Optimization of the PWM power converter using meta-heuristics

KazuyaMorita† and *KenyaJin′no*†

†Nippon Institute of Technology
Email: jinno@nit.ac.jp, e1092423@estu.nit.ac.jp

Abstract—The purpose of this article is to improve the efficiency of the single-phase PWM DC-AC inverter based on the optimization of the switching phase of the PWM. The improvement of the efficiency means to improve the quality of the output waveform, the efficiency of the conversion, and the reduction of the switching-loss. We propose an evaluation function which corresponds to the minimum value of the evaluation function by using the PSO algorithm. We improve the efficiency by setting a criterion of each index. As the result, the proposed algorithm can reduce high harmonic components in the switching times smaller than that of the conventional triangle wave comparison method. Also, we confirm that the algorithm can improve the quality of the output waveform and the efficiency of the conversion.

1. Introduction

In electrical power systems, a lot of electric power switching conversion technologies are used[1][2]. The PWM DC-AC inverter is one of such conversion technologies[3].

The purpose of this article is to improve the efficiency of the single-phase PWM DC-AC inverter. We suppose that the efficiency is improved when the following conditions are satisfied.

(1) Improve the quality of output waveform.

The quality of output waveform is improved if the higher harmonics are eliminated. The ideal output contains only the fundamental component when the higher harmonics are eliminated by LPF(Low Pass Filter).

(2) Improvement of the conversion efficiency.

The conversion efficiency is improved when the energy of the eliminating harmonic components are reduced

(3) Reductive of switching-loss.

In order to reduce the switching-loss, the number of switching times is decreased.

In this article, we propose an evaluation function to evaluate above points. The value of evaluation function depends on the switching phase. We apply the PSO algorithm to optimize the switching phase. We confirmed by numerical experiments the effectiveness of the proposed method, We will compare the results of the triangle wave comparison method.

2. Single-phase PWM DC-AC inverter

Single-phase PWM DC-AC inverter converts direct current into the alternating current by using PWM control[3]. The output V_0 alternates between a positive and a negative values since these pairs switch alternately. Based on the switching phase, we can produce various PWM output. Namely, the high frequency harmonic components of the PWM output are eliminated by the LPF.

The output voltage V_0 is described by Fourier series as follow:

$$V_o = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t) + \sum_{n=1}^{\infty} B_n \sin(n\omega t), \qquad (1)$$

where, A_0, A_n and B_n means Fourier coefficients.

For simplicity, we normalize the period as 2π without loss of generality. Considering the symmetrical property of the periodic output waveform, the output can be regarded as an odd function. We pay attention to only a quarter of the period. We can calculate the corresponding Fourier coefficients B_n as

$$B_n(x_1, x_2, \ldots, x_k) = \begin{cases} \frac{4\mathrm{V_{dc}}}{n\pi}[\cos(n\omega t)]_{x_1,\ldots,x_k}^{x_2,\ldots,\frac{\pi}{2}} (k : odd) \\ \frac{4\mathrm{V_{dc}}}{n\pi}[\cos(n\omega t)]_{x_1,\ldots,x_{k-1}}^{x_2,\ldots,x_k} (k : even) \end{cases} \qquad (2)$$

where, x_i denotes the i-th switching phase, k means the number of switching times in the quarter period. $\mathrm{V_{dc}}$ means the input voltage value. $\mathrm{V_{dc}}$ is normalized as 1 without loss of generality. Power spectrum can be calculated from Eq. (2). Figure 1 shows an example of the power spectrum. The horizontal axis represents the frequency. The vertical axis represents the absolute value of the voltage. PWM output, is composed of many harmonic components as shown in Fig. 1. To improving the quality of the output waveform, reducing the high harmonics components is required. The high harmonic components contains of the PWM output is determined by the switching.

Triangle wave comparison method is one of the most useful design method of the switching phase of the PWM DC-AC inverter[2]. This design method determines the

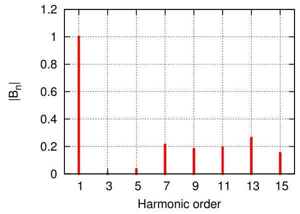


Figure 1: Example of power spectrum.

switching phase comparing with the higher frequency triangle wave and the desired sinusoidal wave. Based on the obtained switching phase, the PWM output is generated. The triangle wave comparison method can reduce the higher harmonic components when the number of switching times is increased. Therefore, in order to suppress the harmonics of the output, the number of the switching times is increased. However, increasing the number of the switching times is the cause to be generated switching-loss. In order to overcome such problem, to decrease the number of the switching times is required. Such problem is called "Optimal PWM problems" [4][5]. In Refs, [4] and [5], the optimal switching phase is solved by numerical solution, but the calculation cost very high. In order to reduce the amount of calculation, some algorithms are proposed which are based on meta-heuristics[6][7]. The PSO method is one of such meta-heuristics. In this article, we propose a design procedure of an optimal switching phase to improve the efficiency the triangle wave comparison method.

3. Evaluation function

Various methods have been proposed to optimize the switching phase of the inverter[8][9]. In Ref. [8], the evaluation function evaluates the filter characteristics under specified conditions. In Ref. [9], it evaluates weighting harmonic components. However, these methods does not consider the conversion efficiency sufficiently. Therefore, we consider the switching phase which satisfies the following three points.

(1) Improve the quality of output waveform.

(2) Improvement of the conversion efficiency.

(3) Reductive of switching-loss.

Reductive of the switching-loss can be achieved if the number of switching times is reduced. Therefore, we propose a evaluation function considering (1) and (2).

(1) Improve the quality of output waveform.

In order to optimize the switching phase of the DC-AC inverter, total harmonic distortion (abbr. THD) is applied to evaluation function[10]. The THD shows the ratio of the harmonic components to the fundamental sinusoidal wave. The small THD value indicates that the output wave contains the small harmonic components. The THD is calculated as the following.

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} B_n^2}}{B_1}, \tag{3}$$

where, n is the order of the harmonics, B_1 is the fundamental component, B_n represents the energy of the n-th harmonic component.

The THD in Eq. (3) is referred the infinity degree. However, the actual output of the DC-AC inverter is passed through a LPF. Namely, the high frequency harmonic components are eliminated by the LPF. The scope of the harmonics of the output waveform is finite. Therefore, we proposed the following evaluation function.

$$F_t = \frac{\sqrt{\sum_{n=2}^{K} B_n^2}}{B_1}, \tag{4}$$

where, K denotes the upper limit of the evaluation range. Namely, F_t is represented in Eq. (4) evaluates between 2nd component and the K-th component. If this value becomes zero, it means that the output is composed of only the fundamental component within the desired frequency range.

Since F_t evaluates only the ratio of harmonic components, the energy of the output waveform may be reduced. Therefore, we have to evaluate the energy of the output waveform. The evaluation function of the energy is the following.

$$F_p = \left| 1 - \frac{\sqrt{\sum_{n=1}^{K} B_n^2}}{\sqrt{2} P_d} \right|, \tag{5}$$

where, P_d means a desired effective value. which is a positive arbitrary constant less than 1.

F_p evaluates the effective value of the output whose upper limit is the K-th component. If this function becomes zero, the effective value of the output is close to the desired effective value P_d.

The quality of the output waveform can be evaluated by the F_t and F_p.

(2) Improvement of the conversion efficiency.

The conversion efficiency is improved when higher harmonic components are removed by LPF. Therefore, we define an index of the conversion efficiency F_e as follow F_e conversion efficiency.

$$F_e = \left| 1 - \frac{B_1}{\sqrt{\sum_{n=1}^{\infty} B_n^2}} \right| \tag{6}$$

If the output contains only fundamental frequency component, Eq. 6 becomes zero. Please note that such situation is impossible.

This optimization problem has above three evaluation functions, F_t, F_p and F_e. Thus this problem can be regarded as a kind of multi-objective problems[11][12][13]. For the multi-objective problems, the construction of the evaluation function is very important. In this article, we apply the linear combination of F_t, F_p and F_e as represented in Eq. (7).

$$F = \alpha F_t + \beta F_p + (1 - \alpha - \beta) F_e, \tag{7}$$

where, $0 < \alpha, \beta < 1$ and $\alpha + \beta \leq 1$ are mixture ratio parameters. The optimum switching phase corresponds to the smallest value of the objective function.

4. PSO

In order to search the optimum switching phase, we use PSO[14][15].

In the PSO, each particle has the location information which gives the best evaluation value before visited locations. The best location information is called ***pbest***. Each particle shares ***pbest*** information in the swarm, and selects the best *pbest* in the swarm. The best information in ***pbest*** is called ***gbest***. The dynamics of the PSO is described as

$$\boldsymbol{v}_j^{t+1} = w\boldsymbol{v}_j^t + c_1 r_1(\boldsymbol{pbest}_j^t - \boldsymbol{x}_j^t) + c_2 r_2(\boldsymbol{gbest}^t - \boldsymbol{x}_j^t), \tag{8}$$

$$\boldsymbol{x}_j^{t+1} = \boldsymbol{x}_j^t + \boldsymbol{v}_j^{t+1}, \tag{9}$$

where, $\boldsymbol{x}_j^t$ denotes the location of the j-th particle on the t-th iteration. $\boldsymbol{v}_j^t$ denotes the velocity of the j-th particle on the t-th iteration. w denotes an inertia weight coefficient. c_1 and c_2 denote acceleration coefficients of the j-th particle. r_1 and r_2 denote the uniform random numbers from 0 to 1.

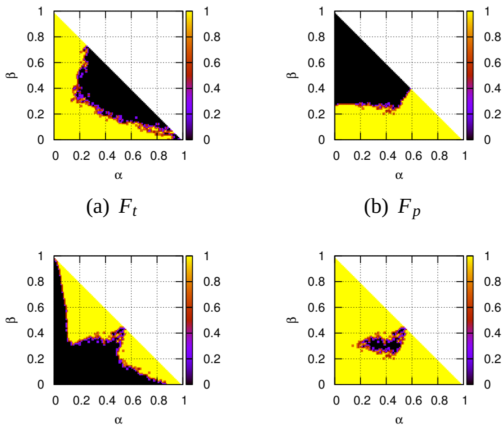
The PSO algorithm searches the optimum solution of Eq. (7). The location $\boldsymbol{x}_{ij}$ which represents the i-th dimensional value on the j-th particle corresponds to the switching phase of the PWM output. Since each dimensional variable corresponds to each switching phase of the PWM, each dimensional variable must satisfy the following condition.

$$0 \leq x_{1j} \leq x_{2j} \leq \cdots \leq x_{kj} \leq \pi/2, \tag{10}$$

where k denotes the maximum number of the switching times in the quarter period. We used to update PSO method of successive asynchronous performs state update for each dimension in this article[16]. Also, for the real circuit, an switching interval is required. Therefore the following condition is applied.

$$x_{ij}^t = \begin{cases} x_{i-1,j}^t + \mathrm{T_{limit}} & \text{, for } x_{ij}^t \leq x_{i-1,j}^t \\ x_{i+1,j}^t - \mathrm{T_{limit}} & \text{, for } x_{ij}^t \geq x_{i+1,j}^t \\ \mathrm{T_{limit}} & \text{, for } x_{1j}^t \leq 0 \\ \pi/2 - \mathrm{T_{limit}} & \text{, for } x_{kj}^t \geq \pi/2 \\ x_{ij}^t & \text{, otherwise} \end{cases} \tag{11}$$

where, $\mathrm{T_{limit}}$ is a constant which is depended on the real circuit.



(a) F_t (b) F_p (c) F_e (d) F_t, F_p, F_e

Figure 2: Setting the α and β.

5. Setting the α and β

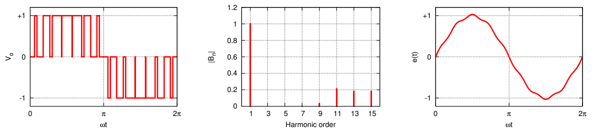
In this article, evaluation function F is a linear combination of F_t, F_p and F_e. The weighted combination parameters α and β is very important. In order to set the parameter values, we define a criterion value of each function. The criterion value of F_t is 0.01, because the output contains less than 1 % error. Finally, the criterion value of F_e is 1.05×10^{-1}, since we want the conversion efficiency more than 89 %. Figure 2 shows the region that each evaluation value satisfy each criterion. The yellow region means the domain of the definition, and the black region means the satisfaction region of the criterion. These results indicate the most effective values that α is around 0.4 and β is around 0.35.

6. Simulation experiment

We carry out the numerical simulations to confirm the effectiveness of proposed switching phase design procedure by using PSO. We assumed the ideal LPF is passed only under nine harmonics. For such ideal LPF, the triangle wave comparison method requires nine or more switching in a quarter period. The parameters of our simulation

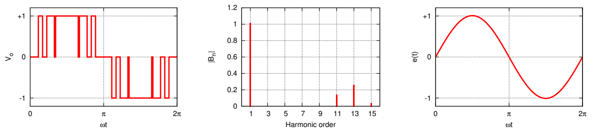
Table 1: Experimental parameters.

The trial number	100
The maximum iteration number	50000
The number of particles	10
The inertia weight coefficient w	0.792
The acceleration coefficient c_1, c_2	1.49
Criterion time $\mathrm{T_{limit}}$	$\pi/1000$
The upper limit of evaluation K	10
The desired effective voltage P_d	$1/\sqrt{2}$
The number of switching (a quarter period) k	5



(a) PWM output waveform (b) Power spectrum (c) Filtered output waveform

Figure 3: Triangle wave comparison method.



(a) PWM output waveform (b) Power spectrum (c) Filtered output waveform

Figure 4: Propose method.

are shown in Table 1. Figure 3 shows an example of the output which is generated by the triangle wave comparison method. In this case the number of switching time in the quarter period is seven. As shown in Fig. 3, some higher harmonic components are remained. Therefore, a ripple is exhibited on the output wave. On the other hand, the results of our proposed method is shown in Fig. 4. In this case, the harmonics in the eliminated band are completely removed. Thus, the quality of the output is improved. The result of each evaluation function are shown in Table 2.

7. Conclusions

In this article, we determined the switching phase of single-phase PWM DC-AC inverter in consideration of the following three items.

(1) Improve the quality of output waveform.

(2) Improvement of the conversion efficiency.

(3) Reductive of switching-loss.

The number of the switching times is less than the triangle wave comparison method. The proposed method can reduce harmonic components of higher order. Also, we confirmed that the quality of the output waveform is improved and the conversion efficiency are also improved. We can say that the switching-loss is also reduced because the number of switching times is less than the triangle wave comparison method.

In the future, it is an object of the present invention examination of the structure of the metrics used in this article, the study of the switching system.

Table 2: Evaluation indexes
(Propose method, Triangle wave comparison method).

	Propose method	Triangle wave comparison method
F_t	0.00	2.80×10^{-1}
F_p	8.11×10^{-3}	3.86×10^{-2}
F_e	1.03×10^{-1}	1.22×10^{-1}

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