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Analysis of Profits/Prices Changes in Formalizing Collaboration among Agents through Double Auctions and Suppression of Fluctuations

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Abstract—As a basic model, we assume that a firm agent produces goods by using support of another outer agents with several cost for labor procurement. Then, the profit of a firm changes under a certain condition despites constant labor price. We extend the model where labors are purchased in a pool market in a competitive manner. If agents try to adjust bid price for the next auction to improve their profits and the utilization of labor stock, then labor prices show fluctuations. Condition for inducing fluctuations and the suppression scheme are discussed.

1. Introduction

Over the past two decades, industries have begun ordering complex assemblies or systems rather than traditional, simple components from their suppliers (called outsourcing) [1]-[4]. However, delegating (outsourcing) tasks to outside firms must be always optimal, and the problems to examine the collaboration based on the mathematical models depending on the situations reveal as another important factors. This paper deals with the analysis of profits/prices changes in formalizing collaboration among agents and its application to the suppression of fluctuation.

At first, we consider a small open economy where a firm agent manufactures goods by using resources provided by outside agents as well as his own capital and labor [3]. Then, it is shown that an increase in wealth raises the investment and the wealth time series bears fluctuations. Secondly, we extend the model to cases where labors are purchased in a pool market in a competitive manner through double auctions[5]-[9]. Agents try to adjust bid price for the next auction to improve their profits and the utilization of labor stock, then the labor prices show fluctuations despites the deterministic scheme of the model, and raises also fluctuations in the wealth time series. Condition for inducing them and the suppression scheme are discussed.

2. Bifurcation observed in basic collaboration

At first, we treat the case where there exits a single firm agent and a single outside agent. Since it is not necessary to distinguish outside agents, we simply define a single agent who provides labor for the production. In the definition of basic model for collaboration, we assume followings [3].

(1) Production by firm agent

A firm agent manufactures goods by using his own capital $K_1(t)$ and labor $L_1(t)$ in time period t. Beside $K_1(t)$ and $L_1(t)$, a firm uses the capital $K_2(t)$ and labor $L_2(t)$ provided by outside agent. Under these conditions, the output of products (goods) $y(t)$ is usually represented by the so-called production functions. We use the production function of the Cobb-Douglus type described as follows.

$$y(t) = A[K_1(t) + K_2(t)]^{\rho}[L_1(t) + L_2(t)]^{1-\rho} \tag{1}$$

where A (a constant value) denotes the total factor productivity, and $\rho(0 < \rho < 1)$ is the elasticity of production (also a constant value).

(2) Purchase of capital and labor

A firm agent obtains the wealth $W(t)$ at the end of production in period t, and then makes the total investment $I(t)$ in period t for the production in period $t + 1$ which is devoted to purchase both capital and labor from the outside agent. For the given level of investment, the optimal demand for the labor input $p(t)$ and for capital $K_2(t)$ in each period rise from the maximization of profit function subject to the budget constraint.

$$I(t) = K_2(t) + p(t)L_2(t) \tag{2}$$

where $p(t)$ is the market specific labor price for a units of purchased labor. Now, we assume that the capital $K_1(t)$ and labor $L_1(t)$ prepared by the firm agent are determined at the beginning of whole production, and are not included in the investment in period t. We also assume that the capital $K_2(t)$ come from outside is purchase in a long range, and is not affected by the market, even though the capital $K_2(t)$ is still remains as variable to be determined in the profit maximization.

(3) Optimal production

We take the derivatives with respect to the variable $L_2(t)$, we have the next relation.

$$L_2(t) = [B + (1-\rho)I(t)]/p(t), K_2(t) = -B + I(t) \tag{3}$$

$$B = K_1(t)(1-\rho) - \rho p(t)L_1(t) \tag{4}$$

It must be noted that the optimal value of the variable $L_2(t)$ includes the price $p(t)$, and depends on it. However, for simplicity we assume that the price $p(t)$ is known as a prescribed value, even though in the latter sections of the paper dealing with multiple-agents systems we change the assumption into fluctuating $p(t)$.

By using the optimal solution to maximize $y(t)$, we have the expression for the optimal value of production as follows.

$$y(t) = A[K_1(t) - B + \rho I(t)]^{\rho}[D + E\rho I(t)]^{1-\rho} \quad (5)$$

where C, D, E are functions of $L_1(t), \alpha, \rho, p(t)$.

We use another expression for the variable $I(t)$, and then $y(t)$ will be further transformed into another form in the succeeding discussions. The firm agent can invest in the productive project where the largest amount he can borrow is given as.

$$I(t) = W(t) + \alpha W(t) = (1 + \alpha)W(t) \quad (6)$$

where r is the interest rate. Hence, in period $t + 1$ the firm agent receives the corresponding profits and pays the cost of debt $rL(t) = r\alpha W(t), r > 1$. Therefore, the dynamics of the wealth of firm is given by

$$W(t + 1) = q[y(t) - r\alpha W(t)] \quad (7)$$

By substituting equation (6) into the above equation, and also substituting $y(t)$ into equation (7), we have the following relation.

$$W(t+1) = [qA[K_1(t)-B)+CW(t)]^{\rho}[D+E'W(t)]^{1-\rho} - r\alpha W(t)] \quad (8)$$

$$E' = (1 - \rho)(1 + \alpha)/p(t) \quad (9)$$

If the interest rate r is greater than the rate of profit obtained by the production, the firm agent feels no incentive to borrow up to the credit limit and use his own current wealth $W(t)$ for production. Then, the firm agent selects productive activity rather than credit. These situations occur if the specific value W^m of W satisfies the relation $y(t) - r\alpha W(t) = rW(t)$, then we obtain the value W^m which satisfies the relation.

By using the value W^m, including the the case where $W(t) \leq W^m$ is satisfied, the asymptotic behavior of wealth is thus determined by the iterate of the following functions.

$$W(t + 1) = \begin{cases} q[y(t) - r\alpha W(t)]; 0 \leq W(t) \leq W^m; \\ rW(t); W(t) > W^m \end{cases} \quad (10)$$

2.1. Fluctuation result for $W(t)$

In the following, we show the result of fluctuations for the wealth time series $W(t)$. Since the functional form for $W(t)$ is complicated, it is hard to show them analytically, and then we use the bifurcation diagram for $W(t)$ depending on the parameter α based on simulation studies.

Fig.1 shows an example for the time series $W(t)$ with $\rho = 1/3, A = 1.5, L_1 = K_1 = 100, r = 1.02, \alpha = 58$. Fig.2 shows the bifurcation diagram for $W(t)$ depending on α. As is seen from these figures, the time series $W(t)$ is stable in the range $\alpha < 10$, but shows bifurcation if $\alpha \geq 10$. We define Region L for $\alpha < 10$ (Region U for $\alpha \geq 10$).

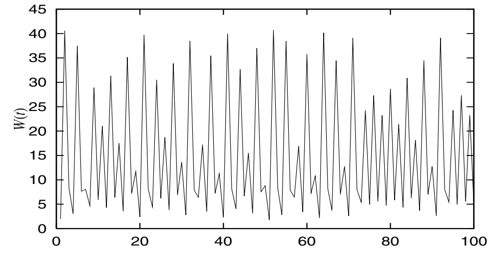

Fig.1 An example of time series $W(t)$.

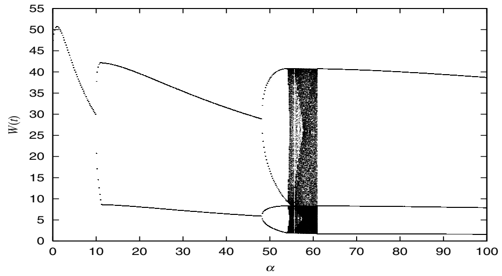

Fig.2 Bifurcation diagram for $W(t)$ depending on α.

Even though the wealth time series $W(t)$ is stable and has a single value if $\alpha < \alpha_B, \alpha_B = 10$ and with constant value of price $p(t)$, but $W(t)$ show fluctuation if $p(t)$ changes. Sources of fluctuation for $p(t)$ is discussed in succeeding sections, and we only show an example of rise of bifurcation in $W(t)$. We only discuss the suppression of fluctuation of $W(t)$ in Region L.

3. Agents' behavior and double auction

Now we extend the model to cases where labors are purchased in a competitive manner through double auctions. Formal description of double auction is complicated to discuss the fluctuation of labor price, then we simplify the model by introducing prediction by firm agent about the labor prices. We assume that there exist multiple outside agents, and agent i possesses multiple pool of human resources $j, j = 1, 2, ..., J_i$ with volume $v_{i,j}$ and price $p_{i,j}(t)$, which is time-varying.

Firm agent

We assume the firm agent is identical to the auctioneer, and then he will forecast the labor price at time $t + 1$ as

$$\pi_{i,j}(t + 1) = (1 - \omega)\pi_{i,j}(t) + \omega p_{i,j}(t) \quad (11)$$

where $p_{i,j}(t)$ is realized bid price in t the auction by outside agents, and $\pi_{i,j}(t)$ is the pervious forecast of labor price about $p_{i,j}(t)$ made by firm agent, including smoothing factor $\omega(0 < \omega < 1)$. Firm agent purchases by using forecast $\pi_{i,j}(t + 1)$ based on the selection from cheaper price. Namely, firm agent selects successively the bid made by outside agent i based on the cheaper price about $p_{i,j}$ until the demand for labor by firm agent is satisfied. If the accumulation of $v_{i,j}$ corresponding to $p_{i,j}$ reaches the necessary demand, then the auction is closed. The last (marginal) price $p_{i,j}$ of this accumulation is called as bid price for Pay Marginal (denoted as p_M).

Outside agent

We assume that the outside agent i revises the bid price by considering results of current auction at time t. The utilization of resource weighted by volumes $v_{i,j}$ denoted as $\lambda_i(t)$ is defined as.

$$\lambda_i(t) = \frac{\sum_{j=1}^{J_i} p_{i,j}(t) v_{i,j} r_{i,j}(t)}{\sum_{j=1}^{J_i} p_{i,j}(t) v_{i,j}} \tag{12}$$

where $r_{i,j}(t) = 1 (r_{i,j}(t) = 0)$ means the group j is included (not included) in the current auction. Then, we assume that outside agents revise the bid prices to the next auction by using simple formula as.

$$p_{i,j}(t+1) = p_{i,j}(t)[1 + c(\lambda_i(t) - \lambda_{i0})] \tag{13}$$

$$p_{min} \le p_{i,j}(t+1) \le p_{max} \tag{14}$$

where c is an appropriate constant common to all agents.

4. Conditions for inducing fluctuation

Then, we discuss the conditions for inducing fluctuations [11].

Condition A

It is easily seen that the system is moved to an equilibrium if the following condition is satisfied, since every agents do not adjust the bid price.

$$\lambda_i(t) = \lambda_{i0} \tag{15}$$

However, we need another condition to keep the steady state of the equilibrium [10].

Condition B

$$\sum_{k=1}^{J_i} |\partial \Pi_{i,j} / \partial \pi_{i,k}| < 1, j, k = 1, 2, ..., J_i \tag{16}$$

$$\Pi_{i,j} = (1 - \omega)\pi_{i,j}(t) + \omega p_{i,j}(t) \tag{17}$$

Then, the boundary state $\sum_{k=1}^{J_i} |\partial \Pi_{i,j} / \partial \pi_{i,k}| = 1$ determine the value for c to induce fluctuations. By using the relation for derivatives $\partial p_{i,j} / \partial \pi_{i,j} = (\partial p_{i,j} / \partial \lambda_i)(\partial \lambda_i / \partial \pi_{i,j})$, we have

$$c \sum_{j=1}^{J_i} p_{i,j} v_{i,j} (r_{i,j} - \lambda_i) = V, V = \sum_{j=1}^{J_i} p_{i,j} v_{i,j} \tag{18}$$

The solution $c = c_m$ for the above equation gives the minimum value of c for inducing fluctuations.

5. Suppression of fluctuation

Different from simple system with few agents, we need following additional procedure to control (suppression) with multiple agents. Since we have multiple outside agents, if a fluctuation is remained in a certain time series in $p_{i,j}(t)$, then it propagates through the system and ultimately destroy the control. Then, we need to simultaneously start imposing external force at all bid prices of outside agents.

As is easily seen the optimal control is reduced by adjusting labor prices so that the following condition is satisfied.

$$\frac{\sum_{j=1}^{J_i} p_{i,j}(t) v_{i,j} r_{i,j}(t)}{\sum_{j=1}^{J_i} p_{i,j}(t) v_{i,j}} = \lambda_{i0} \tag{19}$$

However, the equation includes J_i variables, and we can have no consistent solutions in this form. Therefore, we need some relaxation of restrictions by introducing several relations among variables $p_{i,j}(t)$, and we use an alternative. We propose two schemes to suppress the fluctuations in labor price.

Scheme F

If we know the stable fixed point for a certain variable $x(t)$, we impose an small external force $u(t)$ to the current state $x(t)$ so as to lead the state to the fixed point $\hat{x}(t+1)$ at time $t+1$, so that $\hat{x}(t+1) = f_c[x(t) + u(t)]$.

As is easily seen the optimal control is reduced by adjusting labor prices so that the condition in equation (19) is satisfied. We need some relaxation of restrictions by introducing several relations among variables $p_{i,j}(t)$. By substituting $p_{i,j}(0)$ for $p_{i,j}(t)$, then it is clear that the condition $p_{i,j}(t) = p_{i,j}(0)$ is equilibrium for outside agent i. Now, the suppression of fluctuation (equilibrium) is attained by changing the value of $p_{i,j}(t)$ to $p_{i,j}(0)$ at time t.

Scheme M

The suppression scheme (Scheme F) is seemed to be unrealistic, because the initial conditions are forced to outside agents. Then, the alternative to mitigate the fluctuation is available. We change the clearing in bidding process from Pay Bid to Pay Marginal where every agent can be paid the same price rather than bid prices for labor supply as the marginal price in the bidding process. The effect of suppression scheme is examined in real auctions for electricity trades.

6. Applications

Conditions for simulation studies are summarized as follows.

$N_a = 10 \sim 100$
initial $p_{i,j}$: normal distribution with mean=20, variance=1.2^2
volume $v_{i,j}$: normal distribution with mean=5, variance=1.2^2
No. of resources: $J_i = 3 \sim 8$, selected at random
λ_{i0}: $\lambda_{i0} = 0.1 \sim 0.9$
ω: $\omega = 0.3$
(Fluctuation)

Fig.3 shows an example for fluctuation for labor price $p_{i,j}(t)$ for $N_a = 50, c = 1.5$. We see strict changes (fluctuations) in price time series. Then, the bid price in double auction market is affected by these fluctuation of labor prices. Fig.4 shows an examples for fluctuation for price $p_M(t)$ for Pay Bid auction.

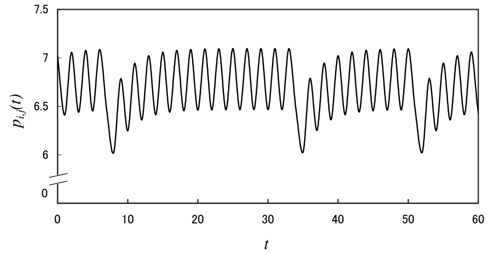

Fig.3 An example for fluctuation for labor price $p_{i,j}(t)$.

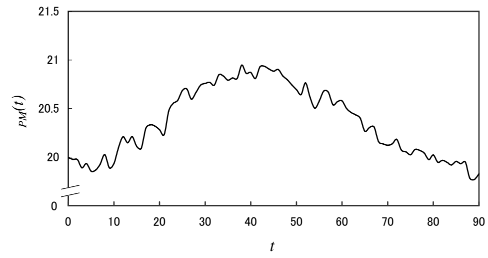

Fig.4 An examples for fluctuation for price $p_M(t)$ for Pay Bid auction.

(Suppression of fluctuation)

Then, we treat the suppression schemes of fluctuation in labor price. As mentioned earlier, the equilibrium is attained if all of the prices satisfy equation (19). However, it is vary hard to obtain solutions, since variables $p_{i,j}(t)$ are arbitrary. Fig.5 shows an example of suppression of fluctuation of $p_{i,j}(t)$ with $N_a = 50, c = 1.1$ using Scheme F (For simplicity, imposed external force is omitted here). Fig.6 shows an example of suppression of fluctuation of $p_{i,j}(t)$ of a certain agent under Scheme M. As is seen from the results, Scheme F seemed to be relevant, however unrealistic. In Scheme M, prices still change, however, become stable, and easily attainable.

7. Conclusion

This paper treated the analysis of profits/prices changes in formalizing collaboration among agents through double auctions and their suppressions. For future works, we will

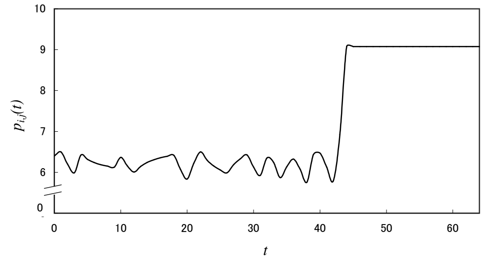

Fig.5 Suppression of fluctuation of $p_{i,j}(t)$ (Scheme F).

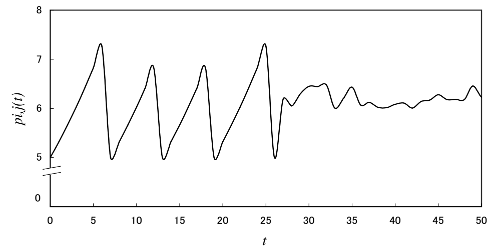

Fig.6 Suppression of fluctuation of $p_{i,j}(t)$ (Scheme M).

examine the real examples of collaborations and usefulness of suppression of fluctuations in labor price.

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