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An Application of Dynamical Robustness Analysis to Power Grids

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Abstract—Power grids have risks that local accidents or sudden fluctuations in the power flow can cause large-scale blackouts due to a cascade of failures. For preventing such phenomena which are socially and economically undesired, it is significant to mathematically understand how power grids are tolerant to errors and attacks. Recently, the dynamical robustness analysis has been developed to argue robustness of networked systems by taking into consideration both network structure and system dynamics. In this study, we investigate the robustness of power grids in terms of the dynamical robustness of complex networks.

1. Introduction

Reduction of CO_2 emissions and depletion of fossil fules are currently major issues to be handled in the world. To deal with these issues, many countries are trying to accelerate introduction of renewable energy sources (RES) for efficient power generation. Power generation by RES, such as wind turbines and photovoltaic (PV) units, is significantly different from that by conventional power plant in many aspects. In particular, the power generation by such new generators are highly time-varying depending on the climate. Also the conventional frequency regulation for synchronous generators cannot be directly applied to these induction generators. If the share of inverter-connected generators increases, it becomes difficult to achieve stable power operation under the traditional system control. Therefore, it is argued how to maintain frequency stability against disturbance in the case of high share of inverter-connected RES.

The conventional synchronous generators based on rotating machinery have large inertias. The power system dynamics in coupled synchronous generators is described with a swing equation. When a disturbance occurs due to a change in the generated power or consumed power, the frequency deviates from the reference value. If the deviation is within an acceptable range, then the system state recovers the steady state and the frequency goes to the reference value again. The inertia plays a role of time constant for the swing equation. Therefore, the more the inertia is, the more slowly the frequency recovers the reference value. The inertia is the first process of frequency control, taking place within several seconds after the disturbance. Since inverter-based power generation is asynchronous, with small or no inertia, the conventional frequency control is not valid in principle. However, recently a concept to equip the oscillatory dynamics with inverters is presented. In that case, the inverter-connected power generators can also be treated as synchronous generators with low inertia [1]. The impact of low rotational inertia on power system stability and operation has been studied. The concept of virtual synchronous machine and virtual inertia has been proposed for controlling the power system in the conventional way, i.e. through self-organized synchronization property [2].

In this study, we consider the problem of low inertia caused by large deployment of RES into the power system from the standpoint of nonlinear dynamics and network science. The tolerance of power grid against local failure has been argued in various frameworks with complex networks [3]. Also the concept of node-wise robustness in power grids has been proposed [4]. Since power system dynamics is involved with both network structure and system dynamics, the framework of dynamical robustness is suitable [5, 6]. We assume that the whole power grid consists of many power systems interconnected each other We denote the fraction of inverter-connected power generators with low inertia by p and investigate how frequency stability is kept as p is increased in netwoks of swing equations. We further examine how to recover the

In Section 2, we introduce the model for inertial response and describe the swing equation. In Section 3, we describe the model for multi-area power systems with heterogeneous inertia. In Section 4, simulation results are shown. In Section 5, this work is summarized.

2. Model

2.1. Swing equation

Here we describe the modeling of inertial response [1]. After a frequency deviation occurs in a power system, kinetic energy stored in the rotating machinery is released.

The kinetic energy is given by

$$E_{kin} = \frac{1}{2} J\omega_m^2, \quad (1)$$

where ω_m represents the angular velocity of the generator machine (i.e. $\omega_m = d\theta/dt$ where φ denotes the phase of the voltage) and J denotes the moment of inertia of the synchronous machine.

The inertia constant H is defined by

$$H = \frac{E_{kin}}{S_B} = \frac{J\omega^2}{2S_B}, \quad (2)$$

where S_B is the rated power of the generator and H denotes the time duration in which the rated power supply is possible due to the stored kinetic energy. Typically the values of H are within the range of 2-10s.

The inertial response is represented as the variation in rotational speed following a power imbalance.

$$E_{\dot{kin}} = Jw_m^2 \dot{\omega_m} = \frac{2HS_B}{\omega_m} \dot{\omega_m} = P_m - P_e, \quad (3)$$

where P_m is the mechanical power supplied by the generator, P_e is the electric power demand, and the last term represents the frequency-dependent load damping with damping coefficient k.

Using an approximation $\omega_m \approx \omega_0$ where ω_0 is the reference angular velocity, the above equation can be rewritten as follows:

$$\dot{\omega_m} = \frac{\omega_0}{2HS_B}(P_m - P_e). \quad (4)$$

This equation implies that a larger value of inertia constant H leads to slower frequency dynamics. In the power system with a high share of inverter-connected generators without inertia, the frequency stability relies on the inertia of the remaining conventional synchronous machines.

2.2. Mutli-area power systems

We consider a local power grid, consisting of G generators and L loads, connected via M transmission lines. By summing up Eq. (4) for all the nodes and defining the center of inertia grid frequency ω, the so-called aggregated swing equation model can be obtained as follows [1]:

$$\dot{\omega} = -\frac{\omega_0}{2HS_B}(-k\omega + P_m - P_{load} - P_{loss}) \quad (5)$$

where

$$\omega = \frac{\sum_{i=1}^{G} H_i S_{B,i} \omega_i}{\sum_{i=1}^{G} H_i S_{B,i}}$$

$$S_B = \sum_{i=1}^{G} S_{B,i}, H = \frac{\sum_{i=1}^{G} H_i S_{B,i}}{S_B}$$

$$P_m = \sum_{i=1}^{G} P_{m,i}, P_{load} = \sum_{i=1}^{L} P_{load,i}, P_{loss} = \sum_{i=1}^{M} P_{loss,l}$$

The aggregated model is valid for highly meshed grid. For simplicity, we assume $P_{loss} = 0$ hereafter.

Now, for an N-area power system, the frequency dynamics of ith area ($i = 1, \ldots, N$) can be described as follows:

$$\dot{\theta_i} = \omega_i, \quad (6)$$

$$\dot{\omega_i} = \frac{1}{M_i}\left(\Delta P_i - k_i\omega_i - \sum_j V_i V_j B_{ij} \sin(\varphi_i - \varphi_j)\right), \quad (7)$$

where the inertia constant is given by $M_i \equiv 2H_i S_{B,i}/\omega_0$, the power imbalance term is given as $\Delta P_i \equiv P_m - P_{load}$, V_i represents the voltage level at node i, and B_{ij} denotes the susceptance between node i and j. Here the voltage level is assumed to be normalized at $V_i = 1$ for all i.

3. Results

3.1. Single area model

First, in order to understand the effect of inertia in the power system, we observe frequency responses following a disturbance in two-area system. Assuming that the 2nd area corresponds to an infinite bus and set $\theta_2 = 0$ and $\omega_2 = 0$. The parameters are set at $S_B = 115[GW]$, $H = 6[s]$, $\omega_0 = 2\pi f_0$ with $f_0 = 50[Hz]$ [Ulbig].

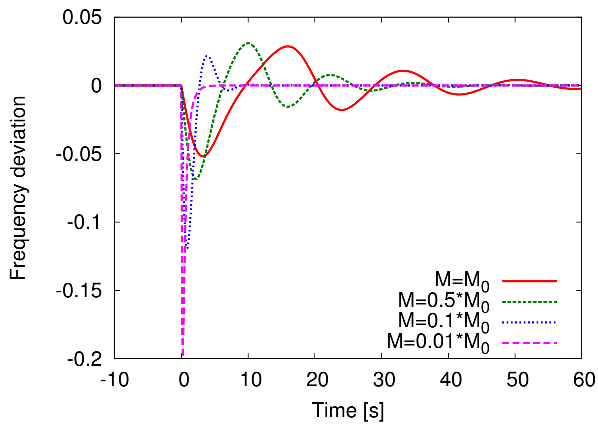

Figure 1: An example of the effect of inertia parameter on the frequency response following a disturbance starting at $t = 0$.

Figure 1 shows the frequency response of the 1st area power system for different values of the inertia constant. We assume a disturbance with $\Delta P = -0.8$ starting at $t = 0$. The frequency is given by $\omega_1/2\pi$. The frequency deviates from the reference value due to the disturbance, but the reference value is recovered by the coupling with the infinite bus within several tens of seconds. We can see that the response time decreases as M_1 is decreased. This is because M_1 corresponds to the time constant of the frequency dynamics. When the inertia decreases, the frequency response tend to be steep and the frequency nadir becomes

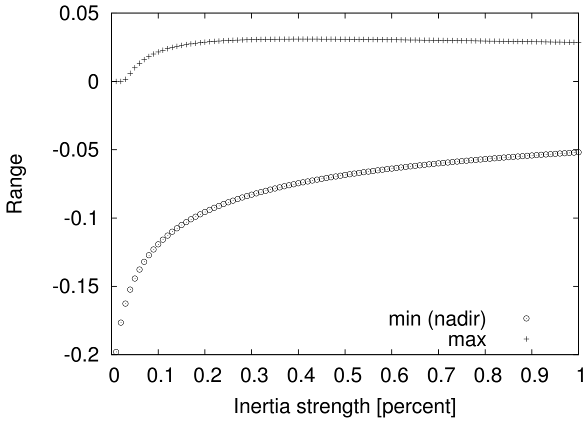

Figure 2: An example of the effect of inertia parameter on the frequency response following a disturbance starting at $t = 0$.

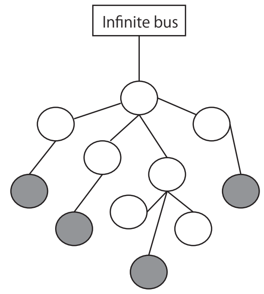

Figure 3: A random grid topology with $p = 0.4$.

smaller. Since large fluctuation of the frequency can cause a mechanical problem for generators, the fluctuation should be kept in a very small range. Therefore, the low inertia could lead to a serious problem about the frequency stability.

The effect of inertia level on the range of frequency variation is shown in Figure 2. The maximum and minimum frequency deviation is plotted against the different levels (percentages) of the inertia M_1. The results shows that the frequency nadir monotonously decreases as the inertia level decreases. The nonlinearity of the curve suggests that an extremely low level of inertia is undesired for maintaining frequency stability.

3.2. A multi-area model

Next, we study the nonlinear dynamics of a multi-area power system model and how an introduction of low-inertia systems influences the frequency response of the whole power grid. Now we focus on a random grid topology as illustrated in Fig. 3. This network consists of one hub node which is only connected to the infinite bus and multiple branch nodes. Some of the branch nodes are assumed to have low inertia as indicated by gray color. Inspired by the dynamical robustness analysis of complex networks [5, 6], we denote the fraction of low-inertia nodes in the branch nodes by p with $0 \leq p \leq 1$.

Figure 4 shows the frequency response following a disturbance to different node: (Case 1) the hub node with normal inertia; (Case 2) a branch node with normal inertia; (Case 3) a branch node with low inertia.

In Case 1, the frequency in the hub node receiving perturbation is largely fluctuating. Although the other branching nodes are influenced by the perturbation to the hub node, their fluctuation is not so large. The undesierd effect of low inertia is not observed in this case.

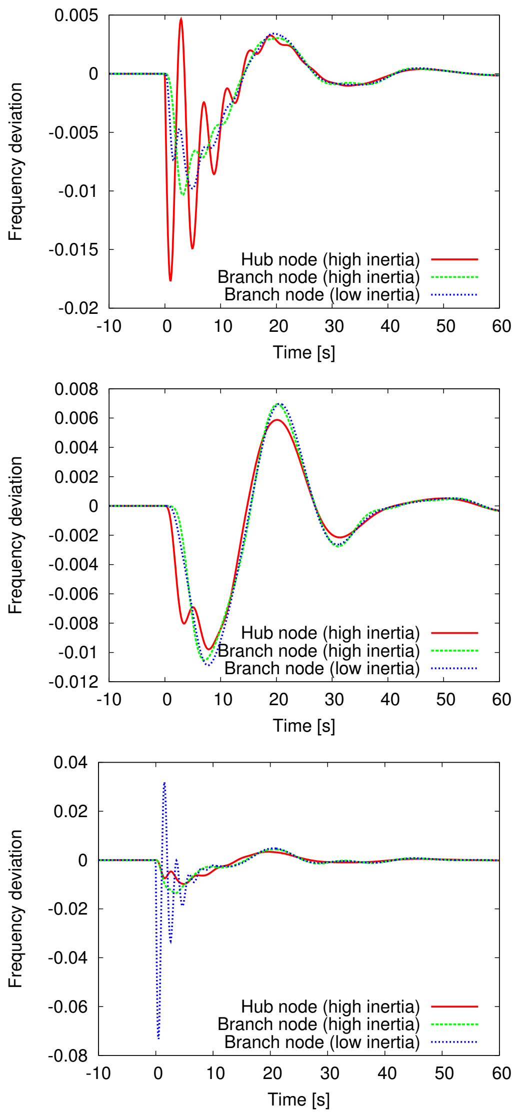

Figure 4: Frequency response in the grid topology in Fig. 3 with $p = 0.4$. (Upper) Disturbance to the hub node with normal inertia. (Middle) Disturbance to a branch node with normal inertia. (Bottom) Disturbance to a branch node with low inertia.

In Case 2, the branch node with normal inertia experiences the disturbance, but more fluctuation is observed for the other two types of nodes. The low-inertia branch node has the minimum frequency nadir. In this case, the self-organized synchronization among the branching nodes is effective.

In Case 3, since the branch node receiving the disturbance has low inertia, the deviation of the fluctuation is much larger than the other cases. The result is similar to that in the experiments of one-area model in Sec. 3.1. It seems that the neighboring nodes are not able to mitigate the rapid and dramatical decrease in the frequency.

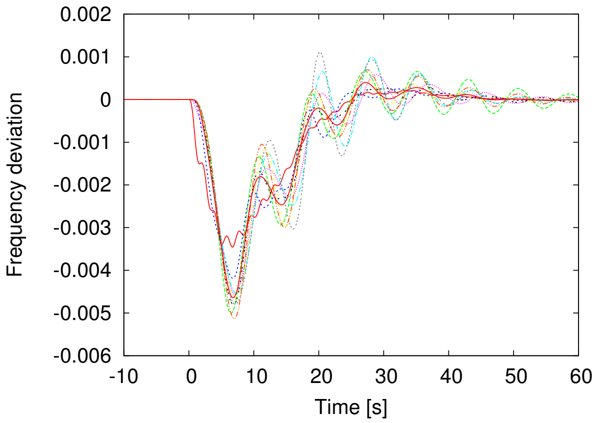

Figure 5: Frequency deviation in $N = 50$ random grid where the level of inertial in each area is randomly determined.

Next, we examine the frequency response in $N = 50$ power grid where the inertia is heterogeneous. Namely, the inertial level in each area is given randomly. The average degree is set at around 8%. In this case, the frequency deviation seems to be small as shown in Fig. 5. Our current issue is to explain in systematic numerical experiments how frequency responses depend on the site in which a disturbance occurs, the distribution of the local power system with low inertia, and the grid topology. For this purpose, we believe that the framework and the theory of dynamical robustness is available.

4. Summary

We have studied a frequency response in power system models described by swing equations. The aggregated swing equation has been introduced to consider multi-area power system. First, we have examined the effect of inertia on the frequency response following a disturbance in the single area power system model. Since the inertia corresponds to the time constant of the frequency dynamics, a lower level of inertia causes a large deviation of the frequency. This is undesired for maintaining frequency stability. We have found that the fluctuation range nonlinearly changes with the inertia level. Next, we have investigated the multi-area power system model. The results have suggested that the frequency response in each local area system differs depending on the site having the disturbance, the distribution of the low-inertia node, and the network topology. Further investigation is required to argue how an increasing share of inverter-connected generators with heterogeneous, low inertia influences the grid stability.

Acknowledgments

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