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Nonlinear Dynamics of Nano-lasers

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Abstract

This paper delineates a range of nonlinear dynamical behaviours, which arise in nano-lasers. Attention will be given to the direct current modulation properties of nanolasers in the large signal regime. The impact of external optical feedback on nano-lasers will be considered. Finally the dynamical behaviour of semiconductor nano-lasers when subject to optical injection will be treated.

Introduction

In recent years, considerable attention has been given to the development of nanolasers due to their potential applications in photonic integrated circuits, optical information processing and system-on-a-chip technologies. A variety of nano-scale lasers have been explored [1-3] including micro-post nano-pillar and bowtie, nanowire and nano-patch lasers where continuous wave lasing has been experimentally studied by optical pumping and electrical pumping.

Such nano-lasers are anticipated to exhibit enhanced dynamical performance which may arise from a combination of physical factors including the Purcell spontaneous emission enhancement factor F, and enhanced spontaneous emission coupling expressed in the factor, β. In recent work, the impact of Purcell enhanced spontaneous emission on the modulation performance of nano-LEDS and nano-lasers [4] has been examined. In complementary work on the dynamical performance of nanolasers it was shown by means of a simple analysis that the direct-current modulation bandwidth of such lasers may suffer deleterious effects due to increased F and β [5]. A number of recent investigations of the dynamical performance of nano-lasers have been made. Ding et. al. explored the dynamics of electrically pumped nano-lasers where the effects of F and β on nano-laser performance were studied [6]. This paper carries forward such work by giving emphasis to nonlinear dynamical aspects of the performance of nano-lasers.

Direct Current Modulation

The response of semiconductor lasers to direct current modulation is a central feature of their behaviour. In the case of semiconductor nanolasers, that response is impacted by the Purcell effect and also due to the availability of enhanced spontaneous emission due to the reduction in the number of laser cavity modes. Such effects appear in modified rate equations provided ,for example, in [4].

$$\frac{dN}{dt} = \frac{I}{eV} - \frac{N}{\tau_n}\big(F\beta + (1-\beta)\big) - V_g g_o (N - N_t)S \qquad (1)$$

$$\frac{dS}{dt} = C_o \beta F \frac{N}{\tau_n} + C_o V_g g_o (N - N_t)S - \frac{S}{\tau_p} \qquad (2)$$

Where, I is the injection current; V is the volume of the active region; τ_n is the radiative carrier lifetime; g_o is the differential gain coefficient, V_g is the group velocity; N_t is the transparency carrier density; C_o is the confinement factor; τ_p is the photon lifetime; N and S are the carrier and photon densities. F is the Purcell Factor and β is the spontaneous emission coupling factor. Gain saturation effects have also been considered by writing the gain in Eq. (1) and (2) in the form $g_o V_g (N - N_t)/(1 + \epsilon S)$. Where, ϵ is the gain saturation factor.

Such equations have been be solved numerically to provide the response of the laser to sinusoidal modulation. Both small-signal and large signal modulation regimes have been explored. It has been shown thereby that for both small and large signal regimes modulation bandwidth of approximately 55GHz can be achieved [7]. In Figure 1 an exemplar of the large signal response is presented where distortions of the response curve are evident. Further details of the modulation response of nano-lasers will be presented.

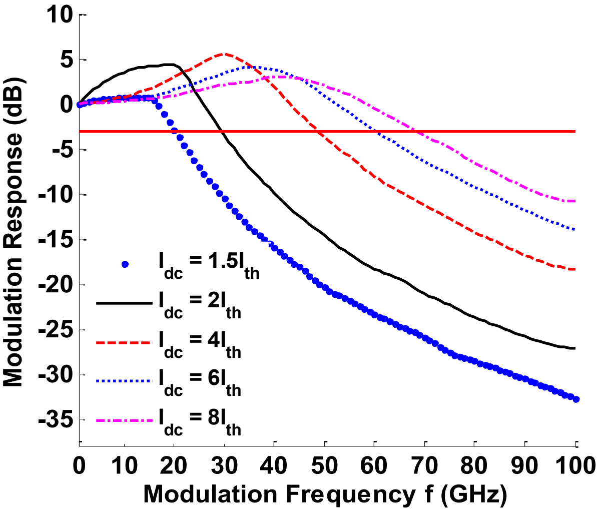

Figure 1. Modulation Response of Nano-laser at β = 1 x 10^{-3} and F = 10 for Large Signal Modulation. The solid red line indicates the -3dB level

Optical Feedback Effects

The response of nano-lasers subject to external optical feedback has also been analysed [8] . Calculations have been performed using rate equations which include the Purcell cavity- enhanced spontaneous emission factor, F, and the spontaneous emission coupling factor β.

$$\frac{dS(t)}{dt} = \Gamma\left[\frac{F\beta N(t)}{\tau_n} + \frac{g_n(N(t)-N_o)}{1+\epsilon S(t)}S(t)\right] - \frac{1}{\tau_p}S(t)$$

$$+2k\sqrt{S(t)S(t-\tau_{ext})}\cos(\theta(t)) \qquad (3)$$

$$\frac{dN(t)}{dt} = \frac{I_{dc}}{eV_a} - \frac{N(t)}{\tau_n}(F\beta + (1-\beta))$$

$$-\frac{g_n(N(t)-N_o)}{1+\epsilon S(t)}S(t) \qquad (4)$$

$$\frac{d\emptyset(t)}{dt} = \frac{\alpha}{2}\Gamma g_n(N(t)-N_{th}) - k\frac{\sqrt{S(t-\tau_{ext})}}{\sqrt{S(t)}}\sin(\theta(t)) \qquad (5)$$

$$\theta(t) = \omega_o\tau_{ext} + \emptyset(t) - \emptyset(t-\tau_{ext}) \qquad (6)$$

Where, $S(t)$ is the photon density, $N(t)$ is the carrier density and $\emptyset(t)$ is the phase, $\theta(t)$ is the phase change, Γ is the confinement factor, τ_n and τ_p are the radiative carrier lifetime and photon lifetime respectively. g_n is the differential gain that takes into account the effect of group velocity, N_o is the transparency carrier density, ϵ is the gain saturation factor and α is the linewidth enhancement factor. I_{dc} is the dc bias current, V_a is the volume of the active region e is the electron charge and N_{th} is the threshold carrier density, ω_o is the optical frequency.

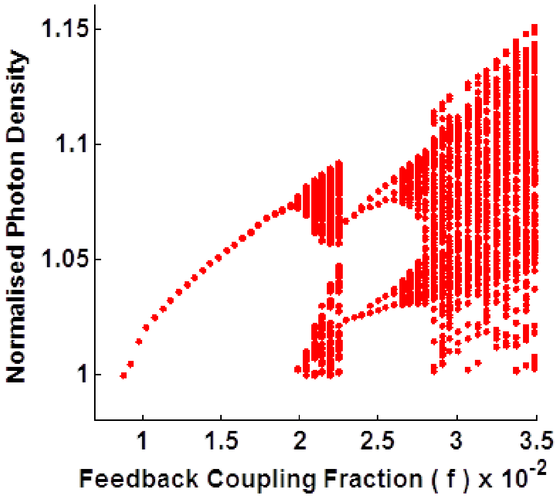

Figure 2 Bifurcation diagram of normalised photon density vs feedback coupling fraction at $I_{dc} = 2I_{th}$, F = 14 and β = 0.1

In the analysis the influence of F and β is evaluated for varying distance from external mirror, current and feedback rate. It is observed that, in general, increased F and β at low bias currents increase the critical feedback for which chaos occurs as compared to conventional lasers, whereas at higher bias currents, chaos occurs at lower feedback rates. It is also found that for larger F, when increasing the distance from external mirror, the feedback rate at which chaos occurs increases.

Bifurcation diagrams are a convenient way to represent the changes in dynamics of lasers subject to optical feedback. In Figures 2 and 3 bifurcation diagrams are given for two values of the Purcell factor,F. The qualitative difference in the response is immediately apparent.

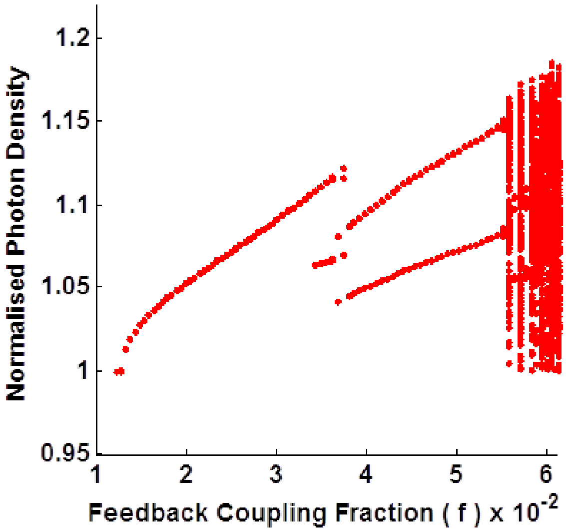

Figure 3: Bifurcation diagram of normalised photon density vs feedback coupling fraction at $I_{dc} = 2I_{th}$, F = 30and β = 0.1,

Optical Injection Effects

The third aspect of the nonlinear dynamics, which is explored in this paper, is the response of nano-lasers to external optical injection. Again, the focus is on the role played by the Purcell effect and the enhanced spontaneous emission coupling as shown in the following rate equations.

$$\frac{dS(t)}{dt} = \Gamma\left[\frac{F\beta N(t)}{\tau_n} + \frac{g_n(N(t)-N_o)}{1+\epsilon S(t)}S(t)\right] - \frac{1}{\tau_p}S(t)$$

$$+2k_{inj}\sqrt{S(t)S_m}\cos(\theta(t)) \qquad (7)$$

$$\frac{dN(t)}{dt} = \frac{I}{eV_a} - \frac{N(t)}{\tau_n}(F\beta + (1-\beta))$$

$$-\frac{g_n(N(t)-N_o)}{1+\epsilon S(t)}S(t) \qquad (8)$$

$$\frac{d\emptyset_s(t)}{dt} = \frac{\alpha}{2}\Gamma g_n(N(t) - N_{th}) - 2\pi\Delta f$$
$$-k_{inj}\frac{\sqrt{S_m}}{\sqrt{S(t)}}\sin(\theta(t)) \qquad (9)$$

$$\theta(t) = \emptyset_m(t) - \emptyset_s(t) \qquad (10)$$

S is the photon density and N is the carrier density and $\emptyset_s(t)$ and $\emptyset_m(t)$ is the phase of slave and master laser. In the analysis $\emptyset_m(t)$ is assumed to be 0. Γ is the confinement factor; τ_n and τ_p are the radiative carrier lifetime and photon lifetime respectively. g_n is the differential gain that takes into account the effect of group velocity, N_o is the transparency carrier density, ϵ is the gain saturation factor and α is the linewidth enhancement factor. I is the dc bias current, V_a is the volume of the active region e is the electron charge and N_{th} is the threshold carrier density. Δf is the frequency detuning between the master and slave laser. The optical injection into the target laser is controlled by the injection rate, k_{inj}.

$$k_{inj} = (1 - R)\sqrt{\frac{R_{inj}}{R}\frac{c}{2nL}} \qquad (11)$$

Where, R_{inj} is the injection parameter, R is reflectivity of the laser, c is the speed of light in free space, n is the refractive index and L is the cavity length of the slave laser.

Figure 4 shows an example of the optical injection response of a semiconductor nanolaser with a Purcell factor of F = 5. The dark blue region is that of injection locking whilst in the red region period-doubled dynamics appears. Preliminary results suggest that the enhanced spontaneous emission in nanolasers serves to stabilise the dynamics. Further results indicating how the response of the laser to optical injection will be discussed.

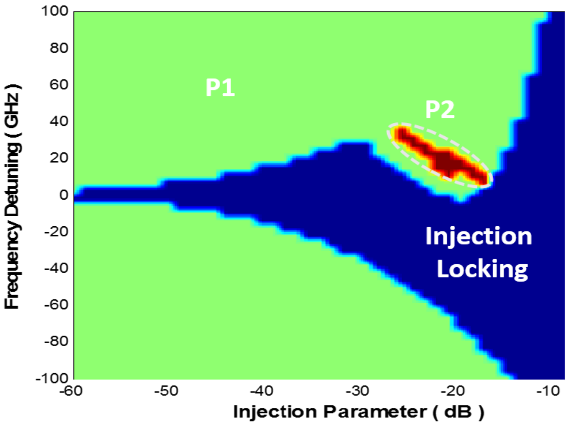

Figure 4 Optical injection response of nanolaser exhibiting injection locking (dark blue region) periodic dynamics (P1) and period doubled (P2 - red region) normalised photon density vs feedback coupling fraction at $I_{dc} = 2I_{th}$, F = 5and $\beta = 0.05$.

Calculations of the enhancement of direct current modulation response of nanolasers subject to optical injection will also be reported.

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