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Stochastic resonance in a unidirectional network of nonlinear oscillators driven by internal noise

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Abstract—Stochastic resonance (SR) is a phenomenon involving noise is used to enable state transitions in a bistable or threshold system driven by subthreshold signals. Recently, SR has been observed in such systems driven by internal noise or fluctuation. In this study, we used a unidirectional circular network with nonlinear oscillators as a potential system to induce SR by using internal noise. These oscillators were connected with four neighboring oscillators that have different connection strengths depending on the distance to a destination oscillator and generated internal noise to induce SR. We observed the classical SR characteristics between the correlation value and internal noise intensity.

1. Introduction

Stochastic resonance (SR) is a phenomenon where dynamical noise is effectively used to induce state transitions in bistable or threshold systems that are driven by subthreshold input signals. Many biological systems process information on the basis of external or thermal noises [1, 2, 3, 4, 5, 6, 7, 8, 9]. Furthermore, arrayed threshold units placed in parallel are known to enhance the correlation value for a wide range of noise intensity levels, thereby exhibiting SR [10].

Recently, a new type of SR that harnesses internal noise has been observed in brain activities [11, 12, 13]. The new SR model has been investigated by developing simulations of neural networks or electric circuits with chaotic internal fluctuation [14, 15]. In this study, to simulate SR by using internal noise, we used a unidirectional circular network, which is based on Collins network, with nonlinear oscillators. These oscillators were connected with four neighboring oscillators that have different coupling strengths depending on their distances to a destination oscillator. The oscillations, which propagate in a single direction and remain in the network, behave as internal noise. Therefore, the propagating oscillations support the detection of subthreshold input signals applied to the network. Based on these simulations, we confirmed typical SR characteristics between a correlation value and internal noise intensity.

This manuscript is organized as follows. Section 2 describes the simulation methods and models. The results of an arrayed FitzHugh-Nagumo (FHN) ring network that has unidirectional coupling are described in Section 3. Section 4 is the summary.

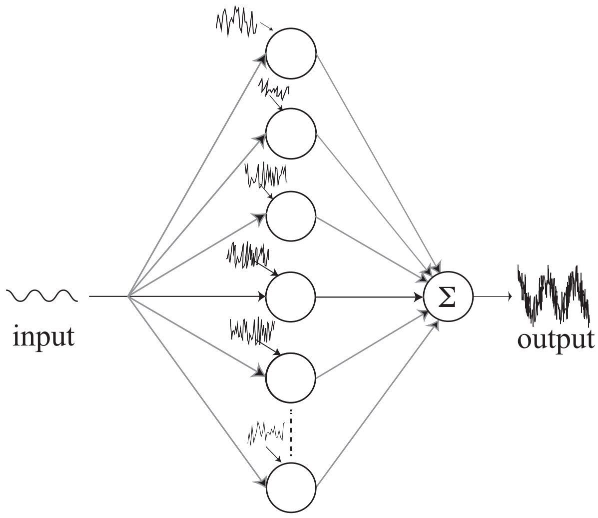

Fig. 1: Collins summing network

2. Methods

In Collins network, which consists of an array of FHN neurons, these neurons need independent external noise to induce SR as shown in the Fig.1. The dynamics of the FHN model is described below

$$\dot{u} = u\,(1-u)\,(u-a) - v + I(t), \quad (1)$$
$$\dot{v} = \epsilon\,(bu - v) \quad (2)$$

where u is the membrane potential, v is a recovery variable, ϵ is a time constant, a and b are system parameters, and $I(t)$ is the input signal.

In this study, we constructed a circular network that is based on Collins network where each neurons follows the FHN model. Figure.2(a) shows a representative destination FHN neuron coupled from four neighboring FHN neurons

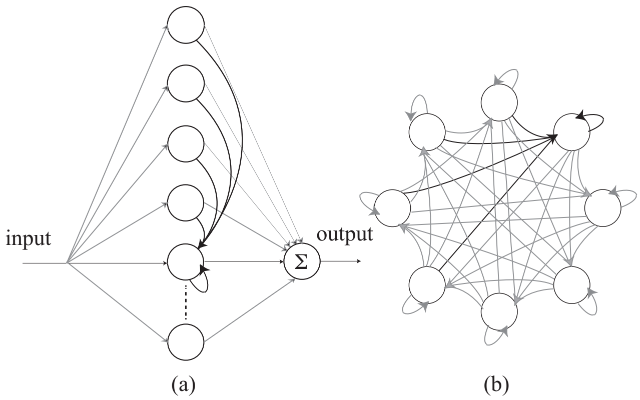

Fig. 2: (a) Proposed circular network that shows one-destination FHN neurons with four unidirectional coupling and auto feedback in one instance, and (b) Ring representation of unidirectional coupling in the proposed circular network. (Only second layer i.e., the input and output neuron are not shown.)

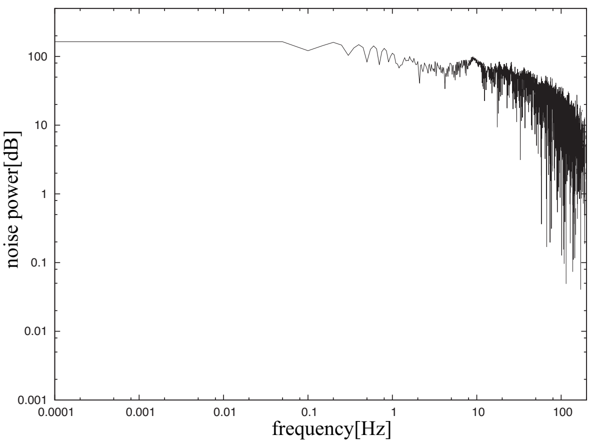

Fig. 3: Internal noise is generated by the summation of output signals of the 500 FHN neuron network driven by an input signal, which is slightly larger than the threshold of the FHN neurons.

and its output. The coupling strengths depending on the distance from the destination neuron Figure 2(b) presents the circular network model of the proposed network when each FHN neuron is unidirectionally connected to four forward neurons.

Whether the propagation of firing waves has a role as internal noise must be confirmed prior to investigating the new SR in the circular network. The parameters of FHN neurons are set to $a = 0.1$ a variation of 5%, $b = 0.24$ a variation of 1%, and $\epsilon = 0.01$ a variation of 3%. When sinusoidal inputs, which are slightly larger than the threshold of FHN neurons, are added to the circular network, we observed that the summation of the network outputs behaves as internal noise as shown in Fig.3. The power spectrum

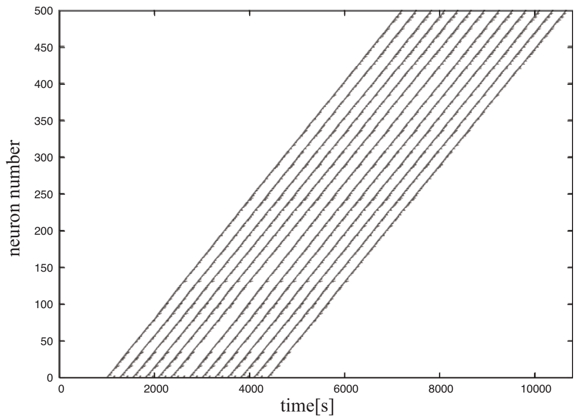

Fig. 4: Increase in the number of propagating firing waves in the unidirectional chain network in which the edge of the chain network is driven by a constant value, which is slightly larger than the threshold of the FHN neurons.

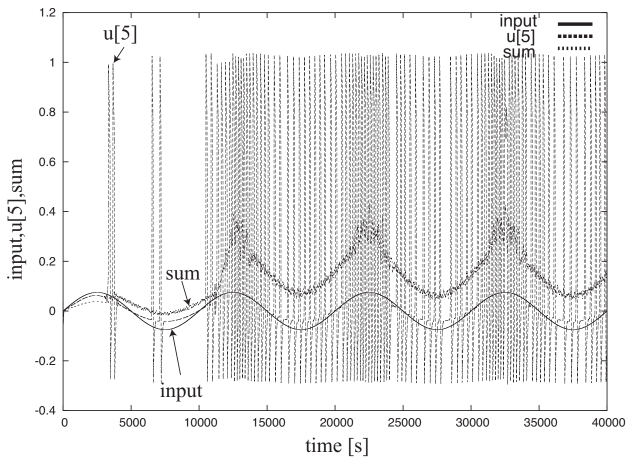

Fig. 5: Time series of subthreshold input signals, the fifth neuron (u[5]), and summation of outputs of the 500 FHN neurons in the network.

was obtained by FFT of the sum of the output signals and indicates that the sum of the network output signals has noise power in a wide bandwidth. In order to determine the number of firing waves in a network, a constant input stimuli is applied, which is slightly larger than the threshold, to an FHN neuron located at the edge of a chain network. As shown in Fig.4, thirteen firing waves simultaneously exist in a chain network consisting of 500 FHN neurons. Subsequently, we suggested that the number of firing neurons that must contribute to generate sufficient internal noise to induce SR.

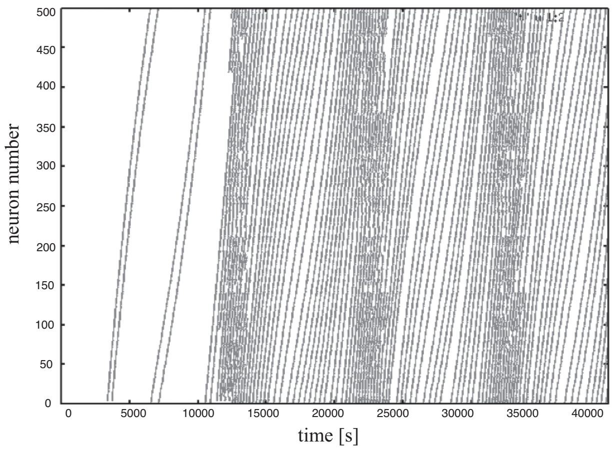

Fig. 6: Raster plot of the 500 FHN neuron network.

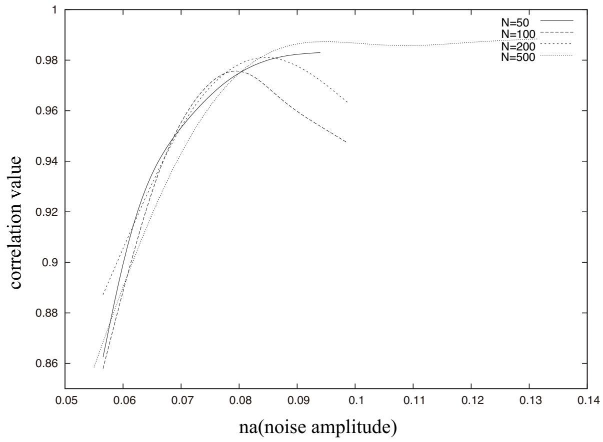

Fig. 7: Correlation value versus noise amplitude ($N = 50, 100, 200, 500$).

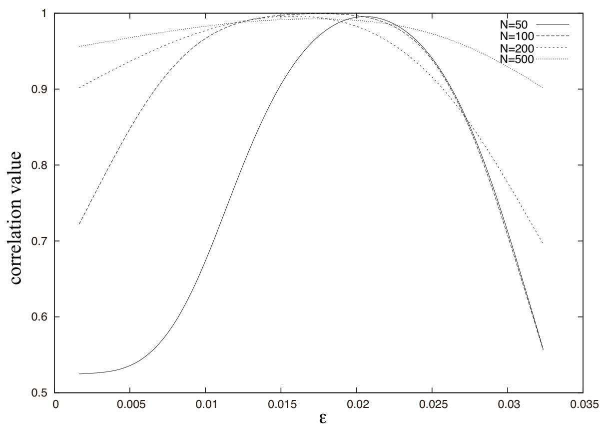

Fig. 8: Correlation value versus time constant ϵ ($N = 50, 100, 200, 500$).

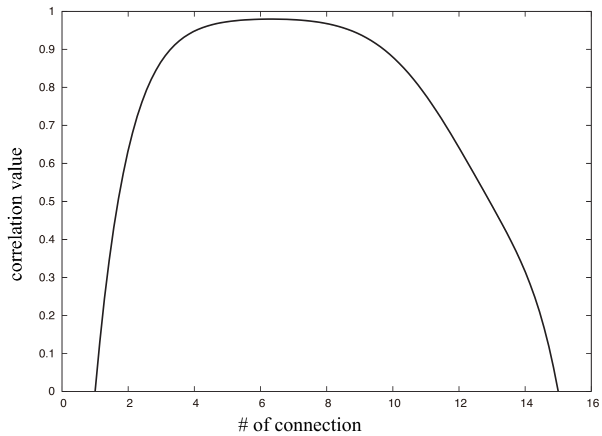

Fig. 9: Correlation value versus the number of coupling ($N = 500$).

3. Results

First, we constructed a circular network consisting of 500 FHN neurons. The dynamics of the FHN neurons in the circular network is described below

$$\dot{u} = u(1-u)(u-a) - v + wI(t) + noise, \quad (3)$$

$$\dot{v} = \epsilon(bu - v) \quad (4)$$

where w is the coupling strength of the input signal and *noise* represents internal noise. The *noise* term is a weighted sum involving coupling FHN neurons and auto feedback, multiplied by a noise amplitude (na). The parameters of FHN neurons are set to $a = 0.1$ with a variation of 5%, $b = 0.24$ with a variation of 1%, $\epsilon = 0.01$ with a variation of 3%, $w = 0.045$ with a variation of 1.8%, $I(t) = 0.075$, and $na = 0.057$.

Figure 5 shows the time series of subthreshold input, the output of the fifth FHN neuron, and summation of outputs of the 500 FHN neurons in the network. When the subthreshold input signals reaches to its maximum value, the firing density of each FHN neuron increased. On the other hand, when the subthreshold input signals reaches to its minimum value, the firing density of each neuron decreased. Therefore, the average of outputs in the network was similar to the input subthreshold signal. Figure 6 presents a raster plot of the FHN neuron network. At first, few waves were generated and propagated. When the input was applied repeatedly, many neurons fired and propagated. Therefore, SR was induced by firing waves as internal noise.

Next, we adjusted the system parameters by changing the number of FHN neurons (N) in the circular network. Subsequently, to measure the performance of the circular network, the correlation value between the input signal ($I(t)$) and the summation of the output signals ($O(t)$) in the network is processed as follows:

$$\text{correlation value} = \frac{\langle I(t) \cdot O(t)\rangle - \langle I(t)\rangle\langle O(t)\rangle}{\sqrt{\langle I(t)^2\rangle - \langle I(t)\rangle^2}\sqrt{\langle O(t)^2\rangle - \langle O(t)\rangle^2}}, \quad (5)$$

$$\langle X(t)\rangle \equiv \frac{1}{T}\int_{t-T}^{t} X(t)dt, \tag{6}$$

where we set T at 250,000. We increased the noise amplitude in some selected networks ($N = 50, 100, 200, 500$) as shown in Fig.7. When the noise amplitude is increased from a low amplitude value, the correlation value is known to increase in any network. When the number of FHN neurons is small ($N = 50, 100, 200$), the correlation value decreases after reaching a peak value. We conclude that the network has a limitation in terms of supplied noise amplitude. Subsequently, the noise amplitude is fixed to $na = 0.08$. Similarly, we increased the time constant ϵ as shown in Fig.8. The size of the network affects the performance when ϵ has small values. Eventually, the following settings of parameters is selected as suitable for further experiments, $na = 0.08$, $\epsilon = 0.02$. We changed the number of coupling FHN neurons at $N = 500$ as shown in Fig.9. When the number of coupling FHN neurons is one, the noise generated by a neuron is not sufficient to fire and propagate. The correlation value is observed to increase with a higher number of coupling neurons. When the number of coupling neurons is larger than twelve, the correlation value decreases. Hence, conventional SR characteristics between the correlation value and internal noise intensity is confirmed.

4. Summary

A circular network of FHN neuron with unidirection and coupling has been constructed. When some neurons are firing, connections assist other neurons to fire as an internal noise source. The network was observed to operate under stochastic resonance regime in total absence of any external noise source. Therefore, the network does not need the external noise source for operation. We obtained the SR curve by changing the amount of noise by increasing the number of coupling neurons. We confirmed that the correlation value increases by optimizing noise.

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