

# Detecting early warning signals for blackouts in power grids from the viewpoint of nonlinear dynamics

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Abstract—More and more renewable energy will be introduced in power grids. It might make the power system unstable, possibly leading to large-scale blackouts. If we can detect early warning signals of the blackouts and predict whether they occur, it would be possible to take necessary measures to prevent the blackouts. Therefore, it is significantly important to detect such early warning signals in power grids. Since the mechanism of the power outages is too complicated to be clarified by modeling all dynamics of power grids, a model-free method is needed for practical detection of the early warning signals for the blackouts. In this paper, we try to detect the early warning signals using the idea of dynamical network marker, which can detect the qualitative change of nonlinear dynamics. In particular, we compare the above method with the method based on Koopman mode analysis and validate the detection abilities of the dynamical network marker.

# 1. Introduction

More and more renewable energy will be introduced in power grids. The output of renewable energy fluctuates a lot because of weather condition. Therefore, it might cause instability of the power system, possibly leading to largescale blackouts.

If we can detect early warning signals of the blackouts and predict whether they occur or not, it would be possible to take necessary measures to prevent the blackouts. Therefore, it is significantly important to detect such early warning signals in power grids.

Since the mechanism of the power outages is too complicated to be clarified by modeling all dynamics of power grids, a model-free method is required for practical detection of the early warning signals for the blackouts. Although several model-dependent methods for detecting early warning signals have been developed so far [1], the model-free methods have not been fully explored.

In the physics community, the studies of detecting the early warning signals have been actively developed recently [2]. Among others, the notion of critical slowing down is a breakthrough to understand the relation between bifurcation and detection of early warning signals. If the system state approaches the bifurcation point, it becomes hard to return to the fixed point when it slightly departs from the fixed point. In Ref. [3], this notion is extended to multi-dimensional time series data. In this study, we apply this method to power grids. To the best of our knowledge, the application of dynamical network marker to power grids has not been investigated up to now. Another model-free method is Koopman-mode analysis, which can clarify the stability of the dynamics in multi-dimensional time series data [4]. In this paper, we try to detect the early warning signals using the idea of dynamical network marker, which can detect the qualitative change of nonlinear dynamics [3]. In particular, we compare the above method with the method based on Koopman mode analysis [4] and validate the detection abilities of the dynamical network marker.

# 2. Model-free Methods for Detecting the Early Warning Signals

We briefly introduce the two model-free methods for detecting early warning signals.

# 2.1. Dynamical Network Marker

The detailed explanation of the method introduced in this subsection is given in Ref. [3]. The dynamical network marker can detect the early warning signals of abnormal state from multi-dimensional time series data. This method is a model-free method. From the viewpoint of detecting the early warning signals, the states of systems can be categorized into three states: normal, pre-abnormal, and abnormal states. In power grids, normal and abnormal states correspond to the ordinary operation and the blackout, respectively. In human body, they correspond to healthy and disease states, respectively. In many systems, it is difficult to distinguish between normal and pre-abnormal states. This method can distinguish between these two states by considering the property of dynamical systems.

In the pre-abnormal state, there exists a group of nodes whose average for Pearson's correlation coefficients (PCCs) drastically increases in their absolute values [3]. We call such a group a dominant group. In this state, the avarage PCCs of the nodes between the dominant group and any others drastically decrease in their absolute values. Also, the average for standard deviations (SDs) of nodes in the dominant group dramatically increases.

The dominant group corresponds to non-zero elements of the eigenvector corresponding to the maximum eigenvalue of Jacobian matrix at the fixed point.

If we know the mathematical model of the system, we derive the dominant group by analyzing the Jacobian matrix of the model at the fixed point. Otherwise, we have to estimate the dominant group in some ways. In order to detect the early warning signals, we calculate the index as follows:

Index 
$$\equiv \frac{\text{SD}_d \times \text{PCC}_d}{\text{PCC}_o}$$
, (1)

where  $SD_d$  is the average for SDs of the dominant group, PCC<sub>o</sub> is the average for PCCs between the dominant group and any others in their absolute values, and PCC<sub>d</sub> is the average PCCs of the dominant group in their absolute values.

#### 2.2. Koopman-mode Analysis

The detailed explanation of the method introduced in this subsection is given in Ref. [4, 5]. This method estimates the stability of the system by decomposing the nonlinear dynamics into some modes. The number of decomposed modes is infinity, so we often use approximation methods for practical application.

Here we introduce an approximation method. Let us consider the set of data given by

$$\{\mathbf{Y}_0, \mathbf{Y}_1, \dots, \mathbf{Y}_{T-1}\},\tag{2}$$

where  $\mathbf{Y}_t \in \mathbb{R}^m$  is the *m* dimensional observed data at time *t* for t = 0, 1, ..., T - 1. We decompose the dynamics in the time series by applying the Arnoldi-like algorithm as follows:

$$\mathbf{Y}_t = \sum_{j=1}^{T-1} \lambda_j^t \mathbf{V}_j, \tag{3}$$

$$\mathbf{Y}_{T-1} = \sum_{j=1}^{T-1} \lambda_j^{T-1} \mathbf{V}_j + \mathbf{r},$$
(4)

where  $\mathbf{r} \perp \operatorname{span}{\{\mathbf{Y}_0, \mathbf{Y}_1, \dots, \mathbf{Y}_{T-2}\}}$ . If  $\lambda > 1$ , it means that the system is unstable in short-term dynamics. The notion of stability in this context is different from conventional ones. The former is the short-term dynamics and the latter is the long-term asymptotic dynamics. We momentarily calculate the eigenvalues and regard the timing when the eigenvalues become larger than one as the precursor of unstability.

#### 3. Numerical Experiments

## 3.1. Toy Model of Gene Regularoty Network and 3node Power Grid

We apply these two methods to a 5-node gene regulatory network and a 3-node power grid in order to demonstrate the ability to detect early warning signals. The gene regulatory network was used as a benchmark model in the validation of dynamical network biomarker [3]. The 3-node power grid can represent the typical case of blackout.

The model equations of the gene regulatory network are written as follows:

$$\begin{aligned} \dot{x}_1(t) &= a_{10} + a_{13}f_n(x_3(t)) + a_{14}f_p(x_4(t)) - a_{11}x_1(t), \\ \dot{x}_2(t) &= a_{20} + a_{21}f_n(x_1(t)) + a_{23}f_p(x_3(t)) - a_{22}x_2(t), \\ \dot{x}_3(t) &= a_{30} + a_{34}f_p(x_4(t)) - a_{33}x_3(t), \\ \dot{x}_4(t) &= a_{40} + a_{45}f_p(x_5(t)) - a_{44}x_4(t), \\ \dot{x}_5(t) &= a_{50} + a_{51}f_n(x_1(t)) + a_{52}f_n(x_2(t)) + a_{54}f_n(x_4(t)) \\ &-a_{55}x_5(t), \end{aligned}$$

where

$$f_p(x) = \frac{x}{1+x}, f_n(x) = \frac{1}{1+x},$$
  

$$a_{10} = 90P - 1240, a_{11} = 30P, a_{13} = 240 - 120P,$$
  

$$a_{14} = 4480/3, a_{20} = 120P - 240, a_{21} = 240 - 120P,$$
  

$$a_{22} = 60, a_{23} = 240 - 120P, a_{30} = -1060,$$
  

$$a_{33} = 60, a_{34} = 4480/3, a_{40} = -600,$$
  

$$a_{44} = 100, a_{45} = 1350, a_{50} = 320/3,$$
  

$$a_{51} = 160, a_{52} = 40, a_{54} = 4480/3, a_{55} = 300.$$

When *P* is larger than 0, the system is stable and P = 0 corresponds to the bifurcation point. In the numerical experiment, we add the additional steady noise with standard deviation 0.05 in every time step. We employ the Euler-Maruyama method with time step 0.005 and iterate 200,000 steps. We set node 1 and 2 as the dominant group. This is derived from the Jacobian at the fixed point. In the simulation of the power grid, we use Kuramoto-like phase model [6–9]. The equation is as follows:

$$M_{i}\ddot{\phi}_{i} + D_{i}\dot{\phi}_{i} = P_{i} - \sum_{j=1}^{N} B_{ij}\sin(\phi_{i} - \phi_{j}), \qquad (5)$$

where *i*, *N*,  $\phi$ , *M<sub>i</sub>*, *D<sub>i</sub>*, *P<sub>i</sub>*, and *B<sub>ij</sub>* represent the index of nodes, the number of nodes, the phase of voltages, damping coefficient, inertia moment, effective power, and coupling strength, respectively. We regard node 1 as generator and nodes 2 and 3 as loads. We set the parameters as follows: N = 3,  $M_i = 1$ ,  $D_i = 1$ , and

$$B_{ij} = \begin{cases} 1, & i \neq j, \\ -2, & i = j. \end{cases}$$
(6)

We set  $P_i$  as follows:

$$P_i = \begin{cases} P, & i = 1, \\ -P/2, & i = 2, 3. \end{cases}$$
(7)

When P < 2, the system has a stable fixed point and P = 2 corresponds to the bifurcation point. In the numerical experiment, we add the additional steady noise to the parameter P in every time step. The standard deviation is set at 0.05. We employ the Euler-Maruyama method with time step 0.005 and iterate 2,000,000 steps. We set node 1, 2, and 3 as the dominant group. This is derived from the Jacobian at the fixed point.

If we can detect the early warning signals when P is near the bifurcation point, the method is effective for the detection. Figures 1 (a) and 2 (a) show the relation between the bifurcation parameter and the indices of the dynamical network marker. When the bifurcation parameter approaches 0, the composite index drastically increases. Therefore, the method can detect the early warning signals of the bifurcation.



Figure 1: Numerical results of the 5-node gene regulatory network. (a) The relation between the bifurcation parameter and the indices of the dynamical network marker. The horizontal axis represents the bifurcation parameter P. The left vertical axis represents the  $PCC_d$ ,  $PCC_o$  and the composite index. The right vertical axis represents the SD. The dash-dotted line, the dashed line, and the thick solid line represent the  $PCC_d$ ,  $PCC_o$ , and the composite index, respectively. When the bifurcation parameter approaches 0, the composite index drastically increases. (b) The relation between the bifurcation parameter and Koopman eigenvalues in their absolute values. The horizontal axis represents the bifurcation parameter P. The dots represent the Koopman eigenvalues in their absolute values. Unlike (a), we calculate the Koopman eigenvalues after exceeding the bifurcation point. We take a sample of 100 time points.

Figures 1 (b) and 2 (b) show the distribution of the Koopman eigenvalues in their absolute values for each parameter. Even if the parameter P approaches the bifurcation point 0, the absolute eigenvalues do not go beyond 1, so we could not detect the early warning signals. This is because the Koopman mode analysis, which captures the exponential growth of the system, is not suitable for this case where the system state remains near the fixed point when the parameter approaches the bifurcation point.



Figure 2: Numerical results of the 3-node power grid. (a) The relation between the bifurcation parameter and the indices of the dynamical network marker. When the bifurcation parameter approaches 2, the composite index drastically increases. (b) The relation between the bifurcation parameter and Koopman eigenvalues in their absolute values. Unlike (a), we calculate the Koopman eigenvalues after exceeding the bifurcation point. We take a sample of 1000 time points.

#### 3.2. Real Data of a Power Grid

We use real data of the System Disturbance in the European Grid [10] and compare the detection abilities of the early warning signals. The data was given from Dr. Yoshihiko Susuki of Kyoto University. The data are power exchange deviations flows in the UCTE grid. Figure 3 shows the time evolution of the indices of the dynamical network marker. The blackout occurs at t = 40. We set node 3, 4, and 6 as the dominant group in this figure. We also tried every pattern as the dominant group, but all the patterns did not show a drastic increase before the blackout. Therefore, the method cannot detect the early warning signals of the blackout.

Figure 4 shows the distribution of Koopman eigenvalues in their absolute values for each parameter. The eigenvalues go beyond 1 even long before the blackout. In order to validate the fact that this result indicates the early warning signals, we have to compare the ordinary operation data.



Figure 3: The time evolution of the indices of dynamical network marker. The horizontal axis represents the time *t*. The left vertical axis represents the PCC<sub>d</sub>, PCC<sub>o</sub> and the composite index. The right vertical axis represents the SD. The dash-dotted line, the dashed line, and the thick solid line represent the PCC<sub>d</sub>, PCC<sub>o</sub>, and the composite index, respectively. At time *t*, we take a sample from time t - 9 to *t*.

#### 4. Conclusion

In this paper, we have numerically demonstrated the detection of early warning signals using the idea of dynamical network marker and Koopman mode analysis using toy models and real data. In the toy models, only the dynamical network marker can detect the early warning signals. In the real data, only the Koopman mode can possibly detect the early warning signals, although further studies are necessary for evaluating the effectiveness of the method.

A possible reason why the dynamical network marker cannot detect the early warning signals in the real power grid example is that the mechanism of blackout is different from bifurcation, or the ordinary qualitative change in nonlinear dynamics.

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Figure 4: The time evolution of the Koopman eigenvalues in their absolute values. The horizontal axis represents the time t. The dots represent the Koopman eigenvalues in their absolute values. At time t, we take a sample from time 1 to t.

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