2014 International Symposium on Nonlinear Theory and its Applications
NOLTA2014, Luzern, Switzerland, September 14-18, 2014

Description Ability of Sound with Three-Rules Set of One Dimensional Cellular Automata with Two States and Three Neighbors

Rika Terai, Jousuke Kuroiwa, Tomohiro Odaka, Izumi Suwa, Haruhiko Shirai

Graduate School of Engineering, University of Fukui
3-9-1 Bunkyo, Fukui, 910-8507, Japan
Email: terai@ci-lab.jp, jou@ci-lab.jp

Abstract—In this paper, we investigate a description ability of three-rules sets in one dimensional cellular automata with two states and three neighbors (referred as the 1-2-3 CA hereafter), in describing digital sound with a rule set of 1-2-3 CA. We have shown that the rule set of (#90, #180) gives the highest description ability of all the two-rules sets. As the number of elements of the rule set increases, the possible states of 1-2-3 CA derived from the rule set increase, suggesting that the description ability could be improved. We employ the rule sets of (#90, #165, #180) and (#75, #90, #180), where #165 and #75 are equivalent with #90 and #180, respectively, under the Left-Right complementation. From computer experiments, the three-rules set of (#90, #165, #180) gives a higher description ability than the rule set of (#90, #180) and (#75, #90, #180). Therefore, three-rules sets is more appropriate in describing digital sound without reproducing errors than two-rules sets.

1. Introduction

Cellular automaton is one of discrete mathematical models which can generate complex phenomena from simple rules. Therefore, cellular automata are applied to investigate universal pattern generator, error correcting code, unsteady flow analysis, traffic stream modeling, and so on [1]. For instance, Chua and his colleagues investigate 1-2-3 CA related to the universal computations [2].

In this paper, we focus on the description of digital sound with rules of 1-2-3 CA. The method is presented by Nara and his colleagues, which enable compressive coding and errorless reproduction of digital sound with a two-rules set of 256 ones in 1-2-3 CA [3, 4]. Moreover, we have shown that the two-rules set of (#90, #180) can give the best description ability in data amount of all the possible pairs of 256 rules. However, it is not unknown that the two-rules set can always reproduce the best coding. In actual, there exists time transitions of bit-patterns in digital sounds, which are difficult to describe with the two-rules set of (#90, #180). As the number of elements of the rule set increases, the possible states of 1-2-3 CA derived from the rule set increasing, suggesting that the description ability could be improved.

Therefore, the purpose of this paper is to investigate a description ability of digital sounds for three-rules sets comparing with the two-rules set of (#90, #180). In this paper, we employ three-rules sets of (#90, #165, #180) and (#75, #90, #180), where #165 and #75 are equivalent with #90 and #180, respectively, under the Left-Right complementation [2].

2. Description Method with Rules of 1-2-3 CA

2.1. 1-2-3 CA

Let us explain 1-2-3 CA briefly. In 1-2-3 CA, N cells are arranged on a one dimensional chain. Each cell takes 0 or 1. The state of the ith cell at time step t is represented by a_i^t, where $i = 1, 2, \cdots, N$.. The state at the next time step is determined by the states of itself and the neighboring two cells. Thus, the updating rule of the states is written as follows,

$$a_i^{t+1} = f(a_{i-1}^t, a_i^t, a_{i+1}^t), \tag{1}$$

where a function $f(\dot{})$ is referred as a transition function. In order to specify updating rule, we introduce,

$$\begin{aligned} &f(0,0,0) = f_0, f(0,0,1) = f_1, f(0,1,0) = f_2, \\ &f(0,1,1) = f_3, f(1,0,0) = f_4, f(1,0,1) = f_5, \\ &f(1,1,0) = f_6, f(1,1,1) = f_7 \end{aligned} \tag{2}$$

where $f_i = 0$ or 1 $(i = 0, \cdots, 7)$. By choosing each of f_i to be 0 or 1, we can determine a certain specified rule.

Moreover, defining the rule number as

$$\begin{aligned} \#r &= 2^0 f_0 + 2^1 f_1 + 2^2 f_2 + 2^3 f_3 \\ &\quad + 2^4 f_4 + 2^5 f_5 + 2^6 f_6 + 2^7 f_7, \end{aligned} \tag{3}$$

one can name each rule "the specified rule number" of $\#r$ from 0 to 255 (totally $2^8 = 256$ rules). If we assign the two state of each cell 1(0) to a black(white) small dot, respectively, the time development of a certain initial state in 1-2-3 CA gives pattern sequences consisting of both black and white dots.

2.2. Basic Idea of Description Method

In general, digital sounds can be expressed as time development of bit-pattern sequences of 16 bits. On the other hand, a time development of 1-2-3 CA gives bit-pattern sequences of N cells. Thus, our idea about describing digital sound by 1-2-3 CA are stated as follows:

- $\boldsymbol{a}^t = \{a_i^t | i = 1, \ldots, 16\}$ at time step t of 1-2-3 CA can be regarded as an amplitude of quantized sound signals in binary coding.

- Time development of digital sound data ($\boldsymbol{a}^t \Rightarrow \boldsymbol{a}^{t+1}$) can be generated by applying rule to $\boldsymbol{a}^t$, that is, $\mathcal{R} \circ \boldsymbol{a}^t = \boldsymbol{a}^{t+1}$, where $\mathcal{R}$ is a certain sequence of rules appropriately chosen from 256 ones in 1-2-3 CA.

Therefore, the problem is how to determine the rule sequences of the 1-2-3 CA which can generate the sequences of 16 bit patterns corresponding to the digital sound.

2.3. Description Method

Let us present a strategy for finding rule sequences of the 1-2-3 CA [4].

1. First, we choose a boundary condition from all the four kinds of fixed boundary conditions, L0-R0, L1-R0, L0-R1 and L1-R1, where L1 means that the boundary cell at the left of the 1st cell is fixed at unity and R0 means that the boundary cell at the right of the 16th cell is fixed at zero.

2. Define the number of elements in the rule set, n, and the rule set, $\mathcal{R}_1 = \{\#1, \#2, \cdots, \#n\}$. Let us introduce a notation $\mathcal{R}_k$, where $\mathcal{R}_k = \{c_1 c_2 \cdots c_i \cdots c_k | c_i \in \mathcal{R}_1\}$, containing all the possible combinations of n rules of $\mathcal{R}_1$. Note that the length of rule sequences in $\mathcal{R}_k$ corresponds to k.

3. An initial state in the 1-2-3 CA of $\boldsymbol{a}^0$ is taken from the initial data of the original digital sound.

4. At each time step, we search for the rule sequence with the shortest length by applying the elements of $\mathcal{R}_k$ $(k = 1, 2, \cdots)$. Starting from $k = 1$, we increase k until getting the target data, meaning that $\boldsymbol{a}^{t+1} = \mathcal{R} \circ \boldsymbol{a}^t$, $\mathcal{R} \in \mathcal{R}_k$. If the correct pattern is not obtained within a certain maximum sequence length $N_{\max}$, then we give up description and employ the original pattern.

5. Repeat the procedure 4 until the end of the sound data.

3. Computer Experiment

3.1. Purpose and Method

In this paper, we investigate a possibility of three-rules set in describing digital sounds, that is, whether could a three-rules set improve description ability in data amount or not? In this paper, we employ (#90, #165, #180) and (#75,#90, #180) as a three-rules set. The rules of #165 and #75 are equivalent with #90 and #180, respectively, under the Left-Right complementation [2]. We expect that the rule of #165 or #75 could generate a time development of the bit pattern sequences which the rules of #90 and #180 could not.

We apply two kinds of digital sounds:

- Spoken word data supplied by ATR(Advanced Telecommunication Research Institute International). In this paper, we employ four words, "sports","knife","news" and "pocket", pronounced by two men and two women.

- 150 music data which are taken from four classic and one Japanese pops music CD. All the intervals of the data are 1 second.

The music data are digitized under the sampling frequency 44.1 kHz and the amplitude at each time step quantized into 16 bits. The data format of music is that of commercial music CD. On the other hand, the pronounced words 22.05 kHz , 16 bits, respectively.

The description ability is evaluated by the quantity of d,

$$d = L \log_2 n, \tag{4}$$

where L denotes an averaged length of the rule sequences which reconstruct the original sound without reproducing errors, and n represents the number of elements in the rule set. In our definition of the description ability, smaller d means less data amount in our coding. We compare the description ability of the three-rule sets with the two-rules set of (#90, #180). In the description procedures, at each time step we search for the rule sequences which perfectly reproduce the original sound. Therefore, let $N_{\max}$ be large, for instance, in this paper, $N_{\max} = 20$. We employ the L1-R0 boundary condition.

3.2. Description Ability

In this paper, we give a part of our results. In table 1, we give an averaged length of rule sequences, which describe target sound without reproducing errors. All the word data in the table are pronounced by a woman. J101, $\cdots$, J104 correspond to Japanese pop music data, and C101, $\cdots$, C104, classic music data. Each averaged length of the three-rules sets is obviously smaller than the two-rules set. The result

Table 1: Averaged length of rule sequences. “J” means Japanese pop music and “C” classic.

	(90,180)	(90,180,165)	(90,180,75)
sports	16.00	10.05	10.30
knife	16.04	10.08	10.29
news	16.10	10.07	10.30
pocket	15.74	10.02	10.30
J101	16.31	10.10	10.29
J102	16.30	10.12	10.31
J103	16.29	10.11	10.28
J104	16.26	10.11	10.30
C101	16.16	10.05	10.27
C102	16.21	10.08	10.29
C103	16.19	10.07	10.27
C104	16.19	10.05	10.30

Table 2: Description ability of d.

	(90,180)	(90,180,165)	(90,180,75)
sports	16.00	15.93	16.33
knife	16.04	15.98	16.32
news	16.10	15.96	16.33
pocket	15.74	15.88	16.19
J101	16.31	16.01	16.38
J102	16.30	16.05	16.34
J103	16.29	16.02	16.30
J104	16.26	16.01	16.32
C101	16.16	15.93	16.28
C102	16.21	15.97	16.30
C103	16.19	15.97	16.28
C104	16.19	15.94	16.32

originates in the fact that the possible states of 1-2-3 CA generated from the rule set increase as the number of elements of the rule set increases.

The description ability of d is given in Table 2. Each description ability of (#90, #165, #180) is slightly smaller than (#90, #180) except for pocket. On the other hand, each description ability of (#75,#90, #180) is corrupted. The description ability of d does not always decreases even if candidate states derived by the rule set merely increases. The fact of the decreasing description ability implies that the rule of #165 plays important roles in describing digital sounds. Thus, the three-rules set of (#90, #165, #180) can improve the description ability in data amount.

Table 3: Averaged ratio that the rule #165 is applied in the rule sequences at the target time steps.

rule number	shorter	almost same
#90	34%	44%
#180	26%	34%
#165	40%	22%

4. Discussions

Now, we consider the reason why the the three-rules set of (#90, #165, #180) can improve the description ability. At first, we evaluate the ratio that the rule #165 emerges in the rule sequences at focusing time steps, where the length of rule sequences with (#90, #165, #180) becomes quite shorter than (#90, #180), or the length is almost same. We employ 100 targets for each focusing time step. The result is given in table 3. For the time steps where the length becomes shorter, the averaged ratio of #165 takes almost 40%

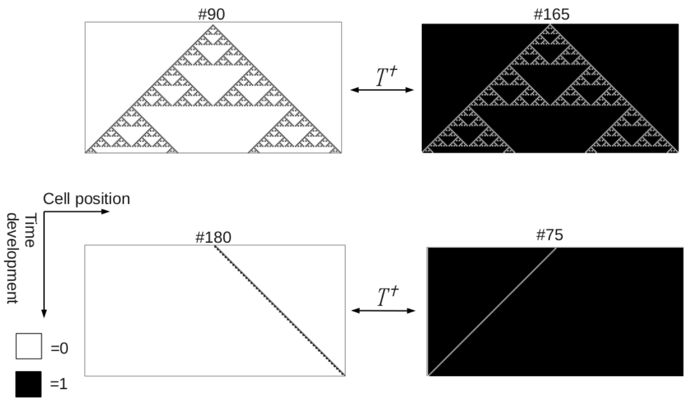

Figure 1: Time development of bit-pattern sequences generated by the four rules of #75, #90, #165 and #180.

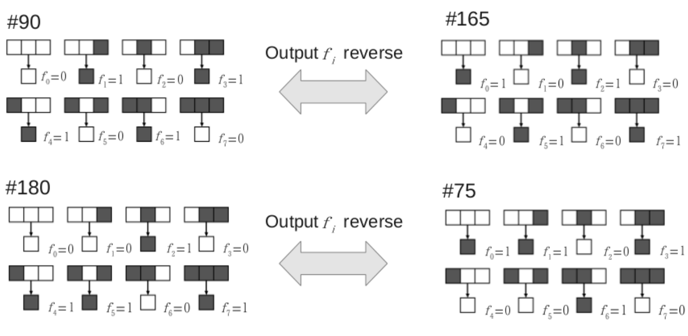

Figure 2: Definition of four rules of #75, #90, #165 and #180.

and is the highest, suggesting that the rule #165 is most frequently applied in the rule sequences. On the other hand, for the time steps where the length is almost same, the averaged ratio of #165 takes almost 22% and is the smallest. Therefore, for specified time steps, the existence of the rule #165 is practical in improving description ability.

At next, we investigate a qualitative feature of the transition function of the rules and generated bit-pattern sequences. The time development of bit-pattern sequences generated by the rule is given in Fig. 1. In the bit-pattern sequences of #165 black and white are reversed to #90. On the other hand, bit-pattern sequences of #75 is obtained by a complementation and reflection of that of #180 about the center line. Thus, #165 and #75 are equivalent with #90 and #180, respectively, under the Left-Right complementation [2]. According to equation (3), the transition functions of the four rules of #75, #90, #165 and #180 are given in Fig. 2. The figure shows that the f_i of #165 (#75) corresponds to complement of #90 (#180), respectively. Thus, the rules of #165 and #75 would complement in reproducing the time development of the bit-pattern sequence for the time steps where the rules of #90 and #180 miss to describe effectively. In addition, the rule of #165 belongs to class 3, meaning that #165 can generate time development of complex bit-pattern sequences as shown in Fig. 1. In the rule of #165, the complementation for #90 and the complexity of generated bit-patterns could enable to improve the description ability in coding digital sounds with the rule set of 1-2-3 CA.

The description method is based on the idea that complex dynamics including chaos, even in systems with large degrees of freedom, could be generated by certain simple rule. The rule sequences which reproduce an original sound corresponds to "rule dynamics" proposed by Aizawa and Nagai [5]. According to the idea of them, the rule dynamics could reflect various features more accessibly than original data. This is our future problem.

5. Conclusions

In this paper, we study a description method of digital sounds with three-rules sets of 1-2-3 CA. We evaluate description ability of the sets, (#90, #165, #180) and (#75,#90, #180), comparing the two-rules set of (#90, #180). Results are as follows:

- Each description ability of (#90, #165, #180) is slightly smaller than (#90, #180) except for one case.
- The three-rules set of (#90, #165, #180) can improve the description ability in data amount.

In near future, we present evaluation results of the description ability for all the possible three-rules sets of 256 rules in 1-2-3 CA.

References

[1] P. P. Chaudhuri, D. R. Chowdhury, S. Nandi, S. Chattopadhyay, *Additive Cellular Automata Theory and Applications*, IEEE Computer Society Press, 1997.

[2] L. Chua, V. Sbtnev, S. Yoon, "A Nonlinear Dynamics Perspecive of Wolfram's New Kind of Science. Part III : Predicting the Unpredictable," *IJBC*, vol.14, pp.3689-3820, 2004.

[3] S. Nara, N. Abe, M. Wada, J. Kuroiwa, "A Novel Method of Sound Data Description by Means of Cellular Automata and Its Application to Data Compression," *IJBC*, vol.9, pp.1211-1217, 1999.

[4] J. Kuroiwa, S. Nara, "Errorless Description with Two Rules of Cellular Automata for Digital Sound Data," *IJBC*, vol.23, pp.135048-1-8, 2013

[5] Y. Aizawa and Y. Nagai, "Dynamics on Pattern and rule – Rule Dynamics, " *Bussei Kenkyu*, vol.48, pp.316–320, 1987.