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Augmented Automatic Choosing Control using Extended Kalman Filter for Nonlinear Systems with Noisy Linear Measurement

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Abstract—This paper deals with a feedback control using automatic choosing functions and the filter-control design procedure for nonlinear systems with noisy linear measurement. A constant term which arises from linearization of a nonlinear equation is treated as a coefficient of a stable zero dynamics. A given nonlinear system is linearized piecewise so as to be able to design the linear optimal controllers with filter. By the automatic choosing functions, these controllers are smoothly united into a single nonlinear feedback controller, which is called an augmented automatic choosing control of filter type by using Extended Kalman filter. Simulation results show that the new controller enables to improve the stability of electric power systems well.

1. Introduction

The problem of nonlinear control design has been studied for many years[1-8]. Most controllers are synthesized by linearizing a given nonlinear system so that the linear estimation and control theories are applicable when some of the state variables of the system are not measurable. One of them is based on a truncation at the first order of the Taylor expansion[1,2]. This control law is easy to implant in many practical nonlinear systems, but is only useful in small region or in almost linear ones. Controllers based on a change of coordinates in differential geometry [3,4] are effective in wider region, but not easy to implant in practical systems. Controllers based on Fuzzy reasoning[5] are more practical, but usually needs a lot of divisions. Controllers using an automatic choosing function[6,7] are superior, but noisy system cases have not yet studied.

This paper is concerned with a nonlinear feedback controller by using the automatic choosing functions and the linear control theory for nonlinear systems with noisy linear measurement. For state estimation, it makes use of an approach of Extended Kalman filter. This controller well works even in nonlinear systems with high nonlinearity and wider region. Considering the nonlinearity, we define some separative variables whose inverse domain associated with the region of system is divided into some subdomains. On each subdomain, the system equation is linearized by the Taylor expansion so as to apply the LQ control theory [2,8]. Constant terms by this linearization are treated as coefficients of a stable zero dynamics[7]. The resulting linear controls are smoothly united by the automatic choosing function to make a single nonlinear feedback control, whose estimator is Extended Kalman filter[2]. This controller is called an augmented automatic choosing control of filter type (AACCF) using Extended Kalman filter.

Experimental results indicate that the stability of electric power systems by AACCF is more improved than by the ordinary linear optimal controller (LOC).

2. Statement of Problem

The plant is assumed to be described by a nonlinear dynamic equation and a linear measurement equation

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathbf{D} \subset R^n \tag{1}$$

$$y = Hx + v \tag{2}$$

where $\cdot = d/dt$, $x = [x_{[1]}, \cdots, x_{[n]}]^T$ is an n-dimensional state vector, $u = [u_{[1]}, \cdots, u_{[r]}]^T$ is an r-dimensional control vector, $y = [y_{[1]}, \cdots, y_{[m]}]^T$ is an m-dimensional measurement vector, f is a nonlinear vector-valued function with $f(0) = 0$ and is continuously differentiable, g is an $n \times r$ nonlinear driving matrix with $g(0) \neq 0$, H is an $m \times n$ constant measurement matrix, v is a white Gaussian noise of $\mathcal{N}(v : 0, V)$, and T denotes transpose.

Considering the nonlinearity of the system (1), introduce a vector-valued function $C : \mathbf{D} \to R^L$ which defines the separative variables $\{C_j(x)\}$, where $C = [C_1 \cdots C_j \cdots C_L]^T$ is continuously differentiable. Let D be a domain of C^{-1}. For example, if $x_{[1]}$ is the element which has the highest nonlinearity of (1), then

$$C(x) = x_{[1]} \in D \subset R \quad (L = 1).$$

The domain D is divided into some subdomains: $D = \cup_{i=0}^{M} D_i$, where $D_M = D - \cup_{i=0}^{M-1} D_i$ and $C^{-1}(D_0) \ni 0$. $D_i (0 \le i \le M)$ endowed with a lexicographic order is the Cartesian product $D_i = \Pi_{j=1}^{L} [a_{ij}, b_{ij}]$, where $a_{ij} < b_{ij}$.

We here introduce an automatic choosing function of sigmoid type:

$$I_i(x) = \prod_{j=1}^{L} \left\{ 1 - \frac{1}{1 + \exp\left(2N\left(C_j(x) - a_{ij}\right)\right)} - \frac{1}{1 + \exp\left(-2N\left(C_j(x) - b_{ij}\right)\right)} \right\} \quad (3)$$

where N is positive real values, $-\infty \le a_{ij} < b_{ij} \le \infty$. $I_i(x)$ is analytic and almost unity on $C^{-1}(D_i)$, otherwise almost zero(see Figure 1).

The aim of the paper is to design a nonlinear feedback control AACCF by smoothly uniting the sectionwise controls and by using Extended Kalman filter.

3. Design of Control

The nonlinear function f of (1) is linearized by the Taylor expansion truncated at the first order about a point $\hat{\chi}_i \in C^{-1}(D_i)$ and $\hat{\chi}_0 = 0$ on each subdomain D_i (see Figure 2):

$$f(x) \simeq f(\hat{\chi}_i) + A_i(x - \hat{\chi}_i) = A_i x + w_i$$

where

$$A_i = \partial f(x)/\partial x^T|_{x=\hat{\chi}_i}, \quad w_i = f(\hat{\chi}_i) - A_i \hat{\chi}_i.$$

Introduce a stable zero dynamics :

$$\dot{\hat{x}}_{[n+1]} = -\sigma \hat{x}_{[n+1]} \quad (4)$$

$$(\hat{x}_{[n+1]}(0) \simeq 1, \quad 0 < \sigma < 1),$$

where the value of σ shall be selected so that $\sigma = -\dot{\hat{x}}_{[n+1]}/\hat{x}_{[n+1]} \le -\dot{x}_{[k]}/x_{[k]}$ holds for all k ($k = 1, \cdots, n$). This tries to keep $\hat{x}_{[n+1]} \simeq 1$ for a good while, when the system (1) is not on $C^{-1}(D_0)$. We approximate f as

$$f(x) \simeq A_i x + w_i \simeq A_i x + w_i \hat{x}_{[n+1]}. \quad (5)$$

Assume that the control is designed by using (3) as

$$u = \sum_{i=0}^{M} u_i I_i(\hat{x}) \quad (6)$$

where $\hat{x}$ is an estimate of x.

Note that $\sum_{i=0}^{M} I_i(\hat{x}) = 1$ for(3). Substituting (5) and (6) into (1), the dynamic equation becomes

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ &= \sum_{i=0}^{M} f(x) I_i(\hat{x}) + \sum_{i=0}^{M} g(x) u_i I_i(\hat{x}) \\ &= \sum_{i=0}^{M} (A_i x + w_i \hat{x}_{[n+1]} + B_i u_i + \varepsilon_i(x)) I_i(\hat{x}) \quad (7) \end{aligned}$$

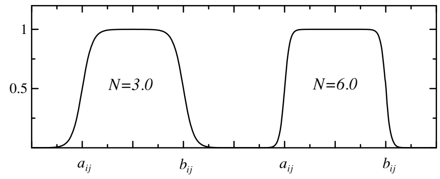

Figure 1: Automatic Choosing Function (N = 3.0,6.0)

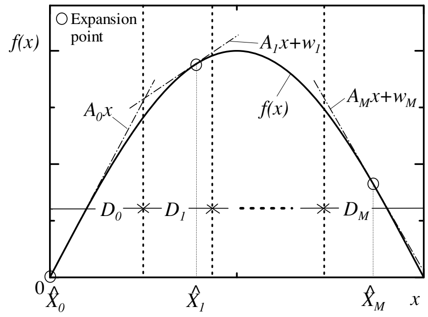

Figure 2: Sectionwize linearization

where $B_i = g(\hat{\chi}_i)$, ε_i is approximation error.

Consider a special case of $I_i(\hat{x}) = 1$ in which $N = -a_{ij} = b_{ij} \to \infty$ in (3).

Put $X = [x^T, \hat{x}_{[n+1]}]^T$, then Eqs.(4) and (7) yield an approximated linear equation:

$$\dot{X} = \bar{A}_i X + \bar{B}_i u_i$$

where
$$\bar{A}_i = \begin{bmatrix} A_i & w_i \\ 0 & -\sigma \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}.$$

Therefore we apply the LQ control theory to get the control formula as follows.

Consider that the system and cost function

$$\Sigma : \begin{cases} \dot{X} = \bar{A}_i X + \bar{B}_i u \\ J_i = \frac{1}{2}\int_0^\infty (X^T Q X + u_i^T R u_i) dt \end{cases} \quad (8)$$

are given. Then an application of the linear optimal control theory [2] yields

$$\begin{aligned} u_i(X) &= -F_i X \\ F_i &= R^{-1} \bar{B}_i^T P_i \end{aligned} \quad (9)$$

where the $(n+1) \times (n+1)$ matrix P_i satisfies the Riccati equation:

$$P_i \bar{A}_i + \bar{A}_i^T P_i + Q - P_i \bar{B}_i R^{-1} \bar{B}_i^T P_i = 0. \quad (10)$$

Here, $Q = Q^T > 0$ and $R = R^T > 0$ which denote positive symmetric matrices. Values of Q and R are properly determined based on engineering experience [8].

4. Design of Filter

We shall make use of an approach of Extended Kalman filter[2] for state estimation.

Assume that the filter equation is given by

$$\dot{\hat{x}}(t) = f(\hat{x}(t)) + g(\hat{x}(t))u + K(t)(y(t) - H\hat{x}(t)) \quad (11)$$

with initial value $\hat{x}(0) = \bar{x}_0$.
If the nonlinear equation of (1) is linearized by the Taylor expansion about an assumed known optimal estimate $x(t) = \hat{x}(t)$, then

$$\dot{x}(t) \simeq f(\hat{x}(t)) + F(t)(x(t) - \hat{x}(t)) + g(\hat{x}(t))u \quad (12)$$

where $F(t) = \partial f(x)/\partial x^T|_{x=\hat{x}(t)}$.
From Eqs.(11) and (12), the difference equation $e = x - \hat{x}$ is derived as

$$\dot{e}(t) = (F(t) - K(t)H)e(t) - K(t)v(t), \quad (13)$$

so the variance $S(t) = var(e(t))$ being zero-mean becomes

$$\begin{aligned}\dot{S}(t) &= (F(t) - K(t)H)S(t) \\ &+ S(t)(F(t) - K(t)H)^T + K(t)VK^T(t)\end{aligned} \quad (14)$$

with initial value $S(0) = S_0$.
We find $K(t)$ such as to minimize an index

$$J(t) = t_r(S(t))$$

where $t_r(\cdot)$ denotes the trace operator.
Thus we have

$$K(t) = S(t)H^TV^{-1} \quad (15)$$

$$\dot{S}(t) = F(t)S(t) + S(t)F^T(t) - S(t)H^TV^{-1}HS(t) \quad (16)$$

as the minimum error variance.
Therefore the filter algorithm is obtained by Eqs.(11)(15) and (16).

5. Synthesis of AACCF

From the above sections 3 and 4, we have the AACCF formula as follows.

[AACCF formula]

$$\dot{\hat{x}}(t) = f(\hat{x}(t)) + g(\hat{x}(t))u + K(t)(y(t) - H\hat{x}(t)) \quad (\hat{x}(0) = \bar{x}_0)$$

$$\dot{\hat{x}}_{[n+1]}(t) = -\sigma\hat{x}_{[n+1]}(t) \quad (\hat{x}_{[n+1]}(0) \simeq 1)$$

$$u(t) = \sum_{i=0}^{M} u_i I_i(\hat{x}(t))$$

$$K(t) = S(t)H^TV^{-1}$$

where

$$A_i = \partial f(x)/\partial x^T|_{x=\hat{\chi}_i}, \quad w_i = f(\hat{\chi}_i) - A_i\hat{\chi}_i$$

$$F(t) = \partial f(x)/\partial x^T|_{x=\hat{x}(t)}, \quad B_i = g(\hat{\chi}_i)$$

$$\bar{A}_i = \begin{bmatrix} A_i & w_i \\ 0 & -\sigma \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}$$

$$u_i = -R^{-1}\bar{B}_i^T P_i \hat{X}, \quad \hat{X} = [\hat{x}^T, \hat{x}_{[n+1]}]^T$$

$$P_i\bar{A}_i + \bar{A}_i^T P_i + Q - P_i\bar{B}_iR^{-1}\bar{B}_i^TP_i = 0$$

$$\dot{S}(t) = F(t)S(t) + S(t)F^T(t) - S(t)H^TV^{-1}HS(t)$$

$$(S(0) = S_0)$$

$$\begin{aligned} I_i(\hat{x}) = \prod_{j=1}^{L} \Bigg\{ 1 &- \frac{1}{1 + \exp\left(2N\left(C_j(\hat{x}) - a_{ij}\right)\right)} \\ &- \frac{1}{1 + \exp\left(-2N\left(C_j(\hat{x}) - b_{ij}\right)\right)} \Bigg\} \end{aligned}$$

Since this formula is of a structure-specified type, each parameter included in the above equations must be properly selected so that the feedback control system (1) by AACCF could stabilize globally.

6. Numerical Example

Consider a control problem of power system.

$$M\frac{d^2\delta}{dt^2} + D\frac{d\delta}{dt} + P_e(1 + \Delta E_{fd}) = P_{in}$$

$$y = \frac{d\delta}{dt} + v, \qquad P_e = \frac{e_l E_{fd}}{X_e}\sin(\delta)$$

where δ: phase angle, $\dot{\delta} = d\delta/dt$: rotor speed, M: inertia coefficient, D: damping coefficient, P_{in}: mechanical input power, P_e: generator output power, X_e: transmission line impedance, e_l: infinite bus voltage, E_{fd}: field excitation voltage, ΔE_{fd}: deviation of E_{fd}. Put x=$[x_{[1]}, x_{[2]}]^T$=$[\delta - \hat{\delta}_0, \dot{\delta}]^T$ and $u = \Delta E_{fd}$.
Then this system is described by Eqs.(1)and(2), where $n = 2, r = 1, m = 1$ and $H = [0, 1]$.

Parameters are $M = 0.06[pu]$, $D = 0.06[pu]$, $E_{fd} = 1.0[pu]$, $e_l = 1.0[pu]$, $X_e = 1.0[pu]$, $P_{in} = 0.8[pu]$, $\hat{\delta}_0 = 0.9276[rad]$, and $\dot{\hat{\delta}}_0 = 0.0[rad/sec]$.

Set $X = [x^T, \hat{x}_{[3]}]^T = [x_{[1]}, x_{[2]}, \hat{x}_{[3]}]^T$, $C(x)$=$x_{[1]}$, $L = 1$, $M = 1$, $N = 8$, $a_1 = 0.6$, $\hat{\chi}_0 = 0$, $\hat{\chi}_1 = [1.51, 0]^T$, $\sigma = 0.1$, $R = 1$, $Q = diag(1, 1)$, $V = 1$, $\hat{X}(0) = [0, 0, 1]^T$, $S(0) = \begin{bmatrix} 5 & 10 \\ 10 & 50 \end{bmatrix}$.

These values are selected by trial and error. Experiments are carried out for the new control(AACCF) and the ordinary linear optimal control (LOC)[1, 2]. Figure 3 depicts the stable regions for AACCF and LOC. Figure 4 makes a comparison between AACCF and LOC for the time responses of $x_{[1]}, x_{[2]}$ and u when $X(0) = [1.3, 0, 1]^T$. Figure 5 shows the state $x_{[i]}$ and its estimate $\hat{x}_{[i]}$ of the AACCF in Fig.4 for $i = 1$ and 2. Experimental results indicate that the stable region and trajectories by the new AACCF are much better than those by the LOC.

7. Conclusions

We have studied an augmented automatic choosing control of filter type (AACCF) using Extended Kalman filter for nonlinear systems with noisy linear measurement. This controller has been applied to a control problem of power system. Simulation results

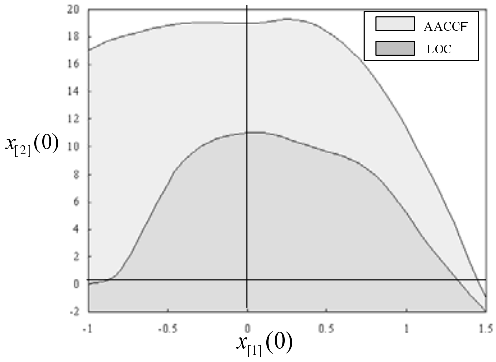

Figure 3: Stable regions

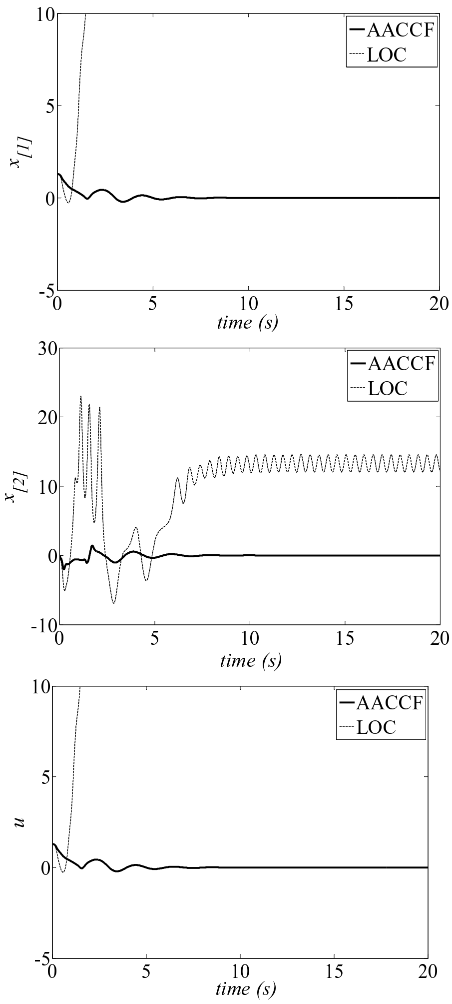

Figure 4: Time responses of x and u

have shown that the new controller is able to improve the stability and trajectories well.

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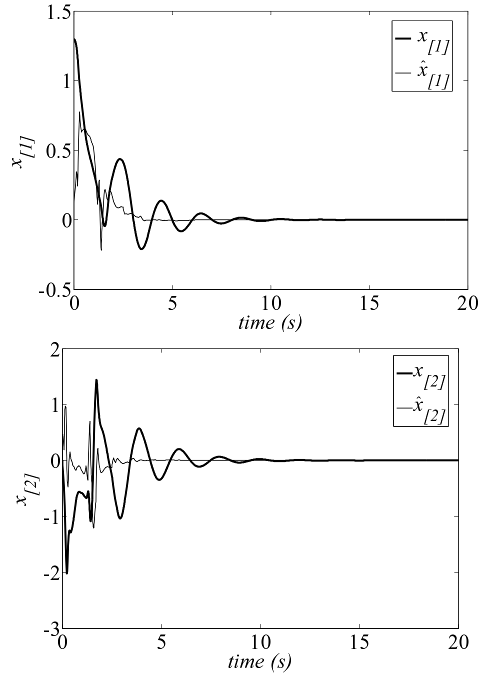

Figure 5: x and $\hat{x}$ of AACCF

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