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Sequential Superparamagnetic Clustering as a Predictor of Visual Fixations

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Abstract—Clustering of different visual patterns from background noise is a task which is easily performed by the human visual system but still proves to be difficult problem for artificial systems. The Sequential Superparamagnetic Clustering (SSC) [1] algorithm can be used to cluster the visual patterns and provides a relative measure of stability for each cluster. By using an eye tracking aparatus which measures the human visual scan paths, a comparisson between statistical properties of the scan paths, and the relative stability for the clusters given by the SSC algorithm is determined. This comparisson of visual scan paths and relative stability provides insight into how the human visual system performs the task of clustering and quantifies how similar the SSC algorithm is to the human visual system.

1. Introduction

When viewing a scene that is larger than the human eyes field of view, the eye will perform rapid eye movements or saccades, followed by time intervals where the eye is stationary on areas of interest which are defined as visual fixations [2], these combined saccadic motions and intervals of visual fixations are collectivly known as the visual scan path. An open problem in understanding visual perception is to model the relationship between the visual scan path and the properties of the visual scene that is being examined. Visual information is supressed during the saccadic motion and gained during the visual fixation periods on areas of interest [3], as such a quatitative measure relating these areas of interest could provide valuable insight into modelling the visual scan paths of human vision. The use of clustering algorithms to segment images into areas of interest is well documented[], however these clustering algorithms do not provide any measure or ordering between these clustered areas of interest which would be required in order to develop a model relating the visual scan path to the properties of the visual scene. The Sequential Superparamagnetic Clustering (SSC) algorithm [1], based on a statistical physics model of magnetic spin systems, provides an ordering of it's output clusters based on a temperature measure of stability. By clustering the visual scene using the SSC algorithm, the ordering of stability induces a ordering on the areas of interest in the visual scene which can be compared to the statistical properties of the visual fixations in these same areas of interest. Thus a model of the visual scan paths based on the output of the SSC algorithm can be defined. Section 2 provides a brief overview of the SSC algorithm and it's application to clustering visual scenes as well as the output of the algorithm for the visual scene used for the experiment. Section 3 is a description of the apparatus used to measure the visual scan paths and the visual scene used during the experiment. Section 4 explains how the statistics of the visual scan paths are related to the stability measure of the SSC algorithm. Finally section 5 shows the experimental results as well as a discussion on future research

2. The SSC Algorithm and Clustering of Visual Images

The task of clustering is defined as the grouping of similar objects. Clustering has found applications in diverse fields such as bioinformatics [4], chemoinformatics [5], neuroscience [6]. As the concept of similarity between objects is ambiguous the clustering problem is inhearently ill defined. The sucess of a clustering algorithm will therefore depend upon the relavance of the similarity measure to the goal of the clustering task. When the similarity is defined by by multiple attributes it is natural to define a fuzzy description of the cluster wherein the mebership of a object to a cluster is determined by a likeihood function. Furthermore clusters of objects inherently emerge in nested hiearchies. While the success of the clustering is ambiguous there are some requirements that are beneficial for a a clustering algorithm to posess. The following list of requirements are all satisfied by the SSC clustering agorithm.

- The SSC algorithm provides a unique clustering hiearchy and provide a measure of of the naturalness of the cluster. The notion of a natural cluster is a group without any significant substructure.

- The SSC algorithm does not assume any apriori information about shape or internal distribution of the clusters or have the number of clusters predefined.
- The SSC algorithm is based on a set of pairwise affinities which allows the optimizing ofresults by optimizing the measure of similarity.
- The SSC algorithm can easily deal with clusters of different shapes, densities and largely unequal distances between clusters.

While other clustering algorithms are often optimized for special situations they do not take the above requirements into account. In particular the fourth requirement is typically overlooked and as such the standard clustering algorithms have great difficulties in finding natural clusters for inhomogeneous distributions of objects [7] which are prevalent in real world applications. As the SSC algorithm has been completly described in [1], the following is only a brief outline of the technical details of the algorithm.
In order to cluster N objects with pairwise affinities d_{ij}, and inhomogeneous grid of of Potts spins is constructed as follows: each object idexed by i is represented by one site of the grid with Potts spin variable s_i where $s_i \epsilon \{1, \ldots, q\}$. q is typically chosen between 10 and 20 [1] (however the choice of q does not effect the number of clustes to be found). Each spin is symetrically coupled to it's k neighboors with a coupling strength

$$J_{ij} = J_{ji} = \frac{1}{K} exp\left(\frac{-d_{ij}^2}{2a^2}\right) \tag{1}$$

where K is the average number of coupled neighboors per site, and a is a scale length. Each spin configuration is characterized by an energy expressed by the Potts spin Hamiltonian

$$H(s) = \sum_{(i,j)} J_{ij} \left(1 - \delta_{s_i, s_j}\right) \tag{2}$$

where s_i, s_j denotes a spin configuration. Using equation (1) and equation(2) the probabilty of a spin configuration is given by

$$p(s) = \frac{1}{Z} exp\left(-H(s)/T\right) \tag{3}$$

For small temperatures T, like spins will be aligned. As T increases the system transitions from a superparamagnetic phase where strongly coupled spins are aligned and weakly coupled spins behave independantly, as so clusters of like aligned spins (or objects) begin to emerge. At a high temperature T the system enters a parametric phase wherewhere only singleton clusters of objects remain. As the temperature T increases existing clusters are broken up into smaller subclusters as so the SSC algorithm satisfies the requirement that a clustering algorithm should provide a hiearchy of clusters of objects. For any given temperature T any two spins or objects i, j belong to the same cluster if the pair correlation

$$G_{ij} = \sum_{s} p(s) \delta_{s_i s_j} \tag{4}$$

exceeeds a given threshold. The calculation of the pairwise correlation given by equation 6 is not feasible for large data sets but can be approximated using a belief propagation algorithm, which results in the approximation

$$G_{ij} \approx \sum_{s_i s_j} b_{ij}(s_i, s_j) \delta_{s_i s_j} \tag{5}$$

The beliefs $b_{ij}(s_i, s_j)$ are given by

$$b_{ij}(s_i s_j) = c \prod_{k \in N(i)/j} m_{k \to i}^{\infty}(s_i) \prod_{l \in N(j)/i} m_{l \to j}^{\infty}(s_j) \tag{6}$$

where c is a temperature based coefficient described fully in []. The messages $m_{k \to i}^{\infty}(s_i)$, $m_{l \to j}^{\infty}(s_j)$ are determined by an iterative message update

$$m_{p \to q}^{t+1}(s_q) = k \sum_{s_p} e^{2 J_{pq} \delta_{s_p s_q}/T} \prod_{r \in N(p)/q} m_{r \to p}^{t}(s_p) \tag{7}$$

with initial messages $m_{p \to q}^0(1) = y$, $m_{p \to q}^0(-1) = 1 - y$ and $0.5 < y < 1$ for all connections $p \to q$.

2.1. Applying the SSC to visual images

The SSC algorith can be used to cluster a image by considering each pixel $i = (x_i, y_i)$ as an object and defing a distance measure between pixels by

$$d_{i,j} = \alpha |I(x_i, y_i) - I(x_j, y_j)| \times |(x_i, y_i) - (x_j, y_j)| \tag{8}$$

which weights the euclidean distance between pixels by the difference in pixel intensities denoted by $I(x, y)$, with an additional weighting factor α. Given the image shown in figure (1) and a distance measure of equation (8), applying the SSC algorithm produced the desired clusters which are the most obvious to the viewer. Figure (2) shows the output heirarchy of clusters for the image shown in figure 1.

3. Measuring the Visual Scan Path

The visual scan path described in section 1 was measured using a "Dr. Bouis" occulometer. The occulometer homogenuously illuminates the eye with infrared light and measures the reflection onto a two dimensional detector which yields separate vertical and

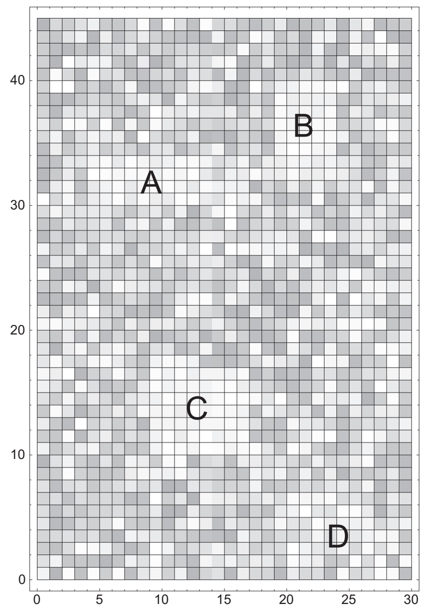

Figure 1: Test Image with four visible clusters labeled A, B, C, and D. The background cluster is not labeled

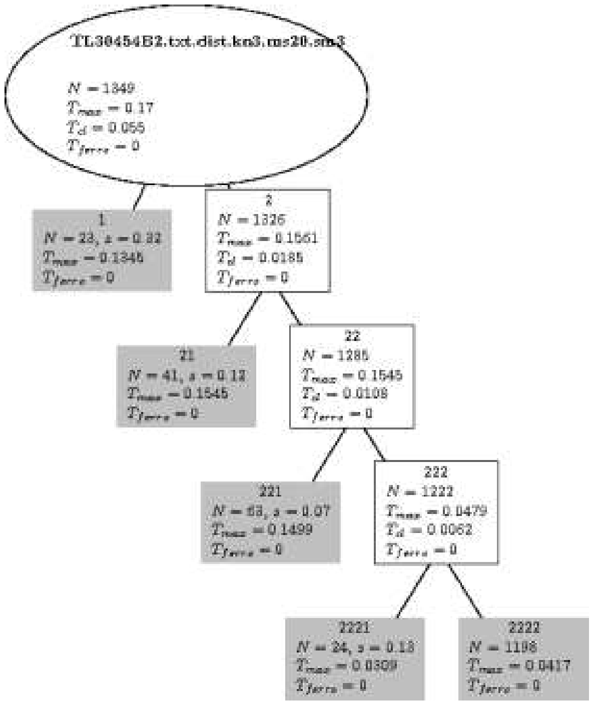

Figure 2: The output hierarchy of clusters given by the SSC algorithm. The realtive ordering of the stability is given by $C > B > A > D$. The fifth cluster represents the background noise.

horizontal position values for the eye movement. Due to inhearent measurment errors in the eye tracking apparatus, each test subject was first required to perfom a calibration phase in which they would visually fixate on clearly defined points of a calibration image with similar dimensions to the test image shown in figure (1). The measured eye positions and the corresponding calibration image is shown in figure (3A), where a piecwise triangular mapping of the measured to desired eye positions is assumed. Using the calibration information, eye position measurments can be corrected using local affine approximations determined by

$$[ab] = M \times [AB] \tag{9}$$

$$M = [ab] \times [AB]^{-1} \tag{10}$$

with vectors a,b,A and B as shown in figure (3B). An example of calibrated output measurments for a test subject viewing the image shown in figure (1) is shown in figure (4). As the pixels of the test image have a size greater than the resolution of the eye tracker the calibrated eye movements were binned to group closely spaced eye movement positions to identical pixel locations.

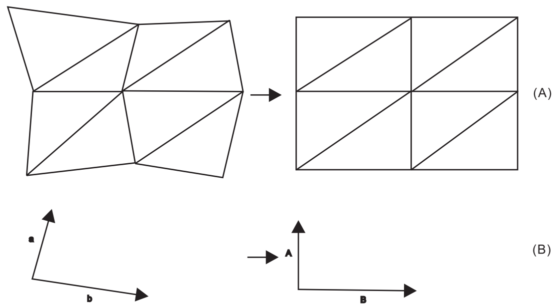

Figure 3: Figure (3A) shows the piecwise triangular mapping between measures and desired eye movements during calibration. Figure (3B) shows the affine mapping of equation 9 and equation 10.

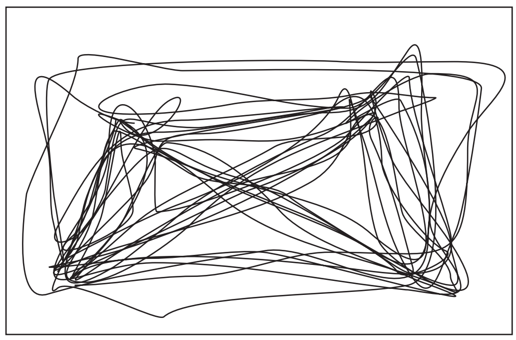

Figure 4: Figure (4) is a calibrated visual scan path for a test subject measured when viewing figure (1).

4. Relating Scan Path Statistics to the Cluster Stability

Every pixel or object in the test image of figure (1) is mapped to a cluster by the SSC algorithm and so every calibrated and binned movement of visual scan path can be mapped to a corresponding cluster. This process creates a sequence of symbols that represent eye movements in terms of cluster labels. The four clusters (excluding the background) of figure (1) were labelled by A, B, C, D, with the background cluster labelled as E. The visual scan path is therefore mapped to an output string with these labels. The first step in relating scan path statistics to the SSC algorithm is to compare the label distribution of the output string to the relative stability of the clusters. As was discussed in section 1 visual information is supressed during saccadic motion. Examining the scan path data indicates that a reasonable assumption is that movements through the background are synonimous to the saccadic motion and can be removed from the analysis. As such the background label E can be removed from any output string defining a visual scan path. The distribution of the labeled states with the background removed provides a relative measure the duration of the visual fixations for each cluster and this measure can be compared to the relative measure of cluster stability produced by the SSC algorithm.

5. Conclusion and Future Work

Given a visual scan path output string the a histogram of the labels is compared to the relative stability of the clusters for the image in figure (1). Figure (5A) shows a sample histogram for a test subject, the figure shows that the distribution of the labels in increasing order is given by $C > B > A > D$ which matches the ordering of the relative stability given by the SSC algorithm. By normalizing the histogram to produce a probability the results of several test subjects can be compared to the relative stability given by the SSC algorithm as is shown in figure (5B). From the experimental results it can be concluded that there is a relationship between the relative stability determined by the SSC algorithm and the duration of the visual fixations. Thus the SSC algorithm can be used to predict duration of the visual fixations for a given test image. There is a caveat to this result in that measuring eye movements using an occulometer is prone to error, and not all test subjects are able to produce reliable eye movement data to to physical imperfections of the eye or problems with the ability to fixate on small regions which is required in the calibration phase. Thus more test subjects will be required before a valid statistical analysis of the results can be obtained, however the initial results are very promising and warrent future research.

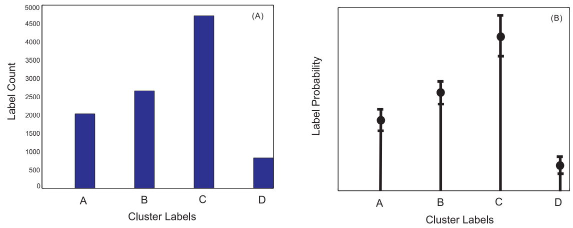

Figure 5: Figure (5A) shows the histogram of the visual fixation labels for a test subject viewing the image shown in figure (1). Figure (5B) shows the the distribution of the visual fixations of the labels for a a group of test subjects viewing theimage shown in figure (1), the large circles represent the mean value of the duration whith the horizontal lines showing the standard deviation. Both figures indicate that the distribution of the visual fixations matched the ordering of the relative stability given by the SSC algorithm.

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