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Adaptive Tracking Control of Uncertain Leader-follower Networks

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Abstract–This work studies the tracking control problem of leader-follower network with uncertain parameters and communication time-delay. An effective control scheme, only involving delay status and adaptive parameter update rules, has been proposed, and its effectiveness on tracking has been well illustrated via simulations. Our studies also show that, with the proposed control scheme, the nearest-neighbor network is the strongest whereas the scale-free network shows is the weakest, in terms of tracking capability.

1. Introduction

Being a classical problem in control theory, the tracking problem has recently received considerable attentions when a network of multiple agents is in concern. Its popularity is not only due to its theoretical importance, but also because of its broad modern applications, such as target tracking, unmanned aerial vehicles alignment and formation control, coordinated control of multiple robots, to name a few.

In some of the research works [1-4], while efforts are endeavored to subtle algorithmic design, the dynamics of the agents are usually ignored for simplicity. However, these dynamics, especially with rich nonlinearities, often affect the tracking performance and should not be neglected. Moreover, many designs are based on the assumption that all the information of the leader can be obtained instantly without any delay [5,6]. However, in practical cases, communication delay and system uncertainties are inevitable.

Hence, in this paper, the tracking control problem of nonlinear agents with communication delay and parameter uncertainty is considered. It is assumed that some parameters of the leader are unavailable for the followers, and a communication delay is existed. As demonstrated in the following sections, it is shown that some control scheme with parameter updating law can be designed so that all the agents can asymptotically track the leader.

2. Main Theorem of the Work

It is supposed that the dynamics of a single node of a network can be described by:

$$\dot{x} = f(x,t) + g(x,t)\alpha \equiv F(x,t) \tag{1}$$

where $f(x,t),\ g(x,t) \in R^n \times R \to R^n$ are some continuous functions and α is a constant vector.

Assumption 1:
$F(x,t)$ satisfies the Lipschitz condition, i.e., for arbitrary vectors x and y, one has

$$\|F(x,t) - F(y,t)\| \le L\|x - y\|,$$

where $L > 0$ is the Lipschitz constant.

Assumption 2:
$g(x,t)$ is bounded, i.e., there exists a constant $M > 0$ such that $\| g(x,t)\| \le M$.

Consider a leader-follower network expressed as

$$\begin{cases} \dot{x}_0 = f(x_0,t) + g(x_0,t)\,\alpha \\ \dot{x}_i = f(x_i,t) + g(x_i,t)\,\hat{\alpha} + \sum_{j=1}^{N} c_{ij} A x_j(t-\tau) \\ \qquad + b_i A(x_i(t-\tau) - x_0(t-\tau)) + u_i,\ i = 1,2,\cdots,N. \end{cases} \tag{2}$$

where the node-0 represents the leader and node-1 to node N are followers; α is assumed to be unknown to all the followers and $\hat{\alpha}$ is the estimation of uncertain parameters by the followers; u_i are the control to be designed. It should also be noticed that a communication delay of τ between leader and follower is incorporated as shown in (2).

The inner coupling matrix $A = (a_{ij})_{n\times n}$ is assumed to be positive definite, $B = diag(b_1, b_2, \cdots b_N)$ represents the connection weight between the leader and the followers, and $C = (c_{ij})_{N\times N}$ is the weighted matrix showing the topological structure of the followers' network subject to the diffusive coupling condition $c_{ii} = -\sum_{j=1, j\ne i}^{N} c_{ij}$.

Let $e_i = x_i - x_0$ and $\delta = \hat{\alpha} - \alpha$ denote the tracking errors and parameter estimation error, respectively, and $e_i(t-\tau) = x_i(t-\tau) - x_0(t-\tau)$, the error dynamical system can be expressed in a form of

$$\begin{aligned} \dot{e}_i &= [f(x_i,t) - f(x_0,t)] + [g(x_i,t) - g(x_0,t)]\alpha \\ &\quad + \delta\, g(x_i,t) + \sum_{j=1}^{N} c_{ij} A e_j(t-\tau) + b_i A e_i(t-\tau) + u_i \end{aligned} \tag{3}$$

Adaptive control law and parameter update rules are to be designed so as to achieve the tracking control of the leader-follower network, that is, these followers will eventually reach the status of the leader.

With the Assumptions 1 and 2, and some existing results in [7], the following theorem can be proved (Due to the page limitation, the proof is omitted here.)

Theorem 1: Consider an undirected network (2) comprising N agents $x_i, i = 1,2,\cdots,N$, with nonlinear dynamics, following an independent leader x_0. With the Assumptions 1 and 2, if there exists positive constants $\beta > 0$ and $k > 0$ such that

$$\beta > \tau\gamma$$

and

$$(L-k)I_N \otimes I_n + (C+B)\otimes A \le -\beta I_N \otimes I_n$$

where $\otimes$ is Kronecker product, $\eta = \|(C+B)\otimes A\|$ and $\gamma = (\eta + k + NM)(L + \eta + k + NM)$, then the following control and parameter update law:

$$\begin{cases} u_i = -k\,e_i(t-\tau), \quad i = 1,2,\cdots,N \\ \dot{\delta} = -\tau\gamma q \delta - \sum_{i=1}^{N} g(x_i,t)^T e_i(t-\tau) \end{cases} \tag{4}$$

guarantee the convergence of the tracking problem asymptotically, i.e. $x_i \to x_0$ when $t \to \infty$, where k is the control gain and $q > 1$ is a constant.

The following corollary can also be induced from Theorem 1.

Corollary 1: Suppose that Assumptions 1 and 2 hold, there exists a upper bound of time-delay

$$\tau^{**} = \varphi(k^*),$$

with

$$\varphi(k) = \frac{1}{\gamma}(k - L - \eta) \tag{5}$$

and

$$k^* = L + \eta + \sqrt{(2L + 2\eta + NM)(L + 2\eta + NM)} \tag{6}$$

such that, for any time-delay $\tau < \tau^{**}$, the control with a gain $k = k^*$ together with the parameter update law in (4) can make the followers track the leader asymptotically.

Figure 1 depicts the relationship between the maximal delay τ^* and the corresponding control gain k based on numerical calculation.

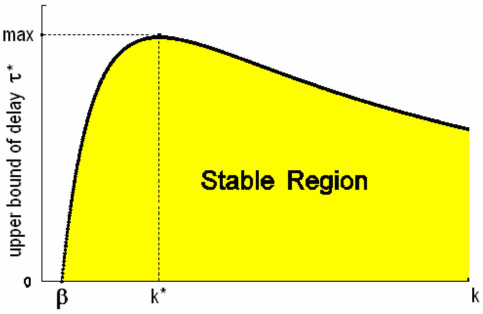

Figure 1. The relationship between the maximal delay τ^* and value of k.

Remark 1: From Figure 1, it is observed that, for a fixed delay $\tau \in [0, \tau^*]$, there exists a stable region for the control gain k which definitely ensures the convergence of the tracking problem. It contradicts with the common view that the larger the control gain is, the better the control effect will be.

Remark 2: The conclusion in Corollary 1 is a sufficient condition, and thereby a much conservative result which admits that some values of k beyond the stable region may still stabilize the tracking problem for some delay $\tau \in [0, \tau^*]$.

3. Simulation Results and Discussions

Some numerical simulations have been carried out to illustrate the feasibility of the design (4) and its effectiveness for tracking problems. It is assumed that each agent is a modified Chua's circuit chaotic system [8], which is described by

$$\begin{cases} \dot{x}_1 = \alpha_1(x_2 - h(x_1)) \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -\alpha_2 x_2 \end{cases} \tag{7}$$

where $h(x_1)$ is a smooth sine-type function given as:

$$h(x_1) = \begin{cases} \frac{b\pi}{2a}(x_1 - 2ac) & \text{if } x_1 \ge 2ac \\ -b\sin\left(\frac{\pi x_1}{2a} + d\right) & \text{if } -2ac < x_1 < 2ac \\ \frac{b\pi}{2a}(x_1 + 2ac) & \text{if } x_1 \le -2ac \end{cases} \tag{8}$$

with $\alpha_1 = 9.5$, $\alpha_2 = 11$, $a = 1.6$, $b = 0.1$, $c = 2$, $d = \pi$. (Note: Assumption 1 is held for (7)).

For the agent i, its dynamical system is represented as

$$\dot{x}_i \equiv \begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = f(x_i) + g(x_i)\hat{\alpha} = \begin{pmatrix} 0 \\ x_{i1} - x_{i2} + x_{i3} \\ 0 \end{pmatrix} + \begin{pmatrix} x_{i2} - h(x_{i1}) & 0 \\ 0 & 0 \\ 0 & -x_{i2} \end{pmatrix} \begin{pmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{pmatrix} \tag{9}$$

Consider a Lipschitz constant L=25, one can easily deduced that $\|g(x_i)\| \le h(x_{i1}) + 2\|x_{i2}\| \le \frac{b\pi}{2a} + 2 < 3$ and hence Assumption 2 is held.

It is assumed that the 50 agents form a small world network, where the network topological matrix $C_{50\times 50}$ is generated following the Watts-Strogatz (WS) algorithm [9] based on a regular nearest-neighbor network, where each node is connected to its 2K neighbors where K=2, and edge is added to each pair of nodes with probability p=0.1. The inner coupling matrix A is chosen as identity matrix, i.e. $A = I_3$.

The design objective is to make all the agents track the leader. It should also be emphasized that the actual parameters α_1 and α_2 of the leader are unknown to the agents.

Suppose that 40 percent of the agents obtain the information directly from the leader with a delay, hence for those agents, $b_i \neq 0$. Using Corollary 1, the maximal value of the time delay can be computed as:

$\tau^{**} = 1.2 \times 10^{-3}$ and $k^* = 234.83$.

Consider that case of having a communication delay of $\tau = 0.9\tau^{**}$ and the control gain is set as $k = k^*$, the tracking errors and the parameters estimation errors against time are shown in Fig. 2.

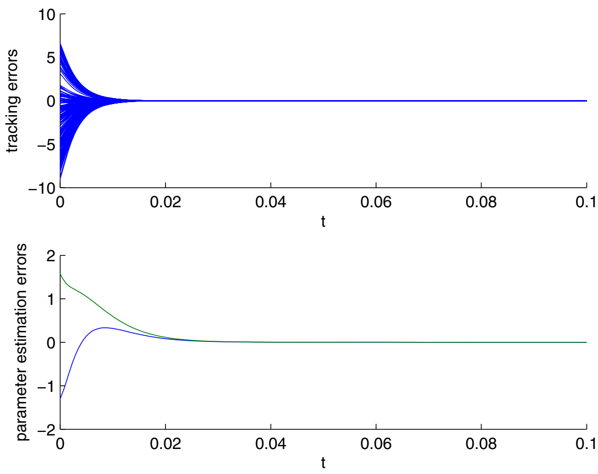

Figure 2. The tracking and the parameters estimation errors tend to zero when $\tau = 0.9\tau^{**}$ and $k = k^*$

On the other hand, as mentioned in Remark 1, if a larger k, for example, k=1450, is adopted, the agents are failed to track with the leader as shown in Fig. 3.

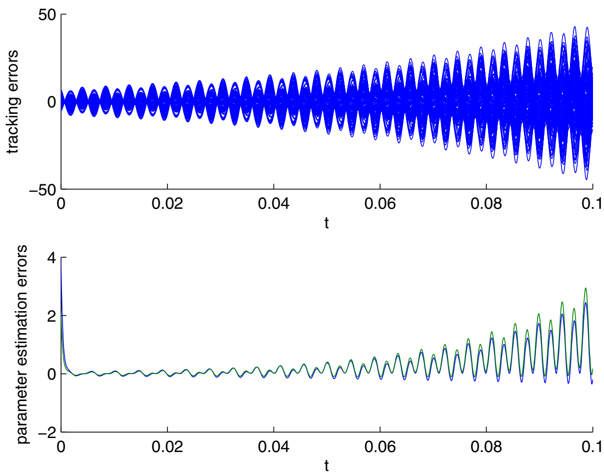

Figure 3. The tracking and the parameters estimation errors become infinite when a very large k is applied.

As revealed in Theorem 1 and Corollary 1, the maximal delay and the corresponding control gain are dependent on the matrix B, implying that the control performance is affected by the number of the agents who can directly obtain information from the leader. In the followings, we will further investigate how it acts with four different types of networks as listed below:

(a) Nearest-neighbor Network with 50 agents, while each one has exactly 4 nearest neighbors;
(b) Small-world Network generated by following the WS algorithm as described in the previous case;
(c) Erdos-Renyi random network with 50 agents and the probability of adding a link between every pair of nodes is 0.08.
(d) Scale-free Network generated by following the BA algorithm [10]. In details, it is to add a new agent on each step and link it with two existing agents using a preferential attachment mechanism on the basis of a fully-connected network comprising 5 agents

For each network, three different cases are configured to show the significance of the type of agents directly linking to leader:

Case I: the agent with larger degree is chosen with higher preference;

Case II: the agents linked to the leader are randomly chosen;

Case III: the agent with smaller degree is chosen with higher preference;

It should be remarked that each network has the same number of nodes and their average degrees are the same (which is 4). Figures 4 and 5 depict the maximal time delay τ^{**} and the corresponding control gain k^*, respectively, against the percentage of the followers link to the leader varies in [0,1], where 1 stands for all agents. The Cases I-III are shown in red, green and blue lines, respectively, while the averaged result of 1000 simulations is used for Case II.

It can be easily observed that, in all cases, the maximal time delay τ^{**} increases and the corresponding gain k^* decreases when more agents obtain information directly from the leader. Furthermore, the more the agents with high degree linking to the leader, the faster the maximal time delay τ^{**} increases and the corresponding control k^* decreases. It implies that agents with higher degree exert a strong influence on the performance of the tracking control. This fact is also illustrated by comparing the tracking capability of Cases I, II and III.

On the other hand, the topological structure of the network also affects the tracking control. As shown in Figs. 4 and 5, the tracking capability of the four types of networks declines successively in the following order: the nearest-neighbor network, small-world network, Erdos-Renyi random network, and scale-free network.

It is noticed that the nearest-neighbor network requires the smallest control gain and can tolerate the largest time-delay no matter what percentage of the agents are linked to the leader. In contrast, the scale-free network resists the smallest time-delay and the largest control gain is demanded. Therefore, the weakest tracking capability is found.

These observations can be explained according to the network topology. In the nearest-neighbor network, each agent has a degree of 4 and can receive the information of the leader rapidly even it is not directly communicated with the leader. For the small-world network, some links of the nearest-neighbor network are rewired but the existence of long range links also accelerates the propagation of the leader's information. Therefore, a satisfactory tracking capability is demonstrated. For random network, each pair of agents leads to a roughly average degree distribution and then, its tracking capability is also acceptable. For the scale-free network, an uneven degree distribution is noticed. Except for a few agents which having large degree, most of the agents have very small degrees and hence the leader's information can only slowly propagated, resulting the weakest tracking capability.

4. Conclusion

In this paper, the tracking control problem of multiple agents is investigated with a more realistic situation, i.e. with the existence of communication delay and parameter uncertainty. As illustrated with the numerical simulations, the problem can be effectively tackled with a control scheme together with some adaptive parameter estimate rules. Moreover, different kinds of networks have also been studied and it is found that the nearest-neighbor network possesses the strongest tracking capability as compared with the small-world network, random network and scale-free network.

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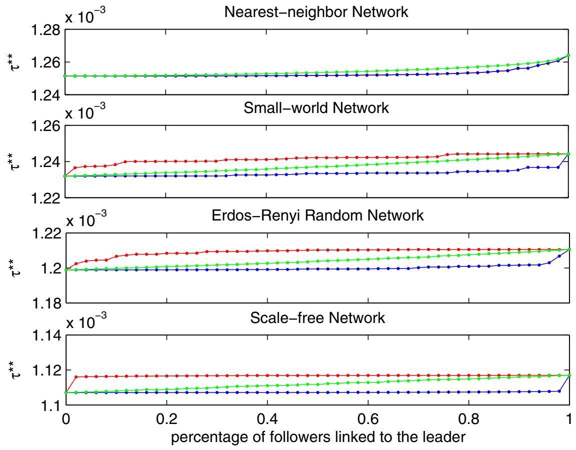

Figure 4. The maximal time delay τ^{**} for Cases I-III against the percentage of followers linking to the leader

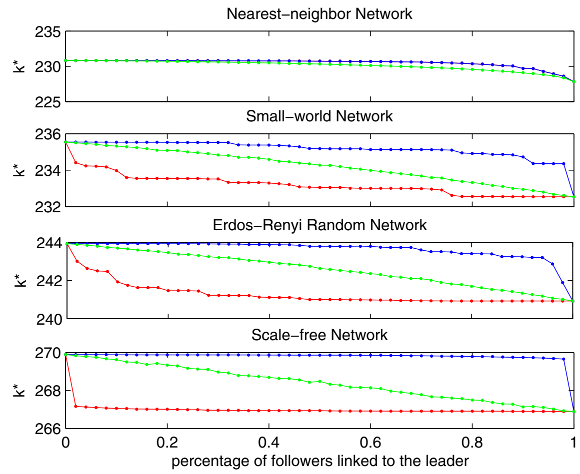

Figure 5. The gain k^* for the Cases I-III against the percentage of followers linking to the leader

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