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Noise-assisted detection in distributed detection system with inhomogeneous signal levels

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Abstract—We analyzed distributed detection of a signal in a system where the signal level is different at each detector. As an example, we consider the case where the signal level is dependent on the distance of the detector from the source. Each detector separately compares the input with its threshold value, and the detection results from all detectors are combined using a K-out-of-N rule. Optimal detection requires adjusting the threshold of each detector to a value suitable for the signal level at that detector. In a suboptimal system consisting of detectors with identical detection thresholds, the detection performance may have a non-monotonic dependence on noise intensity, and performance can be improved by optimizing noise intensity.

1. Introduction

Various noise-assisted effects are known in nonlinear systems. Noise-assisted means that a performance property of the system increases with noise intensity over some range of noise intensity. In particular, stochastic resonance (SR) has been studied in various systems [1]. SR means that the resonance response of a nonlinear system to a subthreshold signal can be optimized by adding noise. It has been suggested that sensors in biological system may use SR to detect signals in noisy environments. Information theoretical approaches have been used to study SR of aperiodic signals, for binary signals [2, 3, 4, 5, 6, 7, 8]. In these studies, bit error probability or mutual information is used to measure transmission of information between input and output.

SR in a threshold system with a signal and noise is closely related to the signal detection problem [5, 6, 8] in classical engineering studies [9]. Here we consider the signal detection problem in which the signal is binary $(0, 1)$ and noise has continuous values. Signal detection determines from the noisy input whether the signal's value is 0 or 1. Full knowledge about the signal level and noise can be used to obtain an optimal detector by calculating the optimal threshold. The optimization criterion is generally the minimal Bayesian risk or the maximal correctness probability.

An optimal detector with an optimal threshold shows monotonic decay of correctness probability with increasing noise intensity. On the other hand, when the threshold of the detector is suboptimal, the dependence on noise-intensity may be non-monotonic, and increasing the noise intensity can maximize the probability of correct detection [10]. We call this noise-assisted detection [11]. One of the reasons why we may need to use detectors with suboptimal thresholds is that we do not have full knowledge about the signal and noise. When the threshold is estimated from only partial knowledge, the threshold may be suboptimal.

Here we consider noise-assisted detection in sensor networks and especially focus on distributed detection (DD) [12, 13, 14]. We consider a DD system, which has multiple detectors working in parallel and decides global output at a data fusion center based on the local decisions gathered from each detector. Each detector separately detects the signal, and the detection results from all detectors are combined using a data fusion rule. When the local detector output is binary, the input-output rule of the data fusion is a Boolean function. In particular, we consider a system in which each detector generates a binary output by comparing the input with its threshold value. In a previous work, we have shown that even with optimal detectors in which the threshold value is optimally matched to the signal level, noise-assisted DD can occur when the fusion rule is suboptimal [15].

In this paper, we consider the system where the detectors are not optimal - the detectors have identical threshold values but the signal level may be different at each detector. As an example, we consider the case where the signal level is dependent on the distance of the detector from the source. Each detector separately compares the signal with its threshold value, and the detection results from all detectors are combined using a K-out-of-N rule. Noise is independent but with identical distribution for each detector. We show that the detection performance may have a non-monotonic dependence on noise intensity, and performance can be improved by optimizing noise intensity.

2. Noise-assisted detection

We consider the following signal detection problem, as in Ref. [7, 8, 9]. An input signal has values 0 (corresponding to bit 0) and 1 (corresponding to bit 1) and prior probabilities for each input bit are defined as p_0 and $p_1(= 1-p_0)$, respectively. Noise is Gaussian with mean 0 and variance σ^2. We define $P_0(x)$ and $P_1(x)$ as two probability distributions corresponding to the inputs 0 and 1 with added noise, respectively. Detection of the signal with noise corresponds to determining that the input signal is 0 or 1 by comparing with a threshold, θ.

Here we define p_{00} as probability that input 0 is detected correctly and also define p_{10} as probability that input 0 is detected as input 1. p_{01} and p_{11} for input 1 are defined similarly. These probabilities can be calculated as follows.

$$\begin{aligned} p_{00} &= \int_{-\infty}^{\theta} P_0(x)dx = 1 - p_{10}, \\ p_{10} &= \int_{\theta}^{\infty} P_0(x)dx, \\ p_{01} &= \int_{-\infty}^{\theta} P_1(x)dx, \\ p_{11} &= \int_{\theta}^{\infty} P_1(x)dx = 1 - p_{01}. \end{aligned} \tag{1}$$

We note that p_{00}, p_{10}, p_{01}, and p_{11} are functions of σ, because $P_0(x)$ and $P_1(x)$ have Gaussian distributions.

To evaluate the dependence of signal detection on noise intensity σ, we can define correctness probability as follows.

$$\begin{aligned} P_{\text{cor}}(\sigma) &= P_{\text{cor}}(p_0, p_1, p_{00}, p_{11}) \\ &= p_{00}p_0 + p_{11}p_1. \end{aligned} \tag{2}$$

We note that the probabilities are functions of σ.

Standard signal detection techniques assume complete knowledge concerning the values (levels, amplitudes) of the binary signals and so we can calculate the optimal threshold θ_{opt} to maximize correctness probability. However, when the values of the binary signals are unknown, we cannot know for sure the optimal threshold.

When the value θ is larger than the true value of input 1 (or smaller than the value of input 0) in the case of $p_0 = p_1 = 1/2$, existence of noise can assist the detection. This is noise-assisted signal detection. For example, when p_1 and p_0 are equal, noise-assists detection when the noise increases the probability p_{11} more than it decreases the probability of p_{00}. We have described noise-assisted detection in a system with a suboptimal threshold in σ [11], showing how finite noise intensity σ can optimize the correctness probability P_{cor}.

When the threshold is optimal (in the example, $\theta_{\text{opt}} = 1/2$), correctness probability decreases monotonically against noise intensity σ. In other words, noise degrades the detection by an optimal threshold. Here we note that the correctness probability for suboptimal threshold never exceeds the correctness probability for the optimal threshold, as expected from the data processing inequality [16].

3. Noise-assisted distributed detection

3.1. Distributed detection

DD [12, 13, 14] in sensor networks gathers local outputs of multiple detectors to detect a signal. Here we consider simple DD that has detectors working in parallel and decides global output at a data fusion center based on the local decisions from each detector as in Fig. 1. We assume that the local detectors are identical and each detector has a common signal and independent noise. When the local detector output is binary, the input-output rule of the data fusion becomes a Boolean function.

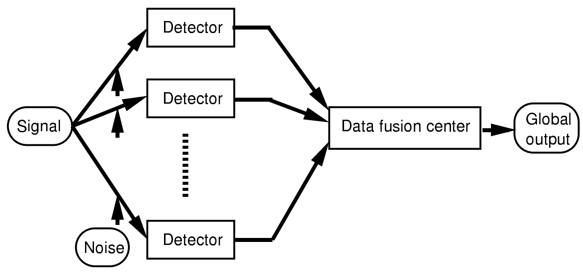

Figure 1: Distributed detection. Simple DD system is composed of multiple local detectors and a data fusion center.

One of the important issues in sensor networks is optimization of the data fusion rule in the case where the local detectors are identical [14]. The assumption of identical detectors is adopted to reduce complexity of design, implementation and control of sensor networks, both in theory and practice. In particular, we consider a simple and basic fusion rule, K-out-of-N rule, i.e. the global decision $u_0 = 1$ if K or more local decisions are equal to 1.

$$u_0 = \begin{cases} 1, & \text{if } \sum_{i=1}^{N} u_i \geq K, \\ 0, & \text{if } \sum_{i=1}^{N} u_i < K. \end{cases} \tag{3}$$

K is an integer threshold for the global decision. For each detector, false alarm probability p_F and detection probability p_D is shown as follows.

$$p_F = p(u_i = 1|H_0) = p_{10}, \tag{4}$$

$$p_D = p(u_i = 1|H_1) = p_{11}. \tag{5}$$

Unknown hypotheses, H_0 and H_1 correspond to input 0 and 1, respectively. From false alarm and detection probability of each detector, we can calculate system false alarm probability p_F and system detection probability p_D as follows.

$$P_F = \sum_{i=K}^{N} \begin{pmatrix} N \\ i \end{pmatrix} p_F^i (1 - p_F)^{N-i}, \tag{6}$$

$$P_D = \sum_{i=K}^{N} \begin{pmatrix} N \\ i \end{pmatrix} p_D^i (1 - p_D)^{N-i}. \quad (7)$$

The probability of correct detection with N detectors is as follows.

$$P_{\text{cor}} = (1 - P_F)p_0 + P_D p_1. \quad (8)$$

Varshney has examined K-out-of-N fusion rules with identical detector and obtained the optimal K value to minimize Bayesian risk [12]. We note that an inhomogeneous system in which the threshold value of each detector can be separately adjusted [17] is a much more difficult problem especially when the number of sensors is large, requiring co-optimization of threshold values and K, and is beyond the scope of this paper.

3.2. Noise-assisted distributed detection

We have explained that noise-assisted DD can be expected when the detectors have suboptimal thresholds with respect to the signal and noise [11]. We also have showed that noise-assisted DD can occur even when the detectors have optimal thresholds, for some types of fusion rules [15]. Specifically, we have showed an example of noise-assisted DD which can occur when the fusion rule is not a monotonic function of the number of detectors. In practice, this case could occur, for example, when receiving too many detections from local detectors is judged by the fusion center to be an indication that local detectors are broken and outputting false detections. This example has showed that we can see noise-assisted DD even when the local detectors have optimal thresholds. Note that the correctness probability is less for the fusion rule where noise-assisted detection is observed, than for the fusion rule where noise-assisted detection is not observed. In this sense, noise-assisted detection occurs for suboptimal fusion rule.

4. Noise-assisted distributed detection for homogeneous thresholds and inhomogeneous signal levels

Here we consider the case where multiple detectors detect signals from the same source, but the level of the signal at each detector is not the same, for example, it depends on the distance between the signal source and the detector. Noise is assumed to be independent but with identical distribution for each detector. We assume that the threshold value is the same for all detectors - i.e. it is not possible for detectors to independently adjust their threshold values.

To be specific, we assume that the signal level $s(r)$ depends on the distance r between the signal source and a detector. The signal level varies as $s(r) = A\exp(-kr)$ with constant k when $s(0) = A$. For simplicity, we consider the sensors are positioned on a one-dimensional line with coordinate x. The signal source is located at $x = 0$. Detectors are located at $x = \ldots, -2, -1, 0, 1, 2, \ldots$. We assume that $p_0 = p_1 = 1/2$. The threshold value, which is the same for all detectors, is taken as $A/2$, which is the optimal value for a detector at $s(0) = A$ when $p_0 = p_1 = 1/2$. We use K-out-of-N fusion rule. In this system, the threshold value is suboptimal for all detectors except detectors with distance 0 and so we can anticipate the possibility of noise-assisted detection with a normal fusion rule, such as K-out-of-N fusion rule.

Figure 2 shows particular examples of noise-assisted detection as manifest in the variation of correctness probability $P_{\text{cor}}(\sigma)$ as noise intensity σ increases. In this example,

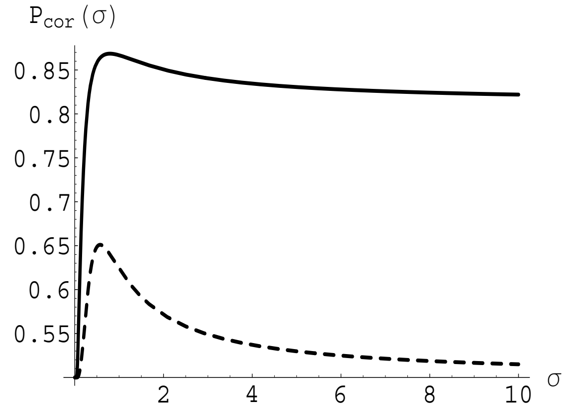

Figure 2: Dependence of correctness probability $P_{\text{cor}}(\sigma)$ on noise intensity σ for a particular system of $N = 5$ detectors with $p_0 = p_1 = 1/2$. The solid line shows when $K = 2$, and the dotted line shows when $K = 3$.

$N = 5$ detectors are located at positions $x = -2, -1, 0, 1, 2$ and the signal source is located at $x = 0$. The key signal parameters are $k = 1$ and $A = 1$. There is a maximum value of correctness probability at a non-zero value of noise intensity. With increase of σ, the correctness probability $P_{\text{cor}}(\sigma)$ increases to a maximum and decreases again. We also can find noise-assisted detection when $K = 2, 3, 4$. We cannot see noise-assisted detection in the case of $K = 1$ as one of the detectors, the one positioned at $x = 0$ has an optimal threshold.

Here we assume that the threshold value is the same for all detectors. We note that noise-assisted detection can also be expected in the case when the detectors are not identical. In general, improvement of detection with noise can occur when the threshold of a detector is not optimal with respect to the signal level [10, 11]. The optimality of a detector depends on both the threshold level and the signal level. In this paper, we assumed identical threshold levels to more clearly illustrate the effect of inhomogeneous signal levels.

5. Conclusions

We analyzed DD of a signal in a system where the detection threshold is the same for all detectors, but the level of the signal is not the same, that is the optimality of the threshold is not the same. We considerd a standard data fusion rule, the K-out-of-N fusion rule. We showed that DD performance can depend non-monotonically on noise intensity and that performance can be improved with noise compared to without noise. This shows that the noise intensity can be used to improve the detection performance. This example shows that in real-world environments, it may be possible to use control of noise intensity to achieve adequate performance when more optimal techniques, such as the separate adjustment of many detector threshold values are not possible or feasible.

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