2014 International Symposium on Nonlinear Theory and its Applications
NOLTA2014, Luzern, Switzerland, September 14-18, 2014

# A Basic Study on Symmetrical Chaotic Dynamics for Population-based Optimization

Yoshikazu YAMANAKA† and Tadashi TSUBONE‡

†‡Nagaoka University of Technology, 1603-1, Kamitomioka, Nagaoka, Niigata, 940-2188, Japan
†Email: y_yama@stn.nagaokaut.ac.jp, ‡Email: tsubone@vos.nagaokaut.ac.jp

**Abstract**—

We propose a novel optimization method that searches for an optimal solution by updating multi points based on chaotic dynamics with information sharing mechanism. In previous work, we have proposed an optimization method based on swarm of chaotic dynamical particles (OSCDP). OSCDP shows better performance than particle swarm optimization (PSO). In this paper, we propose a simplified method governed only by chaotic dynamics with information sharing. The method has two system parameters and does not contain any stochastic terms. Numerical experimental results show that the proposed method has better performance than PSO and the previous proposed method.

## 1. Introduction

Large numbers of heuristic optimization methods are proposed. One of the most simple and powerful methods is particle swarm optimization (PSO) proposed by Kennedy and Eberhart [1, 2]. PSO searches for an optimal solution by population called particles. Each particle has information about position and velocity. The position is a candidate of the solution. They share the information of the best position in own searched history with others. Using the shared information and current particle state, position and velocity are updated for each time-step.

Researches studied about the PSO dynamics to discover the relationship between the behavior of particle and the performance. Ozcan and Mohan derived that the behavior of single particle can be divergence, convergence or sinusoid wave [3, 4]. After these works, some studies showed conditions to ensure that the single particle converges [5–9]. Following the conditions, well-known PSO parameters are set in weak stability condition [10].

In previous work, we have proposed an optimization method with chaotic dynamical particles (OSCDP) [11]. The single particle of OSCDP exhibits chaotic motion. OSCDP denotes better performance than PSO even though it does not contain any stochastic terms. Therefore, OSCDP could be a suitable model to discover the relationship between population behavior and its performance. However, the particles are governed by two dynamics, fixed-point dynamics and chaotic dynamics. It makes the system complex.

In this paper, we propose a novel population-based optimization method that based only on chaotic dynamics with information sharing mechanism. The proposed method is simpler than previous proposed method. In the numerical experiments of 19 benchmark functions, the proposed method derived better performance than PSO and OSCDP.

## 2. Proposed method

This section describes the proposed optimization method. We consider a $D$-dimensional single object optimization problem. The object function is $f$. Proposed method searches for an optimal solution evaluating $n$ points by $f$ for each time-step. The $i$-th searching point at time-step $t$ is denoted by $D$-dimensional vector as,

$$\boldsymbol{x_i}(t) = \{x_{i1}(t), x_{i2}(t), \ldots, x_{iD}(t)\}. \tag{1}$$

$\boldsymbol{x_i}(t)$ is calculated by three information, an independent valuable $\boldsymbol{v_i}(t)$, personal best point $\boldsymbol{pb_i}$, and global best point $\boldsymbol{gb}$. These variables are denoted as $D$-dimensional vector as,

$$\boldsymbol{v_i}(t) = \{v_{i1}(t), v_{i2}(t), \ldots, v_{iD}(t)\}, \tag{2}$$
$$\boldsymbol{pb_i} = \{pb_{i1}, pb_{i2}, \ldots, pb_{iD}\}, \tag{3}$$
$$\boldsymbol{gb} = \{gb_1, gb_2, \ldots, gb_D\}. \tag{4}$$

Proposed method updates $\boldsymbol{x_i}(t)$, $\boldsymbol{v_i}(t)$, $\boldsymbol{pb_i}$ and $\boldsymbol{gb}$ by following steps.

**Step 1, Initialization**

All elements of initial searched point $\boldsymbol{x_i}(0)$, $i = 1, 2, ..., n$, is randomly given. All elements of $\boldsymbol{v_i}(0)$, $i = 1, 2, ..., n$, is set to zero. $\boldsymbol{pb_i}$ is set to $\boldsymbol{x_i}(0)$. $\boldsymbol{gb}$ is set to $\boldsymbol{pb_i}$ that gives best fitness value in $f(\boldsymbol{pb_1}), f(\boldsymbol{pb_2}), ..., f(\boldsymbol{pb_n})$.

**Step 2, Calculating next points of evaluating**

Here, let dependent valuable $\boldsymbol{y_i}(t)$ be $\boldsymbol{x_i}(t) - \frac{1}{2}(\boldsymbol{gb} - \boldsymbol{pb_i})$. The $j$-th element of $\boldsymbol{y_i}(t)$ and $\boldsymbol{v_i}(t)$ are updated by following rule. If $|y_{ij}(t)| < Th_{ij}$ and if $y_{ij}(t)v_{ij}(t) \geq 0$,

$$y_{ij}(t+1) = 2\mathrm{sgn}(y_{ij}(t))Th_{ij} - y_{ij}(t), \tag{5}$$
$$v_{ij}(t+1) = 0, \tag{6}$$

otherwise

$$\begin{bmatrix} y_{ij}(t+1) \\ v_{ij}(t+1) \end{bmatrix} = R \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} y_{ij}(t) \\ v_{ij}(t) \end{bmatrix}, \tag{7}$$

where $R$ and $\theta$ are system parameters and $Th_{ij} = \frac{1}{2}|gb_j - pb_{ij}|$. By this rule, next searching points, $\boldsymbol{x_i}(t+1) = \boldsymbol{y_i}(t+1) + \frac{1}{2}(\boldsymbol{gb} - \boldsymbol{pb_i})$, are given.

**Step 3, Evaluation**

Fitness value is given by $f(\boldsymbol{x_i}(t+1))$. If $f(\boldsymbol{x_i}(t+1))$ is better than $f(\boldsymbol{pb_i})$, $\boldsymbol{pb_i} = \boldsymbol{x_i}(t+1)$. Otherwise $\boldsymbol{pb_i}$ is kept. If $f(\boldsymbol{pb_i})$ is better than $f(\boldsymbol{gb})$, $\boldsymbol{gb} = \boldsymbol{pb_i}$. Otherwise $\boldsymbol{gb}$ is kept.

**Step 4, Checking termination**

If $t+1$ reaches maximum iteration, $\boldsymbol{gb}$ is searched solution. Otherwise let $t$ be $t+1$ and continue to Step 2.

## 3. Results

This section shows comparison results between PSO [12], OSCDP [11] and proposed method. For the comparison, five unimodal and 13 multimodal functions from CEC 2013 test suite [13] are used. The functions are shown in Table 1. Table 2 shows conditions of numerical experiments.

Table 2: Condition of numerical experiments

| Condition | Value |
|---|---|
| Number of Population | 30 |
| Size of dimension | 30 |
| Maximum iteration | 1000 |
| Number of trials | 50 |
| Initial searching points | 50 sets given by the uniformed distributed with range in Table 1 |

PSO, OSCDP and proposed methods have system parameters. For fair comparison, these parameters were desired to adjust for each function. In this paper, the parameters were selected experimentally using almost same computational cost. Compared methods were run by the sets of parameter described in table 3. The number of combination of parameters is 200, 180 and 180 for PSO, OSCDP and proposed method, respectively. Table 4 shows the parameter sets that denoted the best averaged final fitness values under the condition in table 2. Table 5 shows the averaged final fitness values and the standard deviation.

Table 3: Range of Adjusted parameters

| Method | Parameters | Set |
|---|---|---|
| PSO | $\omega$ | $\{0.05, 0.15, 0.25, ..., 0.95\}$ |
| | $c_1 = c_2$ | $\{0.05, 0.15, 0.25, ..., 1.95\}$ |
| OSCDP | $R$ | $\{1.05, 1.25, 1.45, 1.65, 1.85\}$ |
| | $\theta$[deg] | $\{1, 11, 21, ..., 81\}$ |
| | $c$ | $\{0.2, 0.4, 0.6, 0.8\}$ |
| Proposed | $R$ | $\{1.05, 1.15, 1.25, ..., 1.95\}$ |
| | $\theta$[deg] | $\{1, 6, 11, ..., 86\}$ |

Table 4: Adjusted parameters

| $f$ | PSO | | OSCDP | | | SymCDP | |
|---|---|---|---|---|---|---|---|
| | $\omega$ | $c_1 = c_2$ | $R$ | $\theta$[deg] | $c$ | $R$ | $\theta$[deg] |
| $f_1$ | 0.450 | 1.95 | 1.25 | 51.0 | 0.400 | 1.45 | 71.0 |
| $f_2$ | 0.950 | 0.350 | 1.25 | 61.0 | 0.400 | 1.25 | 51.0 |
| $f_3$ | 0.850 | 1.05 | 1.25 | 51.0 | 0.200 | 1.35 | 51.0 |
| $f_4$ | 0.950 | 0.250 | 1.05 | 11.0 | 0.800 | 1.05 | 1.00 |
| $f_5$ | 0.650 | 1.65 | 1.25 | 51.0 | 0.600 | 1.45 | 86.0 |
| $f_6$ | 0.550 | 1.85 | 1.25 | 51.0 | 0.400 | 1.55 | 71.0 |
| $f_7$ | 0.950 | 0.350 | 1.25 | 61.0 | 0.200 | 1.25 | 36.0 |
| $f_8$ | 0.650 | 1.15 | 1.25 | 61.0 | 0.400 | 1.35 | 46.0 |
| $f_9$ | 0.950 | 0.450 | 1.25 | 61.0 | 0.800 | 1.25 | 31.0 |
| $f_{10}$ | 0.650 | 1.65 | 1.25 | 61.0 | 0.800 | 1.35 | 66.0 |
| $f_{11}$ | 0.650 | 1.85 | 1.25 | 41.0 | 0.600 | 1.25 | 26.0 |
| $f_{12}$ | 0.850 | 1.05 | 1.25 | 61.0 | 0.800 | 1.45 | 46.0 |
| $f_{13}$ | 0.950 | 0.450 | 1.25 | 61.0 | 0.800 | 1.25 | 26.0 |
| $f_{14}$ | 0.550 | 1.85 | 1.25 | 71.0 | 0.800 | 1.75 | 86.0 |
| $f_{15}$ | 0.950 | 0.250 | 1.05 | 21.0 | 0.400 | 1.15 | 31.0 |
| $f_{16}$ | 0.550 | 1.95 | 1.25 | 61.0 | 0.800 | 1.45 | 46.0 |
| $f_{17}$ | 0.750 | 1.35 | 1.25 | 61.0 | 0.800 | 1.45 | 66.0 |
| $f_{18}$ | 0.550 | 1.95 | 1.25 | 61.0 | 0.800 | 1.35 | 41.0 |
| $f_{19}$ | 0.550 | 1.65 | 1.25 | 81.0 | 0.200 | 1.45 | 46.0 |

Table 6 shows two sample t-test [14] results at 0.05 significant level between PSO and proposed method. The proposed method denotes significantly better fitness values with 9 functions. PSO denotes significantly better fitness with a function. Table 7 shows the t-test between OSCDP and proposed method. Proposed method and OSCDP denotes significantly better fitness than another with 14 functions and a function, respectively.

## 4. Conclusion

In this paper we proposed a novel optimization method that updates multi searching points based on chaotic dynamics with information sharing mechanism. The basic performance was compared with PSO and OSCDP by 19 benchmark functions. Proposed method derived significantly better performance than PSO and OSCDP with 9 and 14 functions, respectively. The analysis of the chaotic dynamics is one of the future topics.

Table 1: Benchmark functions

| Suite | Type | $f$ | Functions | Optimal fitness | Search range |
|---|---|---|---|---|---|
| CEC13 | Unimodal | $f_1$ | Sphere | -1400 | $[-100, 100]^D$ |
| CEC13 | Unimodal | $f_2$ | Rotated High Conditioned Elliptic | -1300 | $[-100, 100]^D$ |
| CEC13 | Unimodal | $f_3$ | Rotated Bent Cigar | -1200 | $[-100, 100]^D$ |
| CEC13 | Unimodal | $f_4$ | Rotated Discus | -1100 | $[-100, 100]^D$ |
| CEC13 | Unimodal | $f_5$ | Different Powers | -1000 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_6$ | Rotated Rosenbrock's | -900 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_7$ | Rotated Schaffers F7 | -800 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_8$ | Rotated Ackley's | -700 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_9$ | Rotated Weierstrass | -600 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_{10}$ | Rotated Griewank's | -500 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_{11}$ | Rastrigin's | -400 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_{12}$ | Rotated Rastrigin's | -300 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_{13}$ | Non-Continuous Rotated Rastrigin's | -200 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_{14}$ | Schwefel's | -100 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_{15}$ | Rotated Schwefel's | 100 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_{16}$ | Lunacek Bi_Rastrigin | 300 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_{17}$ | Rotated Lunacek Bi_Rastrigin | 400 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_{18}$ | Expanded Griewank's plus Rosenbrock's | 500 | $[-100, 100]^D$ |
| CEC13 | Multimodal | $f_{19}$ | Expanded Scaffer's F6 | 600 | $[-100, 100]^D$ |

## References

[1] J. Kennedy and R. Eberhart, "Particle swarm optimization," *Proc. IEEE Int. Conf. neural networks*, vol. 4, pp. 1942–1948, 1995.

[2] R. Eberhart and J. Kennedy, "A new optimizer using particle swarm theory," *Proc. Int. Symp. Micro Mach. Hum. Sci.*, pp. 39–43, Oct. 1995.

[3] E. Ozcan and C. Mohan, "Analysis of a simple particle swarm optimization system," *Proc. 1998 Artif. Neural Networks Eng. Conf.*, vol. 8, pp. 253–258, 1998.

[4] E. Ozcan and C. Mohan, "Particle swarm optimization: surfing the waves," *Proc. 1999 Congr. Evol. Comput.*, pp. 1939–1944, 1999.

[5] M. Clerc and J. Kennedy, "The particle swarm - explosion, stability, and convergence in a multidimensional complex space," *IEEE Trans. Evol. Comput.*, vol. 6, no. 1, pp. 58–73, 2002.

[6] F. V. D. Bergh, *An analysis of particle swarm optimizers*. PhD thesis, University of Pretoria etd, 2002.

[7] F. V. D. Bergh and A. P. Engelbrecht, "A study of particle swarm optimization particle trajectories," *Inf. Sci. (Ny).*, vol. 176, pp. 937–971, Apr. 2006.

[8] K. Yasuda, A. Ide, and N. Iwasaki, "Adaptive particle swarm optimization," *SMC'03 Conf. Proceedings. 2003 IEEE Int. Conf. Syst. Man Cybern. Conf. Theme - Syst. Secur. Assur. (Cat. No.03CH37483)*, vol. 2, pp. 1554–1559, Oct. 2003.

[9] I. C. Trelea, "The particle swarm optimization algorithm: convergence analysis and parameter selection," *Inf. Process. Lett.*, vol. 85, pp. 317–325, Mar. 2003.

[10] K. Yasuda, N. Iwasaki, G. Ueno, and E. Aiyoshi, "Particle swarm optimization: a numerical stability analysis and parameter adjustment based on swarm activity," *IEEJ Trans. Electr. Electron. Eng.*, vol. 3, pp. 642–659, Nov. 2008.

[11] Y. Yamanaka and T. Tadashi, "An idea of Novel Optimizer using Swarm of Chaotic Dynamical Particles," *Proc. 2013 Int. Symp. Nonlinear Theory its Appl.*, pp. 240–243, Sept. 2013.

[12] Y. Shi and R. Eberhart, "A modified particle swarm optimizer," *Proc. 1998 IEEE Int. Conf. Evol. Comput.*, pp. 69–73, May 1998.

[13] J. J. Liang, B. Y. Qu, P. N. Suganthan, and A. G. Hernández-Díaz, "Problem definitions and evaluation criteria for the CEC 2013 special session on real-parameter optimization," *Comput. Intell. Lab. Zhengzhou Univ. Zhengzhou, China Nanyang Technol. Univ. Singapore, Tech. Rep.*, vol. 201212, 2013.

[14] B. Welch, "The Generalization of 'Student's' Problem when Several Different Population Variances are Involved," *Biometrika*, vol. 34, pp. 28–35, 1947.

Table 5: Benchmark results

The values with * denote the best value for each function.

| $f$ | | PSO | OSCDP | Proposed |
|---|---|---|---|---|
| $f_1$ | ave | * -1.40e+03 | -1.40e+03 | -1.40e+03 |
| | std | * 3.25e-13 | 1.39e-06 | 6.02e-13 |
| $f_2$ | ave | * 3.92e+06 | 8.86e+06 | 3.96e+06 |
| | std | * 2.04e+06 | 5.20e+06 | 2.18e+06 |
| $f_3$ | ave | 4.32e+09 | 8.92e+09 | * 1.42e+09 |
| | std | 5.28e+09 | 6.56e+09 | * 1.66e+09 |
| $f_4$ | ave | 5.15e+04 | * 4.96e+04 | 5.72e+04 |
| | std | 1.49e+04 | 1.54e+04 | * 6.80e+03 |
| $f_5$ | ave | -1.00e+03 | -1.00e+03 | * -1.00e+03 |
| | std | 1.21e-08 | 2.24e-04 | * 4.39e-11 |
| $f_6$ | ave | -8.69e+02 | -8.64e+02 | * -8.69e+02 |
| | std | 1.74e+01 | 2.32e+01 | * 1.60e+01 |
| $f_7$ | ave | -6.54e+02 | -6.56e+02 | * -6.67e+02 |
| | std | 4.63e+01 | * 3.74e+01 | 4.01e+01 |
| $f_8$ | ave | * -6.79e+02 | -6.79e+02 | -6.79e+02 |
| | std | 8.25e-02 | * 6.13e-02 | 7.37e-02 |
| $f_9$ | ave | -5.72e+02 | * -5.73e+02 | -5.73e+02 |
| | std | * 3.11e+00 | 5.00e+00 | 5.46e+00 |
| $f_{10}$ | ave | -4.99e+02 | -4.97e+02 | * -5.00e+02 |
| | std | 7.93e-01 | 1.56e+00 | * 2.13e-01 |
| $f_{11}$ | ave | -3.50e+02 | -3.27e+02 | * -3.51e+02 |
| | std | * 1.50e+01 | 2.03e+01 | 1.76e+01 |
| $f_{12}$ | ave | -1.54e+02 | -1.55e+02 | * -2.08e+02 |
| | std | 4.91e+01 | 4.51e+01 | * 2.87e+01 |
| $f_{13}$ | ave | 3.22e+01 | 2.63e+01 | * -1.21e+01 |
| | std | 4.92e+01 | 5.10e+01 | * 4.31e+01 |
| $f_{14}$ | ave | 1.97e+03 | 2.47e+03 | * 1.85e+03 |
| | std | * 4.17e+02 | 4.70e+02 | 5.19e+02 |
| $f_{15}$ | ave | 4.39e+03 | 4.51e+03 | * 4.23e+03 |
| | std | 7.83e+02 | 7.83e+02 | * 7.83e+02 |
| $f_{16}$ | ave | 4.09e+02 | 4.38e+02 | * 3.93e+02 |
| | std | 2.66e+01 | 3.34e+01 | * 2.01e+01 |
| $f_{17}$ | ave | 6.07e+02 | 6.48e+02 | * 5.59e+02 |
| | std | 5.41e+01 | 7.33e+01 | * 4.47e+01 |
| $f_{18}$ | ave | 5.07e+02 | 5.09e+02 | * 5.05e+02 |
| | std | 2.26e+00 | 4.36e+00 | * 1.29e+00 |
| $f_{19}$ | ave | 6.13e+02 | 6.14e+02 | * 6.13e+02 |
| | std | 1.02e+00 | 8.90e-01 | * 8.50e-01 |

Table 6: t-test between symcdp and pso

| $f$ | Test statistic, $t$ | Probability, $P(t)$ | Significantly better method |
|---|---|---|---|
| $f_1$ | 0.000 | 5.000e-01 | |
| $f_2$ | 0.076 | 5.302e-01 | |
| $f_3$ | -3.706 | 2.348e-04 | Proposed |
| $f_4$ | 2.459 | 9.918e-01 | PSO |
| $f_5$ | -4.407 | 2.854e-05 | Proposed |
| $f_6$ | -0.067 | 4.734e-01 | |
| $f_7$ | -1.515 | 6.656e-02 | |
| $f_8$ | 1.463 | 9.266e-01 | |
| $f_9$ | -1.053 | 1.477e-01 | |
| $f_{10}$ | -3.178 | 1.206e-03 | Proposed |
| $f_{11}$ | -0.253 | 4.003e-01 | |
| $f_{12}$ | -6.703 | 1.377e-09 | Proposed |
| $f_{13}$ | -4.792 | 2.999e-06 | Proposed |
| $f_{14}$ | -1.276 | 1.026e-01 | |
| $f_{15}$ | -0.994 | 1.614e-01 | |
| $f_{16}$ | -3.389 | 5.189e-04 | Proposed |
| $f_{17}$ | -4.808 | 2.868e-06 | Proposed |
| $f_{18}$ | -5.781 | 7.313e-08 | Proposed |
| $f_{19}$ | -1.955 | 2.679e-02 | Proposed |

Table 7: t-test between symcdp and oscdp

| $f$ | Test statistic, $t$ | Probability, $P(t)$ | Significantly better method |
|---|---|---|---|
| $f_1$ | -3.782 | 2.120e-04 | Proposed |
| $f_2$ | -6.147 | 2.622e-08 | Proposed |
| $f_3$ | -7.832 | 8.004e-11 | Proposed |
| $f_4$ | 3.189 | 9.989e-01 | OSCDP |
| $f_5$ | -5.796 | 2.411e-07 | Proposed |
| $f_6$ | -1.229 | 1.113e-01 | |
| $f_7$ | -1.388 | 8.414e-02 | |
| $f_8$ | 0.074 | 5.293e-01 | |
| $f_9$ | 0.437 | 6.685e-01 | |
| $f_{10}$ | -11.201 | 1.229e-15 | Proposed |
| $f_{11}$ | -6.082 | 1.201e-08 | Proposed |
| $f_{12}$ | -6.976 | 3.398e-10 | Proposed |
| $f_{13}$ | -4.069 | 4.871e-05 | Proposed |
| $f_{14}$ | -6.312 | 4.118e-09 | Proposed |
| $f_{15}$ | -1.786 | 3.857e-02 | Proposed |
| $f_{16}$ | -8.118 | 2.282e-12 | Proposed |
| $f_{17}$ | -7.309 | 8.481e-11 | Proposed |
| $f_{18}$ | -7.219 | 6.538e-10 | Proposed |
| $f_{19}$ | -3.628 | 2.283e-04 | Proposed |