMB*. The LLS Granger causality test confirms this observation, since we obtain highly significant values for different orders (Table 1). This is an evidence, that the market capitalization factor strongly depended on the behavior of the excess market return one month ahead. Note, that we use the abbreviation $\sigma_t^{R,S}$ to

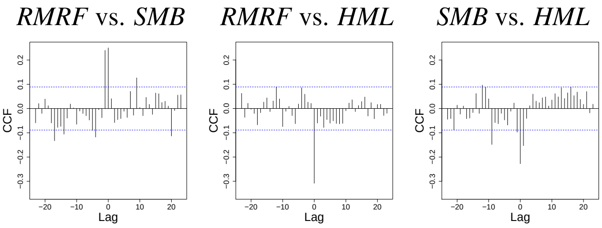

Figure 1: Crosscorrelation plots across the three Fama-French factors.

Order	1	12	24
$\sigma_t^{R,S}$	$3.0 \cdot 10^{-6}$	$6.2 \cdot 10^{-5}$	$2.1 \cdot 10^{-6}$

Table 1: p-values of the LLS Granger causality test, $(RMRF_t)_t \rightarrow (SMB_t)_t$, for different orders.

Order	1	4	12	24
$\sigma_t^{H,S}$	$7.0 \cdot 10^{-3}$	$7.7 \cdot 10^{-2}$	0.15	0.12
$\sigma_t^{H,S,R}$	0.12	0.54	0.79	0.61

Table 3: p-values of the LLS Granger causality test, $(HML_t)_t \rightarrow (SMB_t)_t$, for different orders and sigma-algebras.

Order	0	1	2
$\sigma_t^{R,E_{KO}}$	$1.7 \cdot 10^{-14}$	$2.4 \cdot 10^{-21}$	$1.2 \cdot 10^{-20}$

Table 4: p-values of the LLS Granger causality test (extended by lag 0 values), $(RMRF_t)_t \rightarrow (ER_{KO_t})_t$, for different orders.

denote the sigma-algebra generated by the factors *RMRF* and *SMB*.

Further, the crosscorrelation method applied to *RMRF* and *HML* gives one significant correlation value at lag 0 (Figure 1). The values obtained by the Granger causality approach with *RMRF* being the causing and *HML* the caused variable are nevertheless significant for order 1 and higher orders (Table 2). Therefore, also the book-to-market ratio factor was Granger caused by the excess market return.

Last, the estimated crosscorrelation between *SMB* and *HML* is significant at lag 0 and lag 1 (Figure 1), the later inferring that *HML* influenced the values of *SMB* one month later. The corresponding Granger causality test gives a significant result for order 1, while for higher order numbers they are non-significant (Table 3, first line). Extending the above Granger causality test by including past values of *RMRF* in the regression fits we obtain highly non-significant results already for order 1 (Table 3, second line), confirming the influence of *RMRF* on *SMB*. Hence, we conclude that the market excess return was the driving variable among the Fama-French factors.

The sample crosscorrelation between *RMRF* and the excess return on the Coca Cola stock, ER_{KO}, gives a significant value at lag 0. Together with the highly significant results of the extended version of the LLS Granger causality method, which just takes the contemporaneous values of the causing process into account (Table 4, $order = 0$), it confirms the relevance of the market excess return in a model describing the returns on stocks.

The classical Granger causality test gives highly significant results as well (Table 5), claiming that the excess market return caused excess stock returns. Assuming the efficient-market hypothesis, claiming prices to reflect all past available information, to hold, the last finding would not be of interest. However, this theory may be questioned. The results of the extended LLS Granger causality method for orders 1 and 2 are far more significant than the one for order 0, meaning that predictions for the excess stock return were more accurate when including lagged to the contemporaneous values of *RMRF*.

Results from a Transfer Entropy analysis extend the findings of the LLS Granger causality tests. As described in Sect. 2.3, the estimation of the Transfer Entropy from time series poses algorithmic challenges due to small probabilities. Therefore, the preprocessing is essential. We normalized each of the Fama-French factors onto $[0, 1]$ and then discretized the interval into 20 steps, as depicted in Figure 2. With the preprocessed time series, the Transfer Entropy was then computed (Table 6). We detect the strongest directional influence from HML onto $RMRF$ at time order $s = 12$. The reverse influence, $RMRF$ onto HML is also strong, indicating that HML and $RMRF$ are related, but with unclear causality structure. The directional influence of SMB onto $RMRF$ at time order 12 is also noticeable, here the reverse influence, $RMRF$ onto SMB at the same time order is considerably lower, indicating a causal influence of SMB onto $RMRF$. Note that the time orders 1 and 12 exhibit higher Transfer Entropy values than 24 in all cases except for the $RMRF$ influence onto SMB.

The LLS Granger causality results in Table 3 for time orders 12 and 24, HML onto SMB without inclusion of $RMRF$, oppose the results of the Transfer Entropy anal-

Order	1	12	24
$\sigma_t^{R,H}$	$2.0 \cdot 10^{-2}$	$1.5 \cdot 10^{-2}$	$2.5 \cdot 10^{-2}$

Table 2: p-values of the LLS Granger causality test, $(RMRF_t)_t \rightarrow (HML_t)_t$, for different orders.

Order	1	12	24
$\sigma_t^{R,E_{KO}}$	$3.0 \cdot 10^{-7}$	$1.4 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$

Table 5: p-values of the LLS Granger causality test, $(RMRF_t)_t \rightarrow (ER_{KO_t})_t$, for different orders.

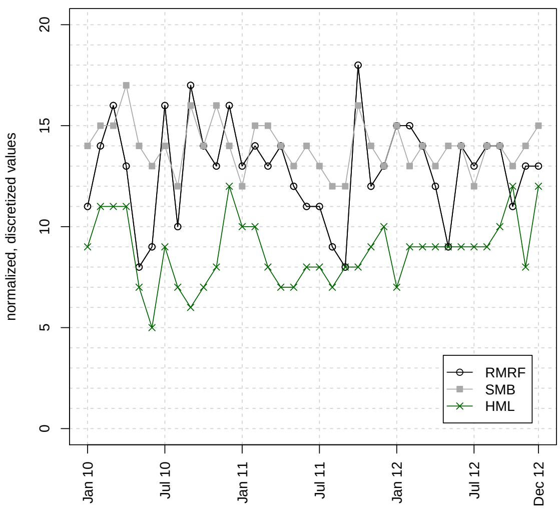

Figure 2: Fama-French factors, normalized to $[0, 1]$, discretized into 20 steps as preparation for Transfer Entropy analysis. Only a fraction of the full time span is shown, not exploiting the whole range of values.

s [month]	1	12	24
RMRF --> SMB	0.2158	0.2058	0.2370
RMRF --> HML	0.2775	0.2887	0.2870
SMB --> RMRF	0.2390	0.2707	0.2505
SMB --> HML	0.2637	0.2611	0.2446
HML --> RMRF	0.2755	0.3014	0.2909
HML --> SMB	0.2040	0.2250	0.2241

Table 6: Transfer Entropy results for different time lags s.

ysis. LLS Granger suggests that orders 12 and 24 show less strong (even non-significant) influences than order 1. Similarly opposing is the ranking of the influences *RMRF* onto HML, which gets high ranking in the Transfer Entropy analysis and *RMRF* onto SMB, getting lower ranking. LLS Granger causality suggests a reverse order of the strength of influence, judging from p-values that differ by orders of magnitude (compare Table 2 and Table 1).

These differences may be due to various effects. Once, Transfer Entropy may have caught non-linear interactions. Secondly, the required assumption of normally distributed residuals required for LLS Granger is not perfectly satisfied (not shown here). This could lead to inaccuracies in the results. Thirdly, the processes look heteroschedastic (Figure 2), and therefore violate the assumption of stationarity (a KPSS test for instationarity does not strictly confirm this, but shows tendencies for HML being instationary, $p = 0.066$).

Our findings can be summarized as follows. Since both approaches, the crosscorrelation and the Granger causality method, are based on linearities, their test results are mainly supported by each other. Applied to the financial time series involved in equation (2), we conclude, that the excess market return was the leading variable among the three Fama-French factors. *SMB* and *HML* depended on *RMRF*, while the later significantly caused the excess stock return. Further, the efficient-market hypothesis, which the Fama-French model relies on, was questioned in this setting, since the test results of the extended Granger causality method improved when including lagged values of *RMRF* in addition to the contemporaneous values in the regression fits. We therefore suggest to incorporate lagged values of the excess market return in describing excess stock returns. Transfer Entropy does not confirm all the findings of LLS Granger. It suggests that $RMRF$ is influenced by SMB, not the other way round. The influence of $RMRF$ onto HML is compatible with the findings of LLS Granger.

4. Outlook

Detecting causality structures is an intrinsically difficult problem. Not only are there strong assumptions on the time series (stationarity and stationarity in dependence structure), but the results typically depend onto parametrization (the time lags involved in LLS Granger and Transfer Entropy) as well as preprocessing (normalization and binning for Transfer Entropy). The authors are aware of the analysis presented being preliminary, there are still many steps to take in order to build reliable causality detection algorithms.

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