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An overview on emerging spatial wave logic for spatial-temporal events via Cellular Wave Computers on Flows and Patterns

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Abstract—After a brief summary of a century of formal logic culminating in the Gödel theorem, the Turing Machine and the stored programmable digital computer of Von Neumann, the Universal Machine on Flows is defined rigorously. Based on this machine, the cellular wave computer and the elements of spatial-temporal wave logic is introduced. It is motivated by the emerging architectures in nanoscale technology: morphic architectures and patterns as data. It becomes clear that these architectures have essential additional constraints and advantages over the general parallel architectures.
A framework is shown how to embed the nonlinear spatial-temporal dynamics as an elementary instruction and how to exploit the emerging many core chips for solving nonlinear problems efficiently.

1. Introduction

Starting with Boolean formal logic in 1845, a century of ingenious insights led to the invention of the digital computer. The Begiffschrift of Frege, the Russel paradox, the Pricipia Arithmetica of Peano, the Principia Mathematica of Russel and Whitehead, the Tractatus logico philosophicus of Wittgenstein, just to mention, a few led to the epoch making discovery of Gödel, his incompleteness theorem, but also to the Gödel numbering and the parallels between logic and computing. Turing's Machine formalized the computational framework. Finally, via introducing stored programmability, as well as by interchanging the data and program representations, John von Neumann invented the (digital) computer, as the cornerstone of computing. This machine is defined on integers as data types, the protagonist instructions are the logic functions on bits and elementary arithmetic, and the algorithms are defined as mu-recursive functions. The advent of the integrated circuit industry made the computer, in a form of a microprocessor, the protagonist of modern science and technology for 50 years.

At the same time, during these 50 years, the data types, the architecture and the elementary instructions did not change significantly. The first "powerful" 8 bit microprocessor, the Intel 8080, consisted about 5, 000 transistors and we can now host about 5 billion on a single chip. Why do we not place a million of 8 bit processors on a chip?

Since about 15 years, a new paradigm of computing has emerged, we call CNN technology or Cellular Wave Computers. Here, the data type is a spatial temporal flow or pattern, the elementary instruction is a spatial-temporal wave, and the algorithms are defined on topographic flows [1,2,3]. The main properties of this new kind of computing are as follows.

- *the geometric place of a signal* (as well as of a „processor") *plays a significant role,*
- *an individual signal is not coding any event, on the contrary, a set of analog signals with their characteristic spatial-temporal patterns code an event, and*
- *the processor) are connected either locally, within a receptive field, or in a sparse global, bus-like way, the time delay of connections could play also a role*.

The archetype of the standard wave instruction is the Turing morphogenesis, with their analog valued oscillatory cells, defined via the Cellular Nonlinear/neural Network (CNN) dynamics, and their logic architecture's is the Cellular Automaton. The combination of analog/arithmetic and binary logic (also called analog-and-logic) operations in space and time is a new possibility that became practical only recently, by using high complexity devices (deep submicron technology). The emergence of chips with over 1000 digital processors and over 25 000 mixed mode processors, even integrated with optical sensors as a visual microprocessor) mark a new era in computing.

2. Cellular Wave Computers

Let us briefly and formally define the Cellular Wave Computer, as an extension of the CNN Universal Machine.

- Data are topographic flows (cell array signals on a 1-, 2- or 3-dimensional grid, e.g a visual, a tactile or an auditory flow, or the states of atoms in a molecular dynamics calculation).

Data type: topographic flow, **Φ (t)**, in R^n (1D), R^{nxm} (2D), or R^{nxmxp} (3D) as a function in time (continuous or, as a special case, discrete) . In a 2D image flow

$$\mathbf{\Phi(t)}: \varphi\{_{ij}(t)\}, \quad t \in [0, T] \text{ in R}, \quad 1 \le i \le n, \quad 1 \le j \le m$$

For example, a 2D image flow could represent the input or the output image flow of a retina.

A nxm *Map (e.g. an image or picture)* P: t = t*, P= **Φ (t***)

if P is binary it is a *Mask* M

if t = t0, t0 + Δt, t0+ 2Δt, …… t0+ k Δt then we say that it is a *map sequence* (e.g. a v*ideo stream).*

- Instructions are defined in space and time, typically as a spatial-temporal wave acting on the image flow data, the cell signals are *continuous* (real) valued (analog or digitally coded) *and binary*; locally (cell by cell) stored in the cells.
- This local storage, providing for stored programmability in von Neumann sense, may be static or dynamic.
- The protagonist elementary instruction, **Ψ** (**Φ**) , also called *wave instruction*, is defined as
  Φ output (t):= **Ψ** (**Φ** input(t), P, ∂); t ∈ T=[0, td] where
  Ψ *: a function on image flows or image sequences*
  P : a map (image) defining initial state and or bias map
  ∂ : boundary conditions, ∂ (t) is a boundary input (might be connected to all cells in a row)
  T is the finite time interval
  A scalar *functional γ on an image flow* is used for branching instructions*:*
  q: = γ (**Φ** input(t), P, ∂);

For example, the so called *global white functional* on a binary mask M is logic 1, if the picture is full white, if at least one pixel is black it is logic 0. The *maximum functional* on a flow is defined by the highest scalar value at any pixel at any time.

We emphasize that the signal and instruction representation in *this architecture may have various physical* realizations*,* it might be analog, mixed mode, digital, using CMOS, optical, etc. fabrication technologies.

As a simple special case for the spatial temporal dynamics, defining the topographic wave instruction, *is the standard Cellular Nonlinear Network (CNN) dynamics* [e.g. 4], given by the well known equation for one first order cell layer.

The output image flow, Y (t) will be calculated from the input image flow U (t) as the solution of the following discrete space, continuous time, nonlinear dynamics (PDDE: partial differential difference equation):

$$\frac{dx_{ij}}{dt} = -x_{ij} + \sum \mathbf{A}(ij,kl)\,y_{kl} + \sum \mathbf{B}(ij,kl)u_{kl} + z_{ij} \qquad (1)$$

$$y_{ij} = f(x_{ij})$$

for all i Є [1,M] and j Є [1,N].

where the spatial summation Σ is made within the rxr neighborhood of the cell ij, and $u_{ij}(t)$, $x_{ij}(t)$, and $y_{ij}(t)$, are the input, state, and output signals, respectively (elements of U, X, and Y array signal flows).

The standard CNN dynamics, representing the simplest *cellular wave instruction*, is defined by a first order *cell dynamics*, *r*=1 neighborhood radius (3x3), feedback (**A**) and feed forward (**B**) linear *local interaction patterns* (templates),with bias map $z_{ij}=z$, and $x_{ij}(t)$ as the state array, $y_{ij}(t) = \sigma(x_{ij}(t))$ being the nonlinear (typically sigmoid type) output function, and an input image flow defined by $u_{ij}(t)$ [3].

A Standard CNN instruction, a template {**A**,**B**,*z*}, is defined by the 19 (9+9+1) numbers.

The *global control of the computation in a Cellular* Wave *Computer, in general,* is performed via well defined *wave algorithms*, as an algorithmic sequence of wave instructions as well as by local and global binary logic instructions. The rigorous definition is given as the *α-recursive function* [3].

The anatomy of the Cellular Wave Computer is described by two types of interconnections:

- the local interaction between the cell processors – each one to their topographic neighbors, and
- the global access – all to one (or row-wise) in a mutual access.

This Machine is able to run the spatial temporal algorithms, having the protagonist instruction as a solution of a nonlinear Partial Differential Difference Equation (PDDE). This is the crucial point when the expertise of the nonlinear science community has a crucial role in the future.

3. Algorithmic challenges

Here, we summarize some recent breakthrough in algorithmic inventions and mention a few open challenges.

The formal introduction of the front propagation and active contour techniques [4], the implementation of the wave metric [5], and the combination of on-chip physical and algorithmic actions [6] highlighted the recent dramatically new ways of thinking of future algorithmic techniques. Furthermore, the combination of various wave instructions in solving difficult problems are paving the way of a new algorithmic design philosophy and principles [7,8].

Some major open challenges

- the determination of dynamic events by the onset of oscillations or fluctuations
- using locally adaptive templates to solve
- transforming non-topographic problems into topographic, mainly locally connected morphic cellular architectures

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