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Bifurcations and Basins in the Complex Logistic Map Including Periodic Parameter Perturbation

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Abstract– Bifurcations, Basins and orbits in the complex Logistic map whose parameter is forced into periodic varying are investigated. From the investigation of bifurcations in the system, combinations of parameters where solutions coexist are observed. From the investigation of Basins, coexisting orbits are clarified. Additionally, fractals are confirmed. From the investigation of orbits, chaotic orbits which have imaginary value are confirmed.

1. Introduction

Chaos is an unpredictable movement that is derived from deterministic nonlinear equations, and it was discovered by the progress of numerical calculations [1]. Chaos is caused in nonlinear systems and it has characteristics such as initial value acuity or strange attractor. It is commonly found in nature systems and studied in several research fields [2]-[5]. Most nature systems are the nonlinear systems, and they have possibilities to cause unpredictable behavior. The nonlinearity makes nature behavior difficult to predict. That can be seen in the movement of the falling leaves, flow of water and earthquake. So in order to clarify the nature behavior and solve the unpredictable problems, it is important to study the nonlinear systems, especially which are chaotic systems. The nonlinear systems can be separated to autonomous nonlinear systems and non-autonomous nonlinear systems. We are interested in the non-autonomous systems whose parameters are oscillatory forced. The parametrically forced systems are still not investigated well, whereas the systems whose state values are forced by external periodic waves are well investigated and many interesting and complicated phenomena are observed in the systems. The Duffing Oscillator whose state value is forced by external sine wave is well known as a non-autonomous nonlinear system and investigated and is analyzed by many researchers. On the other hand, the parametric forcing almost causes periodic oscillation and chaos in a simple oscillator [6]-[8]. The parametric forcing can make structures to unpredictably oscillate and is suggested to lead to the destruction of buildings like Tacoma Narrows Bridge [9]. The purpose of this study is the investigation into influences of periodic parameter change to an asymmetry two-dimensional nonlinear discrete time system. The asymmetry two-dimensional nonlinear discrete time system is the nonlinear system which is consist of two asymmetric equations. In this study, a complex Logistic map is used as the asymmetry two-dimensional nonlinear discrete time system. In order to investigate the influences of periodic parameter change, a parameter of the complex Logistic map is forced into periodic varying.

In this study, we investigate the complex Logistic map including periodic parameter perturbation. From the investigation of bifurcations, basins and orbits, interesting and characteristic phenomenon are confirmed.

2. Parametrically Forced Complex Logistic Map

Complex Logistic map is a Logistic map extended with complex numbers of state values, and is described as;

$$\begin{cases} x(n+1) = \alpha(x(n) - x(n)^2 + y(n)^2) \\ y(n+1) = \alpha y(n)(1 - 2x(n)) \end{cases} \quad (1)$$

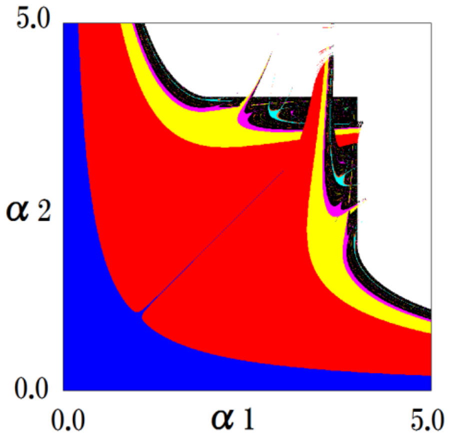
In order to investigate the influence of periodic parameter change, the parameter of the complex Logistic map is forced into periodic varying. The parametrically forced complex Logistic map is described as;

$$\begin{cases} x(n+1) = \alpha_f(n)(x(n) - x(n)^2 + y(n)^2) \\ y(n+1) = \alpha_f(n) y(n)(1 - 2x(n)) \end{cases} \quad (2)$$

and

$$\alpha_f(n) = \begin{cases} \alpha_1 \text{ for each odd value of “n”} \\ \alpha_2 \text{ for each even value of “n”} \\ (n=0,\ 1,\ 2,\ \cdots) \end{cases} \quad (3)$$

The system is investigated according to the following procedure. First, bifurcations in the system are investigated to confirm existence of coexisting solutions by comparing some bifurcation diagrams which have different initial values. Next, basins of the system are investigated to clarify coexisting solutions for several parameter sets. And then, orbits of the coexisting solutions are investigated.

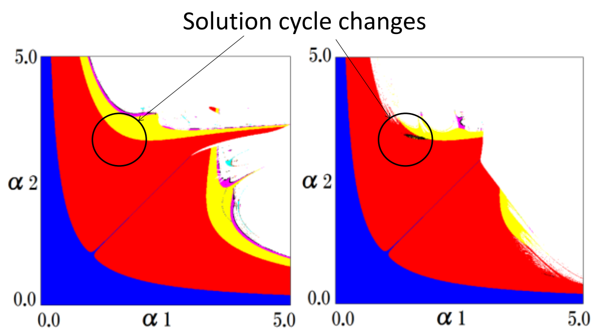


(a)

Solution Cycle	1	2	3	4	5	6	7	8	
Color									
	9	10	11	12	13	14	15	16～	divergence

(b)

Figure 1: Bifurcation diagram. (a)A bifurcation diagram for the complex Logistic map including periodic parameter perturbation. (b)Correspondence of the periods of orbits and color.



($x(0)=0.08, y(0)=0.15$) ($x(0)=0.18, y(0)=0.22$)

Figure 2: Changes due to the initial value.

3. Bifurcations

Fig. 1 (a) shows an example of a bifurcation diagram of the system. Fig. 1 (b) shows the relationship between periods of solutions and colored area in the bifurcation diagram. The white colored area represents divergence of solutions. The other areas represent convergence of solutions. The difference in colors relates to periods of solutions. Solutions are regarded as divergence when the solutions $x(n)$ are greater than 100 because in this case, it is unlikely that $x(n)$ becomes more than 100 if Solutions converge to a certain value. This condition is reasonable.

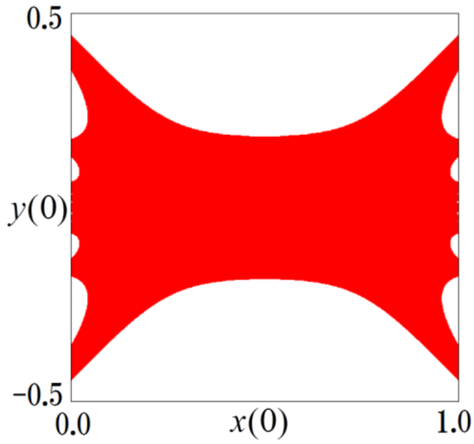


Figure 3: An example of a basin.
($\alpha_1 = 3.0, \alpha_2 = 2.2$)

The general Logistic map has solutions between 0 and 1. If $x(n)$ are greater than 1, $x(n+m)$ tend toward infinity, where m is counting number. Therefore, when $x(n)$ become quite large in the system, the solution diverges to infinity. Solutions do not have possibility to converge to a periodic cycle or chaos when the solutions are greater than 100. Panels in Fig. 2 show bifurcation diagrams when different initial values are set. Comparing the figures, different colored area can be seen. These areas imply possible existence of coexistence of solutions.

4. Basins

Basin is a diagram that shows how the initial values converge to any orbit when parameter value is decided in one set. Fig. 3 shows an example of the basin. The meaning of colored domain corresponds to Fig. 1 (b). The condition deciding divergence is the same as the investigation of the bifurcation. From the investigation of the basin, several patterns of basins that may include chaos can be confirmed. Fig. 4 shows an example of that. Fig. 5 shows an enlarged view of a portion that chaos may occur. The black parts of the figure may include the portion of chaotic. Also, Self-similar fractal forms as shown in Fig. 6 and 7 are found.

5. Orbits

In black colored areas in bifurcation diagrams and basins, the solution has more than 16 cycles. And these areas contain the chaos that does not have a periodic solution. To clarify the chaos occurrence, return maps are investigated on association with the bifurcations and the basins. Fig. 8 shows the orbit when the chaos does not occur and Fig. 9 shows the orbit when chaos occurs. When the chaos does not occur, orbit is converged to countable point as shown in Fig. 8. In contrast, when chaos occurs, the orbit has a complex shape because the solution does not converge to a periodic cycle. In the both case of Fig. 8 and Fig. 9, imaginary values $y(n)$ are 0. Orbits whose $y(n)$ are 0 can be seen in much parameter

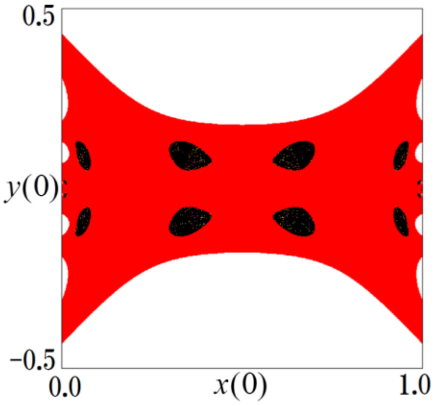


Figure 4: Basin that chaos occurs.
($\alpha_1 = 3.35, \alpha_2 = 1.79$)

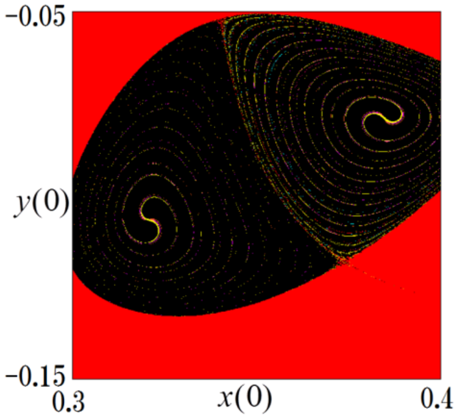


Figure 5: Enlarged view of chaos occurs.
($\alpha_1 = 3.35, \alpha_2 = 1.79$)

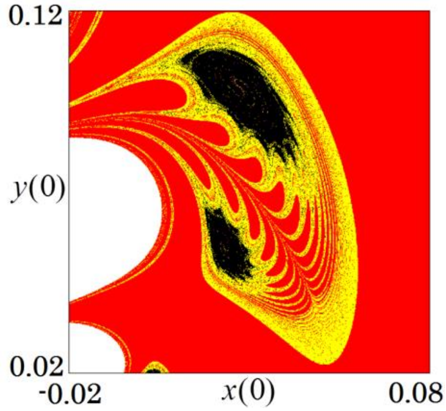


Figure 6: Fractal (1).
($\alpha_1 = 1.60, \alpha_2 = 3.44$)

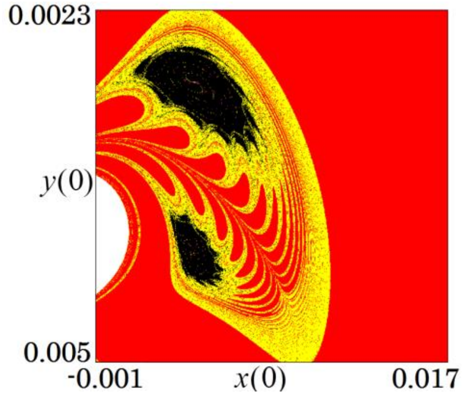


Figure 7: Fractal (2).
($\alpha_1 = 1.60, \alpha_2 = 3.44$)

space and phase space. These orbits are also general in the general complex Logistic map. However, chaos which has imaginary values is confirmed in the system as shown in Fig. 10. This cannot be seen in the general complex logistic map. The chaos including imaginary value is generated by the periodic force.

6. Conclusion

In this study, we investigated bifurcations, basins and orbits in the complex Logistic map whose parameter is forced into periodic varying. From the investigations of the bifurcations and the basins, coexistence phenomena which are generated by the periodic perturbation of the parameter are confirmed. Additionally, fractals are found in the basins. From the investigation of the orbits, chaotic orbits including imaginary value are confirmed. These orbits are only confirmed in the system.

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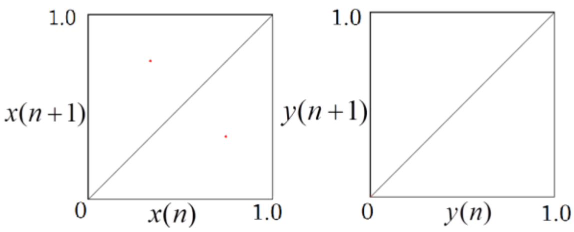

Figure 8: An orbit where chaos does not occur.
($\alpha_1 = 3.35, \alpha_2 = 1.79, x(0) = 0.45, y(0) = 0.10$)

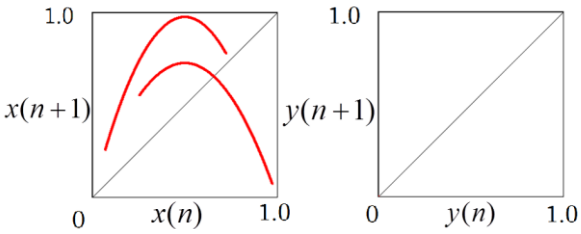

Figure 9: An orbit where chaos occurs.
($\alpha_1 = 3.90, \alpha_2 = 2.90, x(0) = 0.10, y(0) = 0.00$)

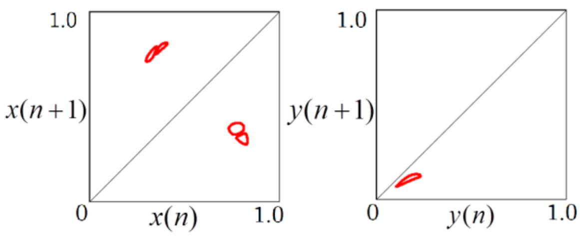

Figure 10: Chaos including imaginary values.
($\alpha_1 = 3.35, \alpha_2 = 1.79, x(0) = 0.35, y(0) = 0.10$)

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