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Asynchronous Temporal Interactions Promote Disparity in Networks

Masaki Ogura† and Victor M. Preciado‡

†Graduate School of Information Science and Technology, Osaka University
1–5 Yamadaoka, Suita, Osaka 565-0871, Japan
‡Department of Electrical and Systems Engineering, University of Pennsylvania
3401 Walnut Street, Office 410B, Philadelphia, PA 19104, USA

Email: m-ogura@ist.osaka-u.ac.jp, preciado@seas.upenn.edu

Temporal networks [1] represent a realistic and high-resolution model of interactions and are used to study various complex dynamics, including social, biological, and physical systems. An important application of temporal networks is to understand the spread of infectious diseases. It has been shown that the timing of interactions can significantly affect the epidemic threshold. On the other hand, the impact of the temporal variability of interactions on the centrality measures of nodes in the network is poorly understood.

The goal of this study is to investigate if and how time variability affects the centrality of nodes in a network. To this end, we first introduce a Markovian model of temporal networks inspired by edge Markovian networks [2]. Within our model, each edge is activated and deactivated by following one of μ independent Markov processes with activation rate q. These two parameters allow us to systematically study the aforementioned effects of time variability by providing us with several different Markovian temporal networks sharing the same time-aggregated static network.

To quantify the effect of the time-variability on centrality measures of nodes within networks, we propose an eigenvector centrality measure for Markovian temporal networks by extending the one for static networks. For a Markovian temporal network $\mathcal{G} = \{\mathcal{G}(t)\}_{t\geq 0}$ having the adjacency matrices $\{A(t)\}_{t\geq 0}$, we define the *eigenvector centrality* of $\mathcal{G}$ by $v = \lim_{t\to\infty} f(E[x(t)])$, where x denotes the solution of the stochastic differential equation $\dot{x}(t) = A(t)x(t)$ with an appropriate initial condition and f is a normalization operator.

In Figure 1, we illustrate how the eigenvector centrality of temporal networks generated from a single Erdös-Rényi network varies with the two parameters. The first row shows the centralities of the nodes for $q = 1, 0.1$, and 0.01. For smaller q, we can observe that as we increase μ, the top few nodes obtain more centralities, showing the promotion of disparity. This phenomenon is illustrated more clearly in the second column, where we show the relative difference of the centrality measures for the cases of $\mu = 2$, 4, and 6 compared to the case of $\mu = 1$. We confirm a rich-gets-richer phenomenon for the case of smaller q.

References

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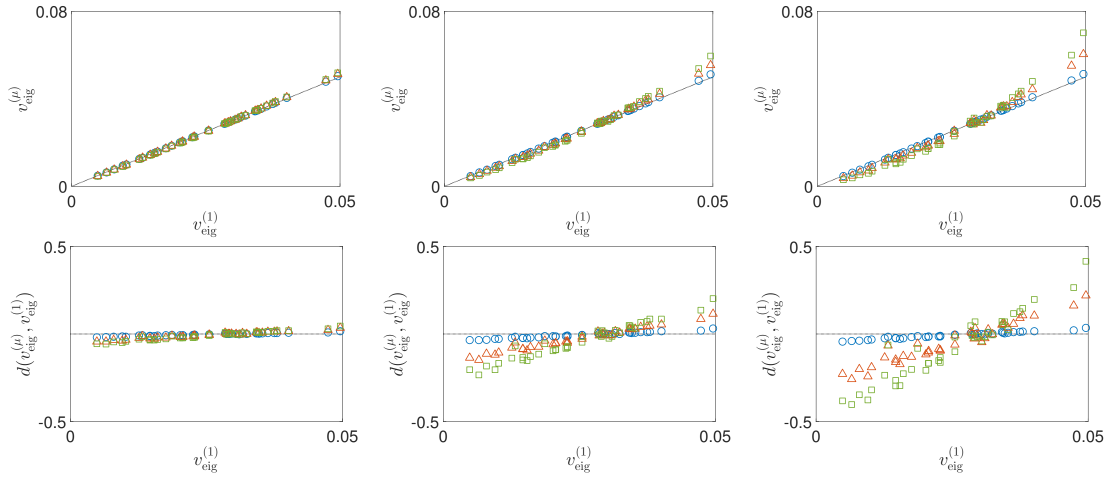

Figure 1: Eigenvector centralities of Erdös-Rényi temporal networks with $q = 1, 0.1$, and 0.01. Circles: $\mu = 2$, Triangles: $\mu = 4$, Squares: $\mu = 6$. In the second row, $d(a, b)$ denotes the relative difference of a from b.

ORCID iDs Masaki Ogura: 0000-0002-3857-3942, Victor M. Preciado: 0000-0001-9998-8730