Border Collisions and Chattering in State Feedback Controlled Boost Converters

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Abstract—Digital state feedback controllers are frequently used in DC-DC converters when high/optimal performance is required. In this paper we investigate the nonlinear behavior of such systems when the feedback control law is a discontinuous function of the location of the state vector in the state space. It is shown that the well known problem of chattering that appears in these controllers (e.g. in Model Predictive Control - MPC) is due to a border collision between the fixed point of the nominal period one orbit and the borders that define the areas that the specific gains are utilized, and forces the converter to exhibit high current ripple that can deteriorate the system's performance. The exact switched dynamical model of the system is described along with the three switching manifolds in the state space and through that a detailed analysis of the border collision that causes the chattering and the high current ripple is presented. Finally, a methodology is proposed that can lead to a design procedure that completely avoids this problem. The latter can be taken into account when advanced controllers are employed in order to guarantee the optimal behavior of the system.

1. Introduction

DC-DC converters are used in numerous applications ranging from simple domestic appliances to mobile phones, laptops and areas that require high performance like military and aerospace systems. The main task of such a power circuit is to regulate the voltage and/or current of its output despite fluctuations in its input. When there is a need to increase the input voltage, a boost converter is usually employed (even though there are other converters that can be used for that) which uses two electronic switches that can either be both set to OFF or they can operate complementary, i.e. when one switch is OFF the other is ON. Hence the converter toggles between three different topologies in such a way that boosts the input voltage [1]. The nonlinear dynamics of DC-DC converters have attracted a lot of interest from the academic community [2, 3] and it is now considered to be a mature topic that is also embraced by industry. The main issue with such systems is the occurrence of high current ripple that appears when a bifurcation happens and this can greatly degrade the efficiency and lifetime of the converter. There are 3 main types of bifurcations that can take place in these converters: a) smooth (period doubling, Neimark-Sacker and Saddle-Node bifurcation) b) an interaction between smooth bifurcations [4] and c) nonsmooth bifurcations (also called border collisions) where a fixed point collides with a border in the state space [5]. Despite the large volume of work on the bifurcation analysis of DC-DC converters, little or no work has taken place when the converter is being controlled by an advanced type of controller like state feedback. These controllers are used when high performance is required and the tuning method is usually based on methods like MPC [6] and Constrained Stabilization [7]. These techniques can lead to either fixed or switching control laws where the vector fields depend on the location of the state vector in the state space. In [8] the authors made a first attempt to explore the various bifurcation phenomena that appear when digital state feedback controllers are used. It was shown that it is possible to have a range of undesired phenomena that can greatly deteriorate the operation of the converter. This work was further elaborated in [9] where various bifurcation phenomena were taken into account in the design of efficient and robust state feedback controllers. In this paper we further explore the nonlinear behavior of boost converters that employ switching state feedback controllers. This creates multiple switching manifolds in the state space where it is possible to have border collisions when a fixed point hits them. This border collision is the explanation behind the well known problem of chattering that frequently appears in converters that use MPC and create large limit cycles of high periodicity. Even though an MPC controller creates multiple switching manifolds in the state space, in this work only one switching manifold is added in order to simplify the analysis and to clearly demonstrate the effect of the aforementioned border collision. The last part of the paper focuses on finding ways to overcome this problem. As a complete design methodology cannot be fully presented (due to space limitations) an analysis of how this work can lead to robust controllers that guarantee the avoidance of chattering is included.

2. System Description

The schematic diagram of the boost converter is shown in Fig. 1 assuming ideal components. Mathematically, and irrespectively of the controller that is chosen, the dynamical model of the converter is given by the piecewise smooth ODE shown in (1), where the two states are chosen to be the inductor current i_L and the output or capacitor voltage v_c, i.e. $x = [v_c \; i_L]^T$.

$$\dot{x} = \begin{cases} A_1 x + B_1 u, & \text{(S,D)=(ON, OFF);} \\ A_2 x + B_2 u, & \text{(S,D)=(OFF, ON);} \\ A_3 x + B_3 u, & \text{(S,D)=(OFF, OFF).} \end{cases} \tag{1}$$

$A_{1,3} = \begin{bmatrix} -1/RC & 0 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix}, B_{1,2} = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, u = V_{in}$

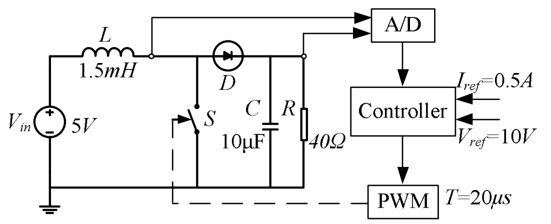

Figure 1: Schematic diagram of a digitally controlled boost converter.

In order to toggle between the three states of the two switches, an external clock is used with period T. At the beginning of each clock cycle the state of the switches is set to (ON, OFF). The switches remain at this state until $t = d \times T$, where d is the duty cycle. In this work a digital state feedback controller is used and hence the duty cycle is defined by (2).

$$d = [k_1 \; k_2] \times (x(nT) - x_{ref}) + d_{ss} \tag{2}$$

where

$$x_{ref} = \begin{bmatrix} V_{ref} + T(V_{ref} - V_{in}/2RC) \\ I_{ref} - V_{in} dT/2L \end{bmatrix} \tag{3}$$

and d_{ss} is the steady state desired duty cycle i.e. if the error is vanished at steady state conditions we have $d = d_{ss}$.

The third state of the switches i.e. (OFF, OFF), takes place if the inductor current becomes zero and due to the diode stays off until the end of the clock cycle; this mode of operation is called "discontinuous conduction mode" (DCM). Regardless of the state of the switches ((OFF, ON) or (OFF, OFF)) at the beginning of the next clock cycle the switch state is set again to (ON, OFF) and the whole operation is repeated. In the state space the transition from the switch state (ON, OFF) to (OFF, ON) and then to (OFF, OFF) is being described by two switching manifolds given by the following smooth scalar equations[1]:

$$h_1 = -t/T + [k_1 \; k_2] \times (x(nT) - x_{ref}) + d_{ss} \tag{4}$$

$$h_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \times x(t) \tag{5}$$

In [8] the gain vector and the value of d_{ss} were assumed to be constant but here they depend on the location of the orbit in the state space, hence this defines another switching manifold given by:

$$h_3 = \begin{bmatrix} a & b \end{bmatrix} \times x(nT) - c \tag{6}$$

and $[k_1 \; k_2] = \begin{cases} \begin{bmatrix} k_{1A} & k_{2A} \end{bmatrix}, & \text{if} \begin{bmatrix} a & b \end{bmatrix} \times x(nT) \le c; \\ \begin{bmatrix} k_{1B} & k_{2B} \end{bmatrix}, & \text{if} \begin{bmatrix} a & b \end{bmatrix} \times x(nT) > c. \end{cases}$

Similarly the value of d_{ss} is d_{ssA} and d_{ssB} above and below the border respectively[2].

Care has to be taken here as the first and third switching manifolds depend on the sampled state vector and the second on the continuous state vector. As a switching occurs when the continuous orbit hits a manifold these expressions have to be modified; in order to do that we use the uniqueness and existence theorem of differential equations [9]:

$$x(t) = e^{A_1 t} x(nT) + \int_{nT}^{t} e^{A_1(t-\tau)} \begin{bmatrix} 0 \\ V_{in}/L \end{bmatrix} d\tau$$

or

$$x(nT) = (e^{A_1 t})^{-1} \left(x(t) - \int_{nT}^{t} e^{A_1(t-\tau)} \begin{bmatrix} 0 \\ V_{in}/L \end{bmatrix} d\tau \right) \tag{7}$$

Therefore, the complete mathematical model of the system is given by (1), along with the expressions of h_1, h_2 and h_3 and (7). In this work the chosen gains are $k_{1A} = 0.0443$, $k_{2A} = -0.2324$ and $k_{1B} = 0.0482$, $k_{2B} = -1.5886$, while $d_{ssA} = 0.5$ and $d_{ssB} = 0.1$. The switching manifold h_3 for simplicity reasons was chosen to be parallel to the $y - axis$, i.e. $a = 0$, $b = 1$; the value of $c = I_{max}$ is chosen as the bifurcation variable.

[1] At $t = 0$ there is a transition from (OFF, ON) or (OFF, OFF) to (ON, OFF) but as this depends on time and not on the value of the state vector, it does not need to be included in the analysis.

[2] In (6) the switching manifold is defined as a straight line (based on the values of a, b and c) since all advanced switching controllers such as MPC employ a state space partitioning where all regions are polytopes.

3. Chattering and Border Collision

As it has been described in the previous section the normal boost converter has one switching condition based on the location of the state vector when the switch state changes from (ON, OFF) to (OFF, ON) (switching manifold h_1) and another one when it goes into DCM (switching manifold h_2). A third condition is imposed in this work as the feedback law changes when the inductor current is higher than I_{max} (switching manifold h_3). In order to see the effect of the last switching manifold the bifurcation diagram of I_{max} is plotted for values between 0.481A and 0.485A. In Fig. 2 we see that for high values of I_{max} there is a unique period 1 orbit and then at approximately 0.483A there is a sudden transition to a much higher periodic orbit with a much bigger limit cycle. From this figure we can also see that this bifurcation happens when the fixed point hits the switching manifold defined by h_3, i.e. when the fixed point hits the border I_{max}. In Fig. 3a we see the limit cycle in the state space and as I_{max} is decreased (in the figure shown from 0.5A to 0.49A and then to 0.4835A) we see that the border described by h_3 comes closer and closer to the fixed point. At some point there will be a collision and as it can be seen by Fig. 3b after that we have a new large limit cycle with high periodicity. A similar picture can be seen in Fig. 4. This phenomenon in the nomenclature of control systems theory is called chattering and as it can be seen here it is due to a border collision.

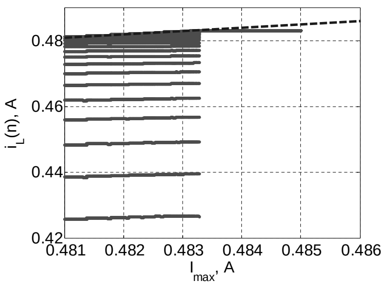

Figure 2: Bifurcation diagram. The black dashed line is the line $i_L(n) = I_{max}$

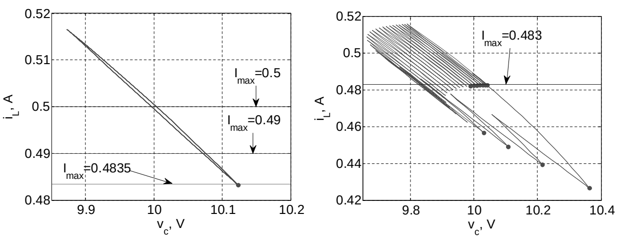

Figure 3: Limit cycle as I_{max} approaches the fixed point.

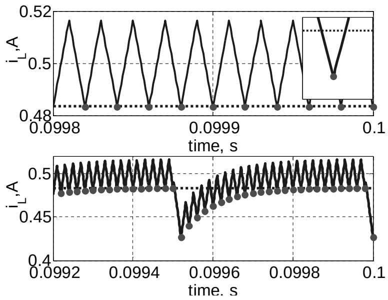

Figure 4: Current response when I_{max} is 0.4835 and 0.483.

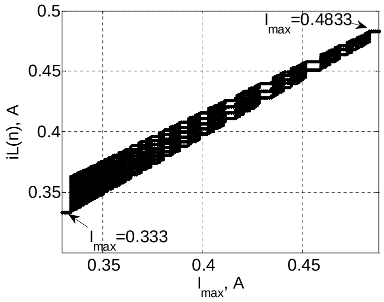

Figure 5: Bifurcation diagram with $d_{ssA} = 0.5$ and $d_{ssB} = 0.4$.

4. Avoidance of Chattering

In this section we will present ways to avoid the chattering in this simple case and discuss how this can be extended to more complicated controllers (like MPC) that create multiple manifolds and hence the phenomenon of chattering can be even more severe. In order to do that we will start by answering the following four questions: a) can we predict exactly where/when the border collision occurs? b) for what values of the bifurcation variable does this occur? c) will we always have chattering when the border collision happens? and d) what can be done in order to avoid this phenomenon.

The first two questions are easily answered by calculating the fixed points of the converter for the two values of d_{ss} before and after the switching[3] using (3), which are $\begin{bmatrix}10.1250 & 0.4833\end{bmatrix}^T$ and $\begin{bmatrix}8.4167 & 0.3339\end{bmatrix}^T$. Hence, the borders where the chattering occurs are between 0.3339 and 0.4833 which is also shown in Fig. 5. The third question can be answered by investigating the direction of the vector fields before and after the switching. As in this case both vector fields are directed towards h_3 it is expected that sliding will occur on that surface. Based on this observation, it is possible to answer the fourth and probably the most important question, i.e. how can we avoid the occur-

[3] In order to improve the visibility of the figures, in this section we use $d_{ssB} = 0.4$ instead of 0.1.

rence of chattering. The simplest answer is to avoid hitting the border, i.e. setting I_{max} to a value that we do not have a border collision, but in this case the benefit that we have of using multiple control laws is lost. Another approach is to make sure that when the border collision occurs, the vector fields do not point at the same switching manifold. In Fig. 6 we see the response of the system similarly to Fig. 3 where the values of d_{ssB}, x_{ref} are modified appropriately so that that the two vector fields have the same direction, for $I_{max} = 0.4$ [4].

This approach could be utilized by controllers such as MPC, where multiple switching manifolds are created in order to optimally drive the trajectory in the state space.

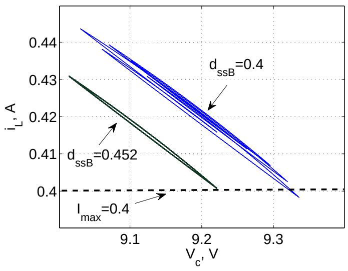

Figure 6: Limit cycles for $I_{max} = 0.4$ and two different values for d_{ssB}, x_{ref}.

5. Conclusions

In this paper, we present the bifurcation behavior of a boost converter under a digital switching state feedback controller. This controller can be found in demanding applications that employ advanced types of control strategies, such as MPC. It is proven that the well known problem of chattering (that can greatly deteriorate the performance of the system) is due to a border collision bifurcation. In order to avoid this problem we either have to ensure that the collision does not occur or that chattering conditions are not met. Further work will include the study of the interaction of this border collision with the smooth bifurcations previously investigated by the authors in [8] with an ultimate goal of producing a new design methodology similar to [9] that can guarantee an optimum behavior of the system in wide operating region. Also, the incorporation of the aforementioned strategy of avoiding chattering in MPC algorithms must be further studied.

[4] It can be easily found from (3) that, for $I_{max} = 0.4$, in order to change the second vector field's direction we need to set $d_{ssB} \geq 0.452$, provided that x_{ref} is also modified accordingly.

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