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Proposal and Evaluation of Pursuit Formations based on Cyclic Pursuit Dynamics

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Abstract— A multiagent system comprises many agents that autonomously make decisions to complete a task. Such a system can perform tasks that are difficult for a single agent. Formation control is such a task in which multiple agents perform the task of forming an entire shape. Among various strategies for formation control, cyclic pursuit is notable for its simplicity and ability to enable agents in forming a circle. However, it remains unclear whether the strategy can be used to realize formations other than circles. Motivated by this observation, in this study, we propose a method for enabling agents that adopt the cyclic pursuit strategy to realize a specific shape. We examined the effectiveness of the proposed method through numerical simulations. In the simulations, the proposed method succeeded in forming a square, star, and heart shape.

1. Introduction

A system in which multiple agents interact with each other to make decisions and accomplish a task is called a multiagent system [1]. This system is expected to be applied to various situations because of its ability of accomplishing tasks that are difficult to be achieved by a single agent and its resilience to failures and disturbances [2, 3]. A typical task for multiagent systems is formation control [4], in which multiple agents form various shapes or move while maintaining the shape. Although various control algorithms for multiagent tasks have been proposed in the literature [5, 6], many of them rely on the assumption that agents have dense interaction dynamics. For example, the method proposed by De Marina et al. [5] requires the interaction topology of agents to be rigid. One exception is the cyclic pursuit strategy [7, 8, 9], in which each agent is assumed to be able to only observe information about the relative position of the agent moving ahead of it. This strategy is proved to be able to enable the agents to form a circle by enabling them to "pursue" the agent moving ahead of the them. Although this strategy can achieve formation control with limited information, it remains unclear whether the strategy enables agents to form other shapes.

Motivated by the aforementioned question, in this study, we proposed a method for forming desired shapes based on the cyclic pursuit strategy. In the proposed method, the dynamics of the agents is based on cyclic pursuit. At each time, agents calculate its ideal position based on the position of the agent moving ahead of it, and shifts to the ideal position. By repeating this process, all agents can form the complete required shape.

This paper extends the theory of formation control in multiagent systems by showing that the cyclic pursuit strategy, which has been regarded to be able to realize only circles, can realize various kinds of shapes. In addition, we can gain better understanding of the dynamics of cyclic pursuit. Furthermore, because the dynamics of cyclic pursuit is simple, we expect that formation control using inexpensive agents will become possible in the future.

This paper is organized as follows. First, we formulate the problem studied in this paper in Section 2. In Section 3, we describe the proposed method. Then, in Section 4, we evaluate the effectiveness of the proposed method using three representative shapes. Finally, Section 5 summarizes the paper and discusses future issues.

2. Problem Statement

In this section, we describe the problem studied in this paper. We consider a multiagent system in a two-dimensional plane $\mathbb{R}^2$. We assume that the system consists of N agents. We assign the numbers $1, \ldots, N$ to the agents. In addition, $x_i(k)$ denotes the position of the ith agent at time k. The objective of the system is to form a complete required shape by the agents' distributed decision. The desired shape is assumed to be described by a Jordan closed curve $\gamma\colon [0,1] \to \mathbb{R}^2$. For example, if the desired shape is a square, then we can use γ given by

$$\gamma(\xi) = \begin{cases} (1, 4\xi), & \text{if } \xi < \frac{1}{4}, \\ (1 - 4(\xi - \frac{1}{4}), 1), & \text{if } \frac{1}{4} \le \xi < \frac{1}{2}, \\ (0, 1 - 4(\xi - \frac{1}{2})), & \text{if } \frac{1}{2} \le \xi < \frac{3}{4}, \\ (4(\xi - \frac{3}{4}), 0), & \text{otherwise.} \end{cases} \tag{1}$$

We provide the space $[0, 1]$ the distance $d(\cdot, \cdot)$ defined by

$$d(x, y) = \min(|x - y|, |x + 1 - y|) \tag{2}$$

for all $x, y \in [0, 1]$.

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It is well-established [7] that the aforementioned objective can be achieved by a multiagent system performing a cyclic pursuit when γ is the unit circle, that is, the system converges to the state where all the agents form a circle by moving on it with the same speed while maintaining equal distances between two consecutive agents. Notably, the circle formed by the agents does not necessarily coincide with the unit circle specified by γ. Therefore, the cyclic pursuit strategy guarantees the existence of a Jordan closed curve $\tilde{\gamma}$ representing a circle, potentially different from γ, and functions $\tau_1, \ldots, \tau_N$ from the set $\{0, 1, 2, \ldots\}$ of nonnegative integers to the interval $[0, 1]$ such that

$$\lim_{k\to\infty} (x_i(k) - \tilde{\gamma}(\tau_i(k))) = 0, \tag{3}$$

and

$$\lim_{k\to\infty} d(\tau_{n(i)}(k), \tau_i(k)) = \frac{1}{N} \tag{4}$$

for all $i = 1, \ldots, N$, where $n(i)$ represents the index of the agent i's predecessor agent and is defined by

$$n(i) = \begin{cases} i+1, & \text{if } i < N, \\ 1, & \text{otherwise.} \end{cases} \tag{5}$$

The first equation (3) implies that, asymptotically, an agent i will move on $\tilde{\gamma}$ with its position specified by $\tau_i(k)$. The second equation (4) implies that, under the parametrization of $\tilde{\gamma}$ by $[0, 1]$, any adjacent pair $(i, n(i))$ of agents travels an equal distance of $1/N$ (i.e., the length of interval $[0, 1]$ divided by the number of agents) after a sufficient time from the start of the formation control.

This observation leads to the following question: can a multiagent system with cyclic pursuit-based interactions among agents realize a formation other than a circle? We formulate the problem as follows. First, the dynamics of the agents are assumed to be based on the cyclic pursuit. Therefore, each agent is required to determine its own moving vector using the information about the agent ahead of it. Specifically, A1) each agent is assumed to be able to observe the relative position of its predecessor agent. In this paper, in addition to this information, we assume that A2) each agent i is aware of its own orientation $\phi_i \in [0, 2\pi)$ (i.e., the angle of the direction vector measured as a counterclockwise rotation from the positive x axis) at any time. Furthermore, we assume that A3) each agent has knowledge of the desired shape γ. Under these assumptions, we formulate the problem that we investigate in this study as follows:

Problem 1. *Let γ be a closed Jordan curve. Under assumptions A1-A3, Design a distributed movement law for each agent such that the positions x_i of all the agents satisfy* (3) *for a closed Jordan curve $\tilde{\gamma}$ similar to γ and a set of functions $\tau_i \colon \{0, 1, 2, \ldots\} \to [0, 1)$ $(i = 1, \ldots, N)$ that satisfy* (4).

The following notation will be used throughout this paper. For a real number $a > 0$, and $\lfloor a \rfloor$ denotes the largest integer that does not exceed a.

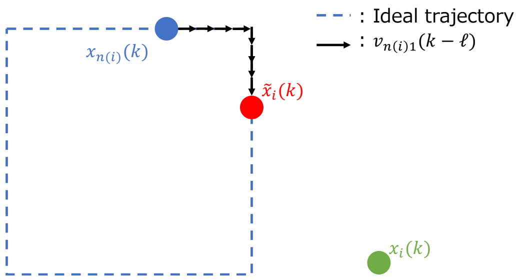

Figure 1: Schematic of Eq. (12). The black arrow indicates the movement vector $v_{n(i)1}(k - \ell)$.

3. Proposed Method

In the proposed method, we first run the cyclic pursuit algorithm [7] for sufficiently long time. When the multiagent system reaches its stationary state, we move to the next phase where the system attempts to form the given shape γ. Without loss of generality, we set the time of moving into the next phase as 0. Therefore, at time $k = 0$, the agents are equally spaced. The difference in orientation between agents is geometrically $2\pi/N$. In other words, the following equation holds for all agents:

$$\phi_{n(i)} - \phi_i = \begin{cases} \dfrac{2\pi}{N}, & \text{if } \phi_i < \phi_{n(i)}, \\ \dfrac{2\pi}{N} - 2\pi, & \text{otherwise.} \end{cases} \tag{6}$$

We then describe the configuration of τ_i such that the Eq. (4) is satisfied. First, agent i initializes its own τ_i as

$$\tau_i(0) = \frac{\phi_i}{2\pi}. \tag{7}$$

Subsequently, each agent updates τ_i at each time as

$$\tau_i(k+1) = \begin{cases} \tau_i(k) + \eta, & \text{if } \tau_i(k) + \eta < 1, \\ \tau_i(k) + \eta - 1, & \text{otherwise,} \end{cases} \tag{8}$$

where $\eta \in (0, 1)$ is a tunable parameter. From (6), we can easily confirm that τ_i satisfies $d(\tau_{n(i)}(k), \tau_i(k)) = 1/N$ for all k. Therefore, Eq. (4) is satisfied.

Next, we describe the movement law of the agents after time $k = 0$. One of the movement laws of an agent that satisfies Eq. (3) is $x_i(k) = \gamma(\tau_i(k))$. However, this movement law cannot be used because the agent cannot obtain its own absolute coordinates. Therefore, within the proposed algorithm, each agent determines its moving vector $v_i(k)$ in a distributed manner and updates its own position as

$$x_i(k+1) = x_i(k) + v_i(k) + \epsilon_i(k), \tag{9}$$

where $\epsilon_i(k)$ represents possible disturbances. The vector consists of two distinct vectors as

$$v_i(k) = v_{i1}(k) + v_{i2}(k), \tag{10}$$

where $v_{i1}(k)$ is the vector by which the agents individually draw the desired shape, and $v_{i2}(k)$ is the vector for achieving coordination among agents.

First, we define $v_{i1}(k)$ as

$$v_{i1}(k) = \dot{\gamma}(\tau_i(k)), \tag{11}$$

where $\dot{\gamma}$ is the derivative of γ. Although this vector $v_{i1}(k)$ enables each agent to achieve the formation individually, it is not guaranteed that the trajectories on which each agent moves coincide. The compensation of the potential difference is the role of the second vector $v_{i2}(k)$. Within the proposed algorithm, the agent i assume that the agent $n(i)$ is on the correct trajectory, and construct $v_{i2}(k)$ in such a way that the agent i can create its own trajectory close to the one of its predecessor agent, $n(i)$. The agent 1 approaches the trajectory of the agent 2, the agent 2 approaches the trajectory of the agent 3, and subsequent agents follow suit, with the agent N moving to approach the trajectory of the agent 1, resulting in the eventual synchronization of all agents' trajectories. Assuming $v_{n(i)2}(k) = 0$ and $1/N\eta$ are integers, agent i can calculate its own "ideal position" as

$$\tilde{x}_i(k) = x_{n(i)}(k - (N\eta)^{-1}). \tag{12}$$

Using this ideal position, we define the movement vector $v_{i2}(k)$ as

$$v_{i2}(k) = \alpha(\tilde{x}_i(k) - x_i(k)). \tag{13}$$

Because the ideal position defined in (12) is not accessible to the ith agent, we employed the following approximation in our implementation:

$$\begin{aligned}\tilde{x}_i(k) \approx x_{n(i)}(k) - (v_{n(i)1}(k-1) + v_{n(i)1}(k-2) + \cdots \\ + v_{n(i)1}(k - \lfloor (N\eta)^{-1} \rfloor)).\end{aligned} \tag{14}$$

Our rationale behind this approximation is illustrated in Figure 1, where the ith agent determines an ideal position $\tilde{x}_i(k)$ by inverting $v_{n(i)1}(k)$, as shown by the black arrow in Figure 1. Furthermore, agent i can compute the approximation using only $\dot{\gamma}$ and $\tau_i(k)$ because a simple calculation shows

$$v_{n(i)1}(k - \ell) = \dot{\gamma}(\tau_i(k) + N^{-1} - \ell\eta) \tag{15}$$

for all $\ell \geq 1$.

4. Numerical Simulations

In this section, we evaluate the effectiveness of the proposed method. As described in Section 3, we assumed that the agents are placed on a circle with equal spacing at the initial step $k = 0$. Within the simulation, we set the center and radius of the circle to be the origin and 15, respectively. The total number of the agents N was set to 3, η was set to 0.01, and α was set to 0.01. In addition, the maximum step of the simulation was set to 300. We used the noise term $\epsilon(k)$ in Eq. (10) expressed by

$$\epsilon(k) = 0.1[\cos(\omega_i(k)), \sin(\omega_i(k))], \tag{16}$$

where $\omega_i(0), \omega_i(1)$, and ... are independent and identically distributed random variables with uniform distribution on $[0, 2\pi]$.

For the desired shape γ, we consider the following three shapes: a square, star, and heart (see Figure 2). In Figure 3, we show the trajectories of the agents for each of the desired shapes. We can observe that each agent gradually moves to draw the desired shape nearly on the same trajectory.

To quantitively evaluate the accuracy of the formation control, we measured the distance between the desired

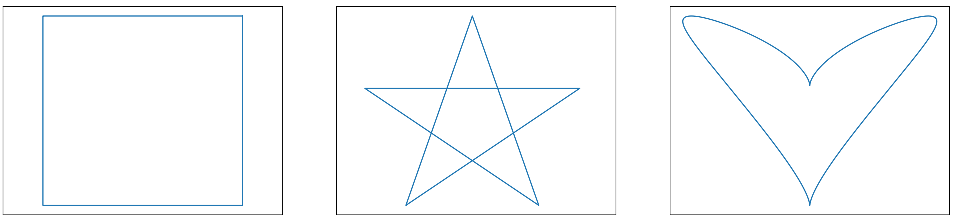

Figure 2: Desired shapes.

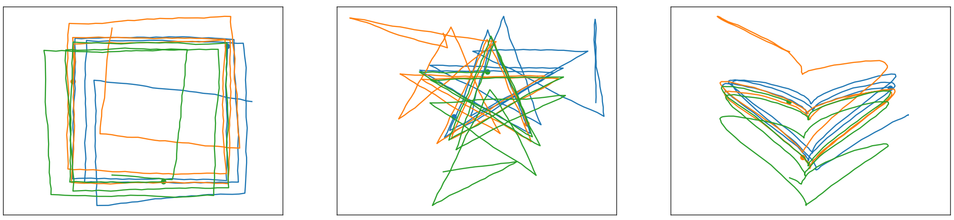

Figure 3: Trajectories of the agents when the desired shapes are square, star, and heart. The dots indicate the position of each agent at the final step.

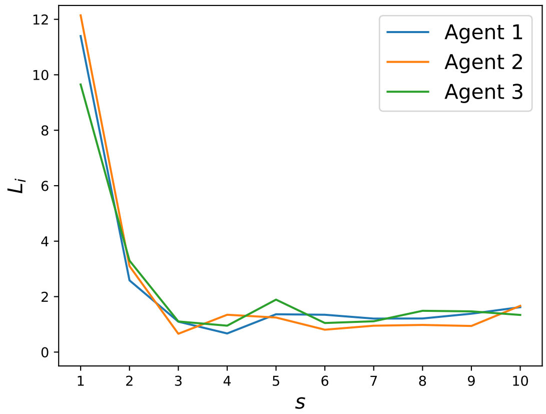

Figure 4: Discrete Fréchet distance. The horizontal and the vertical axis represent the cycle and distance, respectively.

shape and the trajectory drawn by the agents. For this purpose, we used the discrete Fréchet distance [10]. In this study, because the shape can move parallelly, the distance was calculated after translating the desired shape to enable the center of all agents per cycle and the center of the desired shape to be at the same point. One cycle was 100 steps because the value η was equal to 0.01. In addition, in Figure 4, we show the distance between the desired shape and actual trajectory in each cycle when the desired shape is a square. The horizontal axis indicates the number of cycles and the vertical axis indicates the discrete Fréchet distance. We can observe that for all agents, the distance is decreasing. In other words, the trajectories of all agents are attaining the desired shape. The diagram also suggests that the trajectory on which each agent moves almost coincide. This is because the center of gravity of the shape drawn by *all* agents is aligned with the center of gravity of the desired shape (i.e. all agents are translating the same amount). Therefore, if the discrete Fréchet distance is small for all agents, all agents are considered to be on the same trajectory.

We note that $(N\eta)^{-1}$ is not an integer within the current simulation. Therefore, the approximation (14) is not necessarily accurate. Nevertheless, the agents were able to complete the task of drawing a required shape. This observation suggests a certain amount of robustness in the proposed algorithm. The reason for this robustness remains an open problem for future research.

5. Conclusion

In this study, we propose a method for formation control based on a cyclic pursuit strategy. In the proposed method, the agents draw the desired shape using only limited information about themselves and the agent moving in front of them. Simulations confirmed that the proposed method was able to draw some shapes.

Future work will involve demonstrating that arbitrary shapes can be drawn. In addition, we plan to investigate the robustness of the system when the number of agents changes, or when the desired shapes change during the drawing process.

Acknowledgments

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