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A Method for Solving Asymmetric Traveling Salesman Problems Using Neural Networks and Block Shift Operations with Tabu Search

Toshihiro TACHIBANA†* and Masaharu ADACHI†**

†Department of Electronic Engineering, Graduate School of Engineering, Tokyo Denki University
2-2 Kanda–Nishiki–cho, Chiyoda–ku, Tokyo, 101–8457, Japan
Email: * 07gmd09@ms.dendai.ac.jp, ** adachi@eee.dendai.ac.jp

Abstract—In this paper, a method for solving Asymmetric Traveling Salesman Problems (ATSP) is proposed. Where the asymmetric TSP means that the costs for travel between a city and another one are not symmetric. The proposed method uses Hopfield Neural Network and Block shift Operations combined with Tabu rules. The decisions of the cities to be exchanged are made by indices called " Asymmetric Index" focused on symmetry and asymmetry. In addition, this operation does not change the combination of the same cities with certain conditions represented as tabu rules. The proposed method can obtain the exact solution of for a 13-cities problem.

1. Introduction

Traveling Salesman Problem (TSP) is one of the famous problems belongs to the NP hard problems. It is a combinatorial optimization problem which appears quite frequently in various fields such as transit and control [1]. Its combinations number increases by $\frac{(n-1)}{n}!$ as the number of city n increases. It is called combinatorial explosion [1, 2]. In other words, to solve this problem with large scales of the cities, it is difficult to obtain a good solution by simple techniques such as a round robin method[3].

Previously, the costs of TSP were dealt as symmetry because of the low performance of computers to solve it. However, in the everyday various scenes, it is rare that the costs are symmetric. Therefore, there are demands for dealing with asymmetric cost problems [2]. At the present, symmetric TSPs have been studied extensively by many researchers. However, asymmetric TSPs are not considered well.

In this paper, we focus on the asymmetric TSPs and propose a heuristic method to solve them by using neural networks and Block Shift operations with tabu search.

2. Asymmetric Traveling Salesman Problems

TSP is an optimization problem to determine a tour to visit all cities once with a minimum cost as possible [1, 2]. Variables, constants, and a function to be used in this paper are listed as follows.

d_{ij} : Cost to travel from city i to city j ;
$f_{TSP}(\sigma)$: Total cost of a tour ;
σ : The order of cities visiting in a tour ;
V : Set of the cities to visit of a given problem ;
n : Number of the cities of a given problem ;
u_{im} : Internal state of the unit im ;
θ_{im} : Threshold of the unit im ;
x_{im} : Output of the unit im;
$w_{im,jn}$: Synaptic weight from the unit jn to the unit im ;

A set of the cities of a given problem in the case of the n cities is denoted as follows:

$$V = \{v_1, v_2, \cdots, v_n\} \tag{1}$$

Furthermore, we describe the total cost $f(\sigma)$ of a tour σ can be evaluated from a cost $d_{ij}(v_i, v_j \in V)$ for the travel from city j to city i as follows:

$$f_{TSP(\sigma)} = \sum_{k=1}^{n-1} d_{\sigma(k),\sigma(k+1)} + d_{\sigma(n),\sigma(1)} \tag{2}$$

In other words, TSP is a problem to minimize the cost that we showed in the Eq. (2). In the case of asymmetric TSP, the cost of the travel between the city i and the city j becomes $d_{ij} \neq d_{ji}$ and such costs are called asymmetry. In addition, with the asymmetry of the travel cost between cities to be treated in this paper, we do not include the case that two cities are connected to only in one direction.

3. Hopfield Neural Networks

3.1. A brief review of Hopfield Neural Networks

The Hopfield neural network (HNN) was proposed in 1982 by J. J. Hopfield. It is a mutual connection type neural network [4]. Its state can be represented by an energy function that is proposed by an analogy of a physical system. When a unit updates its state the energy function becomes smaller. Then it reaches the local minimum and stops automatically.

3.2. Solution representation of TSP using HNN

In order to solve TSPs using HNN, a solution of the tour of TSP has to be represented as a state of HNN. Therefore we arrange units of HNN in a two-dimensional grid and the output states of the units are represented as a matrix. Each row of the matrix corresponds to a city to visit. The columns of the matrix represent the order of visiting cities [4] (Fig. 1(a)). Where, the values of the elements of the matrix represent the visiting order of a specific city, i.e. , unity and naught represent, visiting and not visiting, respectively. Therefore, a tour can be represented by a state of the HNN as shown in Fig. 1(b) [4] where the filled circles represent firing neurons.

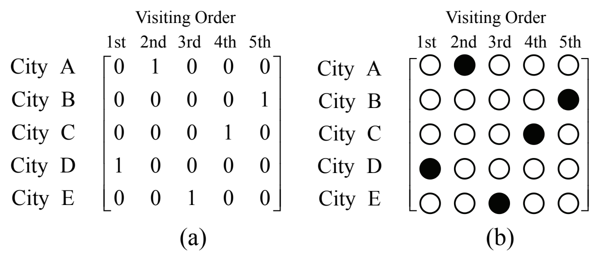

Figure 1: Example of the solution expression of TSP using HNN.

Next, the object function can be determined in order to make the HNN represent feasible solutions of TSP satisfying the following conditions. ‘The agent visits a city only once’, ‘The agent visits only one city at one moment’, and ‘The cost of the tour is as smaller as possible’. The internal state u_{im}, and the output x_{im} of the unit im are represented by Eqs. (3), and (4), respectively.

$$u_{im} = \sum w_{im,jn} x_{jn} + \theta_{im} \quad (3)$$

$$x_{im} = \frac{1}{2}\left(1 + \tanh\left(\frac{u_{im}}{0.5}\right)\right) \quad (4)$$

The connection weights $w_{im,jn}$ among units in HNN to be used in this research are determined by relating the energy function of HNN and the object function of TSP as in Ref. [4].

4. Proposed method

4.1. Overview of the proposed method

We show the flow chart of the present algorithm in Fig. 2. In the present algorithm at first, an index of asymmetry of each city is evaluated by Eq. (5). We define the following two expressions:

$$A_i = \{v_j \mid \text{with cost satisfies } d_{ij} \neq d_{ji}\},$$

$$a_i = \#(A_i) \quad (5)$$

Where # denotes the number of elements in the set. The asymmetric index is calculated with A_i, and a_i. The index

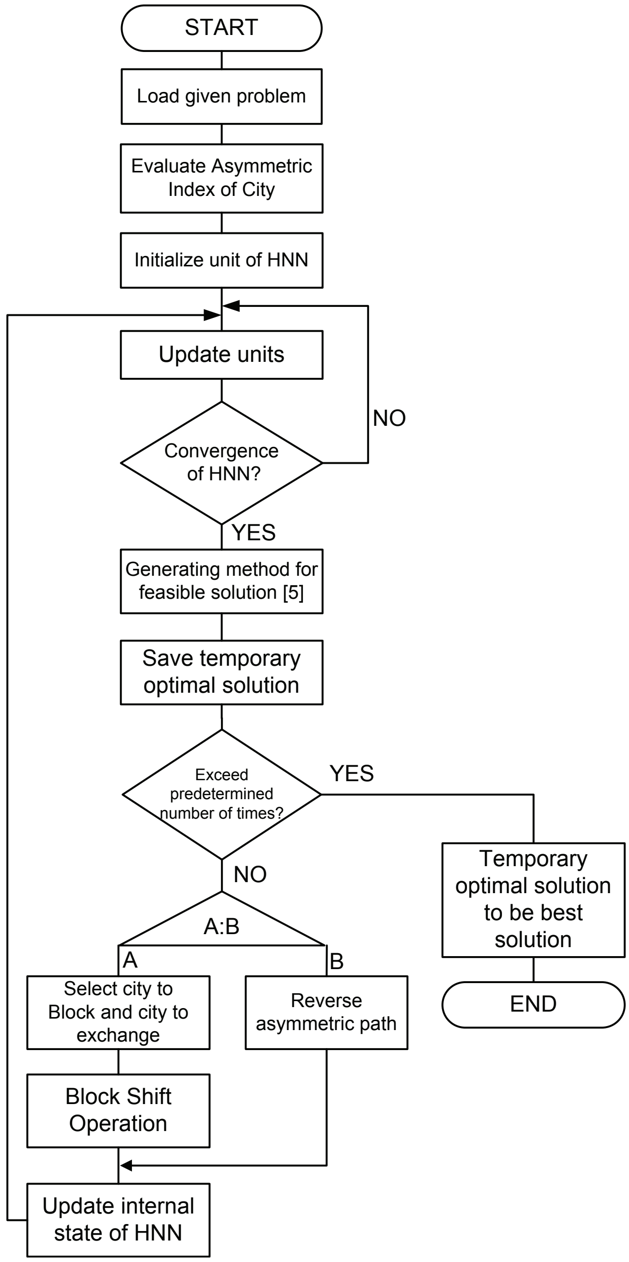

Figure 2: Flow chart of proposed method

is used for determine whether a city is to be included in Block shift operations.

Next, the present algorithm treats a tour obtained by HNN as the initial solution. Then one of the following two procedures is carried out with predetermined ratio for selecting which procedure to be executed.

One procedure is called Block Shift operation. In this procedure a set of cities that are neighboring in the tour is treated as one city which is called a “Block”. Then the Block and other cities become candidate for exchange in the tour. During the exchanging a city and the Block, the tour inside the Block is maintained. The city to be exchanged with the Block is determined according to the value of the asymmetry index. In concrete, the city whose asymmetry index is smallest is exchanged with the Block in the tour. However, this procedure may be trapped to a local minimum or cyclic exchanges. Therefore, a method to escape from such situations called a tabu search is used.

The other procedure is to reverse the direction of a part of the tour for the path whose asymmetry index is highest. This procedure aims to escape from a local minimum which can not be escape with the above mentioned Block Shift operation. In addition, the result of one of the above two procedures is represented as the state of each unit of HNN. Then in the next cycle, the HNN state is updated until it reaches to other local minimum. In the following sections the details of each process are described.

4.2. Asymmetric Index

In the proposed method in this paper, the index of asymmetry is used for improving a tour. When a tour is formed by the HNN, the asymmetric index B_i for the tour is calculated from the index of asymmetry a_i of the city in the tour as Eq. (6).

$$B_i = a_i + a_{i+1} \tag{6}$$

where a_i denotes the asymmetry index of the ith visiting city in the tour.

4.3. Tabu Search

A tabu search is a technique to prevent the network state being trapped at local minimum and periodically repeating the same tour by taking off a city and the branch from candidates of spot exchange for a certain period of time. This proposed method has lists of two independent tabus. One is a tabu applied to the city that is already used for spot exchanges more than the number of times that a user predetermined. This tabu list has a queue structure. Namely, when the number of the cities in the tabu list becomes larger than the size of the list, the oldest city in the list returns to a spot exchange candidate. The Second tabu list is for the Block which is already used for spot exchanges more than a predetermined number of times. In other words, branches included in the set of city are put to the tabu list. This tabu list has a queue structure too. Namely, when the number of the Blocks in the tabu list becomes larger than the size of the list, the oldest Block in the list returns to a spot exchange candidate.

4.4. Block Shift Operation

Symmetry of the costs for traveling between cities back and forth is an important key to a cost reduction, the 2-opt method as the fundamental spot exchange method solving TSP. When a 2-opt operation is executed, a direction of a part of the tour is reversed by the spot exchange. However, the change of the cost with the direction change is never occurred in a symmetric TSP. In the asymmetric TSP, a 2-opt operation may cause cost rise because of the reverse path. As one of the methods to overcome such a disadvantage of the 2-opt method in asymmetric TSP, a Block Shift method was proposed. This method considers certain plural cities to be one city, which is called a Block, and the Block is exchanged its order in the tour with other city. During the exchange, the order of the city in the Block is maintained.

In the present algorithm a section of the tour whose cities are with low a symmetric index is selected as a Block. Furthermore, in the proposed algorithm the city whose asymmetry index is largest is selected as a partner of the spot exchange with the Block. However, if the candidate of spot exchange is in the tabu lists it can not be exchanged. In this case, only the tabu lists are updated.

4.5. Reversing of the asymmetric path

When the present algorithm continues carrying out Block Shift operation many times, the solution of this algorithm may be trapped in local minimum. In such state, we can not obtain a better solution. In order to overcome such problem we introduce an operation called the reversing the asymmetric path.

In the reversing procedure, a city i is chosen at random in the tour. Then the asymmetric index δ which represents the degree of the asymmetry of a path between city i and j as in Eq. (7). When the cost of the path is reduced replacing the visiting order of the cities, the spot exchange is executed. Otherwise, the procedure is terminated.

$$\delta_{ij} = d_{ij} - d_{ji} \tag{7}$$

When the exchange is executed, the corresponding state change is applied to the HNN.

5. Numerical Experiments and Results

In this paper, we modify the benchmark problems of TSPLIB [6] which is a list of benchmark problems of symmetric TSPs to be asymmetric TSPs. The problem that we call ATSP13 in this paper is modified by mixing two problems of eil76 and ftv47 listed in TSPLIB. In the experiments, we performed 30 trials for each problem.

In the experiments, the conventional methods are as follows. The 2-opt method cuts off two branches and connects two new branches. It is a basic city exchange method in Symmetric TSP. The Point Shift method exchanges two cities. Note that, it does not exchange the branches, but exchanges the order of the cities in the tour. In the proposed method, the order of a city and the cities that became a Block in the tour is exchanged. Namely, the proposed method executes the same exchanges as the Block Shift methods. The difference between the proposed and the Block Shift methods is that the proposed one exchanges a tentative solution obtained by the HNN. We show comparisons of the methods in Table 1.

Table 1. Comparisons of the exchange methods.

Method	Exchange
Proposed	order of Block and city
2-opt	two branches
Point Shift	order of two cities
Block Shift	order of Block and city

We evaluate the proposed method and conventional methods for solving symmetric TSP including 2-opt method. The solutions are evaluated by the minimum cost solution, the maximum one, and the mean one for each method. We show the results in Table 2. Proposed method shows the best result as shown in Table 2. From these experiments, we confirmed that the proposed method can achieve the exact solution. Whereas the conventional methods could not achieve the exact solution with our experimental condition. In comparison of mean solutions among the proposed and conventional methods, the solution of the proposed one is again the best among them.

6. Conclusions

In this paper, we proposed a method for solving an Asymmetric TSP using HNN and Block shift Operations combined with tabu rules. As a result, the proposed method achieves the exact solution for 13-cities asymmetric TSP though the conventional methods do not achieve it with a certain experimental condition.

A future problem is to apply the proposed method to lager scale problems than the present one.

Acknowledgments

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Table 2. Experimental Results of Proposed and Conventional Methods for ATSP13.

NAME	PROPOSED METHOD	2-opt	Point Shift	Block Shift		
BLOCK SIZE				n=2	n=3	n=4
MINIMUM	**191.48**	249.20	293.60	261.79	331.97	231.45
MAXIMUM	253.68	321.91	461.80	374.03	480.43	413.64
AVERAGE	232.14	302.08	365.09	328.68	386.21	285.76

EXACT SOLUTION : 191.48

NUMBER OF UPDATING SOLUTION : 5,000