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Combinatorial optimization problem on neural network with dynamic synapses

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Abstract—Strength of synaptic connection changes quickly due to short-term plasticity mechanism depending on presynaptic neural activities. Such dynamic synapses have much influence for neural network dynamics. In an associative memory network, the depression-dominant dynamic synapses cause destabilization of the memory retrieved state and represent ongoing state transition among memory patterns. In conventional associative memory network model or Hopfield model, state of the network changes so that its energy function always decreases; this corresponds to the process of the memory recall. This minimization of the energy function with neural dynamics is applicable to solve combinatorial optimization problems, e.g. traveling salesman problem (TSP). The network structure can be configured so that the optimal solution corresponds to the lowest energy state. In general, there are multiple local energy minimum, and the state of the network tends to be trapped on a local minimum in the conventional network model. Stochastic and/or chaotic dynamics are applicable to avoid to be trapped on a local minimum. Here we use the transitive dynamics induced by dynamic synapses to search the space of solution of TSP. The network model composed of the stochastic neurons and depression-dominant dynamics synapses. We show that the transitive dynamics with dynamic synapses enhances performance of solving STP rather than a network with mere stochastic noise.

1. Introduction

Combinatorial optimization is a problem to find an optimal object from a finite set of objects and is intensively studied in the field of information science. One of the combinatorial optimization problem, the traveling salesman problem (TSP) is a typical nondeterministic polynomial (NP)-hard problem. Various methods are proposed for solving the TSP. One of the heuristic approach is based on neural network dynamics, which is proposed by Hopfield and Tank [1]. They applied gradient descent dynamics on a neural network with symmetric mutual connections [2] to the TSP. The decreasing property of the energy function is utilized for finding a local minimal of an objective function of the TSP. This decreasing process of the energy function corresponds to the convergence of the state of the network to an attractor in the dynamical system. This properties are closely related to the model of the associative memory network, which implements multiple memory patterns on the connection weights of the neural network. The process of the convergence to a memory pattern and decrease in the energy function correspond to the process of memory recall [2]. However, the Hopfield-Tank neural network has a notorious local minimal problem, namely, there are many local minimum, the state of the neural network is trapped on a local minimum, and thus, the state of the network cannot reaches the global minimum.

In order to overcome the difficulty of the local minimal problem, several methods are proposed. The simulated annealing is know to be efficient with its stochastic dynamics. The chaotic dynamics on the neural network also contribute to avoid to be trapped on a local minimum. The attractors of the chaotic dynamics usually has fractal structures, and thus, the chaotic search is efficient on the TSP [3, 4]. In the associative memory network, this chaotic dynamics causes state transitions among the stored memory patterns and shows sequential memory association [5].

In the above-mentioned conventional neural network models, the strength of the recurrent synaptic connections are assumed to be static. However, recent physiological studies revealed that the the strength of the synaptic connections changes largely and quickly with short-term plasticity mechanism; these synapses are called dynamic synapses[6]. Properties of the neural network with the dynamic synapses are intensively investigated [7, 8, 9]. In the associative memory network with the dynamic synapses, the network shows transitive dynamics among the stored memory patterns [9]. The synaptic connections are depressed depending on the state of the neurons, and the memory recalled state becomes unstable.

In the present study, we use the transitive dynamics induced by the dynamic synapses to improve the performance of neural network for solving the TSP. The proposed neural network model is composed of stochastic neurons and depression dominant dynamic synapses. We evaluate the influences of the noise on the neurons and the dynamic synapses.

2. Model

For a given set of cities and the distances between each pair of cities, the TSP required to find a shortest route that visits each city once and returns to the original city. The Hopfield-Tank network implements this problem to the recurrent neural network with $N \times N$ neurons for N city TSP. The possible route on the TSP is represented by the state of the neuron s_{ij}, where i and j respectively represents the index of the cities and the order of visiting, namely, $s_{ij} = 1$ represent to visit i th city at j th step of the route.

In the present study, we use the network model with N^2 stochastic binary neurons, and the neurons are connected via the dynamic synapses. The state of the (i, j)th binary neuron is denoted by the variable $s_{ij}(t)$ and takes an active state $[s_{ij}(t) = 1]$ or a resting state $[s_{ij}(t) = 0]$ according to the following equation

$$\mathrm{Prob}[s_{ij}(t+1) = 1] = \frac{1}{2}\left(1 + \tanh[\beta h_i(t)]\right), \quad (1)$$

where $h_{ij}(t)$ is the total input for the neuron (i, j). and $1/\beta = T$ represents the noise intensity.

We use two types of recurrent connections, namely connections with dynamic synapses and static synapses. The strength of the connection on the static synapses is fixed, and the input for the neuron (i, j) is described by

$$h_{ij}(t) = \sum_{j\neq i}^{N} W_{ijkl}(2s_{kl}(t) - 1) + \theta_{ij}, \quad (2)$$

whereas the input with dynamic synapses are given by

$$h_{ij}(t) = \sum_{j\neq i}^{N} J_{ij}[2s_{kl}(t)x_{kl}(t)u_{kl}(t)/U_{se} - 1] + \theta_{ij}. \quad (3)$$

In this network, weight values for both directions of each pair of neurons is same, namely $W_{ijkl} = W_{klij}$, and zero weights for the self-connection $W_{ijij} = 0$. θ_{ij} specifies a bias input. Changes in the strength of the synaptic connection is determined by the fraction of releasable neurotransmitters $x_i(t)$ and the utilization parameter $u_i(t)$ [6]. The state of the neuron and the dynamic synapses changes according to the following equations [9]:

$$x_i(t+1) = x_i(t) + \frac{1 - x_i(t)}{\tau_R} - s_i(t)x_i(t)u_i(t), \quad (4)$$

$$u_i(t+1) = u_i(t) + \frac{U_{se} - u_i(t)}{\tau_F} + U_{se}(1 - u_i(t))s_i(t). \quad (5)$$

U_{se} represents the steady state value of the variable $u_i(t)$. If the neuron is active, the x_i is decreased depending on the utilization parameter, whereas x_i recover its steady state $x_i = 1$ with time constant τ_R. The utilization parameter u_j increases with the activation of the neuron and recovers its steady state $u_j = U_{se}$ with time constant τ_F. The strength of synaptic transmission is given by the product of $x_j(t)$ and $u_j(t)$; the strength decreases (depression) or increases (facilitation) depending on the parameters τ_R, τ_F, and U_{se}.

If $T = 0$, the network with the static synapses is equivalent to the Hopfield-Tank network. In the Hopfield-Tank network, the energy function

$$E = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} W_{ijkl}s_{ij}s_{kl} + \sum_{i=1}^{N}\sum_{j=1}^{N}\theta_{ij}s_{ij}. \quad (6)$$

always decreases by updating the state of the network asynchronously. This property can be utilized to find minimal value of an objective function.

The energy function can be configured for solving the TSP [1]. The length of the route can be described by

$$E_1 = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} d_{ik}s_{ij}(s_{k,j+1 mod n} + s_{k,j-1 mod n}), \quad (7)$$

where d_{ik} is the distance between i th and j th cities.

The constraint for visiting each city once is described by

$$E_2 = \sum_{i=0}^{N-1}(\sum_{j=0}^{N-1} s_{ij} - 1)^2. \quad (8)$$

The constraint for visiting one city at once is described by

$$E_3 = \sum_{j=0}^{N-1}(\sum_{i=0}^{N-1} s_{ij} - 1)^2. \quad (9)$$

In order to obtain an appropriate solution, above constraint terms E_2 and E_3 should be zero. To find the optimal solution of the TSP, the objective function is defined by

$$E_{TSP} = AE_1 + BE_2 + CE_3 \quad (10)$$

with coefficients A, B, and C.

This objective function can be transformed to the form of Eq. (6) by setting

$$W_{ijkl} = -Ad_{ik}(\delta_{l,j+1} + \delta_{l,j-1}) - B\delta_{i,k}(1 - \delta_{j,l}) \quad (11)$$

$$-C\delta_{j,l}(1 - \delta_{i,k}), \quad (12)$$

$$\theta_{i,j} = -\frac{B + C}{2}. \quad (13)$$

In the Hopfield-Tank network, the objective function is monotonically decreased, but the state often trapped on a local minimal solution. In the present model, the stochastic dynamics and the transitive dynamics induced by dynamic synapses avoid to be trapped on the local minimal.

3. Results

In the present paper, the performance of the model is evaluated with $N = 10$ cities TSP with the city map shown in Fig.1. We set values of the coefficient as $A = B = C = 1$. For the network with dynamic synapses, we fixed $\tau_F = 2$

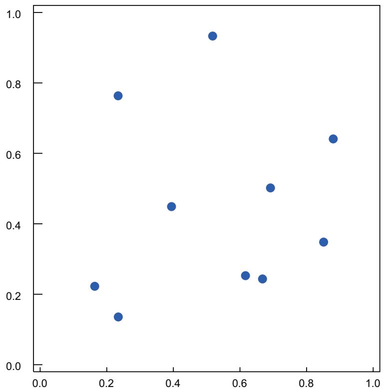

Figure 1: The city map used in this paper, which is appeared in [1]. The dots indicate the position of the cities. The distances between each pair of cities d_{ij} are based on this map.

and $U_{se} = 0.1$, which means that the synapse undergoes depression.

The typical time courses of the state of the network and the energy function are shown in Fig. 2.

In the case of the network with static synapses and without noise, which is equivalent to Hopfield-Tank model, the state of the network quickly converge to a (local) minimal solution, and the state is trapped on the (local) minimal solution. There exist many local minimal solution, and the state converge to one of them depending on the initial state of the neural network.

By adding the noise ($T > 0$), even once the state converge to a local minimum, the state escapes the local minimum and move to another solution. By continuing this process long enough, the state can reach the global minimum.

In the case of the network with dynamic synapses, we choose values of parameters that the synapse undergoes depression, and that the network make the state transition. Once the state converges to a local minimal, the synapses connected from active neurons are depressed, and this makes the state converged on the local minimal unstable and makes the state transitions.

Figure 3 compares the performance of these networks. The performance is quantified by the minimal distance when the network runs 200 step updates; the average value of the minimal distance is calculated by 20 times simulation with different initial state.

The performance largely depends on the intensity of the noise. In the network with static synapses, the performance is improved (the average distance is decreased) by increasing the intensity of the noise till $T \approx 1.5$, but the performance get worse if the noise intensity is further increased. Appropriate strength of the noise contribute to avoid to be trapped on local minimal solution and enhances the search ability. However, the too much noise disrupt the tendency to converge the local minimum.

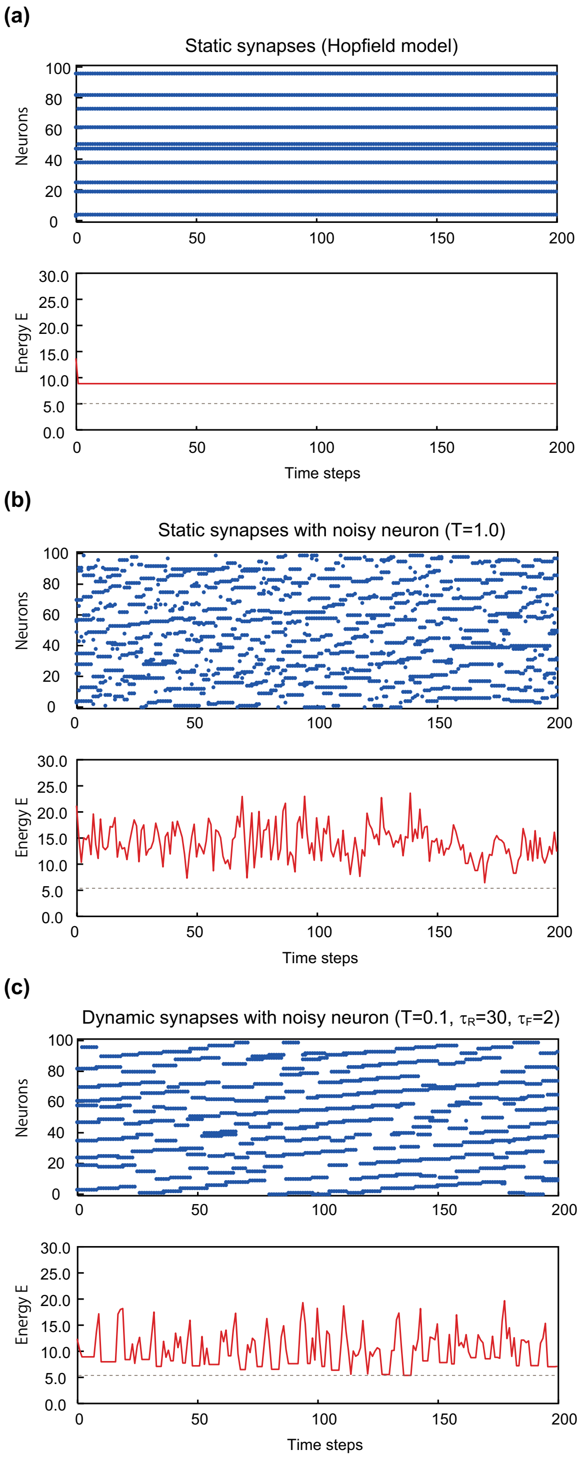

Figure 2: Typical time courses of the state of the network and the energy function. Blue dots indicate $s_{ij} = 1$. Red curves indicate the energy function. (a) The network with static synapses and without noise ($T = 0$). (b) The network with static synapses and with stochastic neurons ($T = 1$). (c) The network with dynamic synapses. The energy corresponding to the optimal solution of the TSP is indicated by the dashed line.

The performance of the network with dynamics synapses depends on both T and τ_R, which specifies the noise intensity and the influence of the synaptic depression. If the $\tau_R = 2$, the property of the dynamic synapses close to the static synapses, and its performance is similar to that of the network with static synapses. If the τ_R takes an appropriate value e.g., $\tau_R = 30$, the performance predominate the network with static synapses.

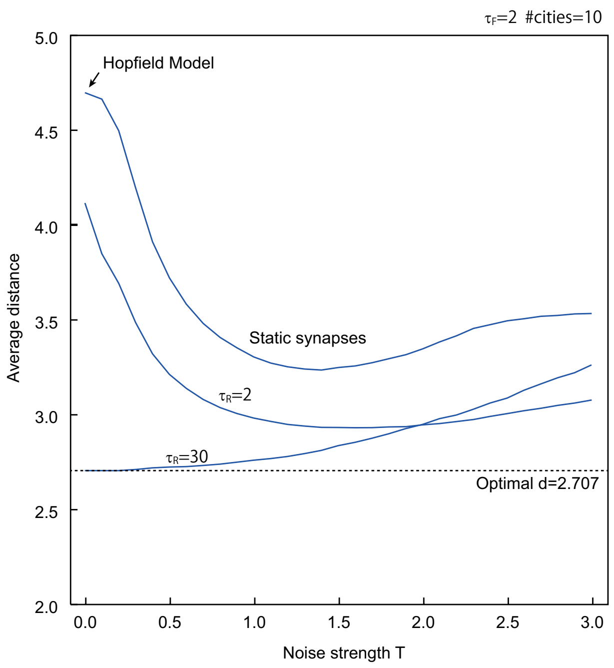

Figure 3: Performance of the model. Average distances of the route on the TSP as a function of the noise intensity is indicated by the solid curves. The optimal distance (shortest route on the TSP) is indicated by the dashed line.

4. Conclusion

The transitive dynamics induced by dynamic synapses improve the performance of searching the optimal solution on the TSP. The network with dynamic synapses (particularly depression synapses) predominate the network with static synapses with/without noise. The transitive dynamics induced by the dynamic synapses has the tendency to actively escape from the local minimum rather than the mere stochastic noise.

As a future study, we should compare the proposed model to other heuristic methods, e.g. the network with chaotic dynamics. Although we evaluated the performance of the network on the small size problem ($N = 10$) on the TSP in the present study, the performance should be evaluated on larger size problem and on other combinatorial optimization problem e.g. quadratic assignment problem, MAX-CUT problem.

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