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A Method for Transforming Complex Networks to Time Series Using Classical Multidimensional Scaling

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Abstract—Novel methods have been proposed to analyze nonlinear dynamical systems using the complex network theory. In these methods, time series are transformed to complex networks and the networks are analyzed by the methods based on the complex network theory. In this paper, we propose an opposite direction of these methods: we transform complex networks to time series by using classical multidimensional scaling which is one of the multivariate analysis methods. Then, we analyze transformed time series using the nonlinear time series analysis methods. In numerical simulations, using two mathematical models which can generate regular, random, and small-world networks, we examine the characteristics of the transformed time series from these two network models. We show the structural difference between two small-world networks generated from the two different models. This result indicate that our method can reveal a hidden structural property in the complex networks.

1. Introduction

The complex network theory has emerged in 1998 as a new theory for analyzing various real networks from several fields, for example, biology, sociology, physics, and so on. Recent studies on complex networks have revealed structural features of real networks [1, 2]. Then, various methods for analyzing complex networks have been proposed [1, 2].

On the other hand, the nonlinear dynamical system theory has already established essential philosophy and methodology to analyze complex phenomena in the real world: complicated behavior could be produced from a low dimensional nonlinear dynamical system. Then the nonlinear time series analysis methods based on the nonlinear dynamical system theory have been applied to various complex phenomena observed in the real world.

Recently, novel methods for analyzing the nonlinear dynamical systems using the complex network theory have been proposed [3–7]. In Refs. [3–7], nonlinear time series are transformed to complex networks, and the transformed networks are analyzed by the methods of the complex network theory. These results suggest that analyses of time series through the complex network theory offer a different viewpoint from conventional methods in the nonlinear dynamical theory.

In this sense, we can expect that an opposite direction of these methods [3–7], which transforms complex networks to time series, can also give important but different viewpoints to elucidate hidden properties in complex networks.

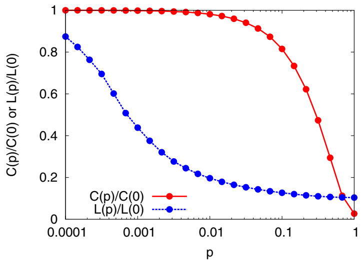
In Ref. [8], we have already proposed a method to transform complex networks to time series using the classical multidimensional scaling (CMDS) [9, 10] and reported preliminary results with the linear spectral analysis. CMDS is one of the multivariate analysis methods. It provides m-dimensional coordinate values from the distance information between each element. Then, we use the adjacency information of two nodes in complex networks as the distance information and obtain m-dimensional coordinate values from the distance information through CMDS. We treat the transformed coordinate values from the complex networks as time series.

In this paper, we use two mathematical models of the small-world network: the Watts and Strogatz (WS) model [11] and the Newman and Watts (NW) model [12]. We first transform these two models into time series and analyze the transformed time series using a nonlinear time series prediction method. We apply the method of analogues [13] to the time series generated from the WS and NW models and investigate the difference between the WS model and the NW model from the viewpoint of nonlinear predictability.

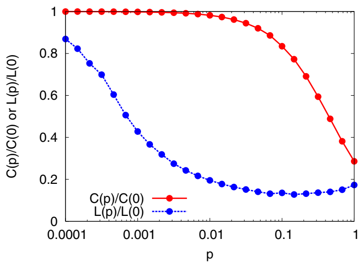
The results show that the transformed time series exhibit different nonlinear predictability. If we use conventional measures in the complex network theory, the clustering coefficient and the characteristic path length, it is shown that a network generated from the WS model and that from the NW model have almost the same structural property. Then, these results indicate that the proposed method could be a possible tool to distinguish a slight structural difference in complex networks which would appear the same tendency only through the complex network theory.

2. Two mathematical small-world network models

In this section, we introduce two mathematical complex network models, the WS [11] and the NW [12] models, and explain their differences. These two mathematical network models can reproduce the small-world property. At the initial state, these two mathematical models are equivalent: namely, they are regular networks with n nodes and k edges. However, the procedure for generating the small-world networks are different. In the WS model, each edge in an initial regular network is randomly rewired with a probability p. On the other hand, in the NW model, new edges are randomly added to an initial regular network with a probability p. Namely, no edges are removed from the initial regular network. Then, the average degree of the NW model becomes $k(1 + p)$. In Fig. 1, we show the clustering coefficient $C(p)$ and the characteristic path length $L(p)$ of the WS model and the NW model for the probability p, where $C(p)$ and $L(p)$ are normalized by $C(0)$ and $L(0)$.



(a)



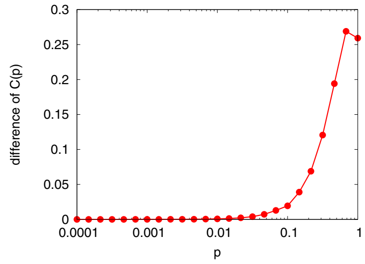
(b)

Figure 1: The results of $C(p)$ and $L(p)$ of (a) the WS model and (b) the NW model for the probability p.

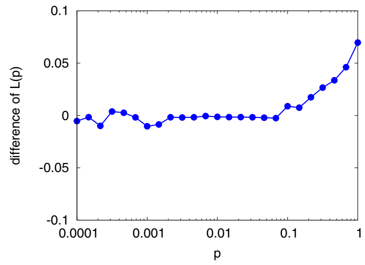
In Fig. 2, we show the difference of $C(p)$ and $L(p)$ between the WS model and the NW model. From Figs. 1 and 2, $C(p)$ and $L(p)$ of the two models take very close values when $p \leq 0.01$. However, the values of $C(p)$ of the WS model decrease when $p > 0.01$ and the difference of $C(p)$ and $L(p)$ between two models increases as p increases. The main reason is that the initial regular network is almost destroyed in the WS model when the probability p approaches unity but completely remains in the NW model.

3. Proposed method

We use the CMDS [9] to realize the transformation from complex networks to time series. If the distances between elements are given, the CMDS determines multi-dimensional coordinates for each element so that the distance information between any two elements in the Euclidean space is preserved. Here, the distance information is given by a matrix form, called a dissimilarity matrix $D = (d_{ij})$. In our method, we produce the dissimilarity matrix D from the adjacency matrix of a complex network, $A = (a_{ij})$, according to the following rules:



(a)



(b)

Figure 2: Difference of (a) $C(p)$ and (b) $L(p)$ between the WS model and the NW model. The abscissas represent the probability p, and the ordinates represent the difference of $C(p)$ and $L(p)$.

$$d_{ij} = \begin{cases} 0 & (i = j) \\ w & (a_{ij} = 0,\ i \neq j) \\ 1 & (a_{ij} = 1), \end{cases} \quad (1)$$

where w is a weight of distance between each pair of disconnected node and must be larger than 1. Applying the CMDS to the dissimilarity matrix D generated by the rules in Eq. (1), we can obtain the m-dimensional coordinate values of each node which preserve the adjacency information of the original complex network. In CMDS, the m-dimensional coordinate values are given as m eigenvectors of the matrix $-\frac{1}{2}JD^{(2)}J$, where J is the centering matrix and defined by $J = I_n - n^{-1}\mathbf{1}_n\mathbf{1}_n^{\top}$, I_n is the identity matrix of size n, $\mathbf{1}_n$ is a vector of n ones, and $D^{(2)} = (d_{ij}^2)$. After the above procedure, we regard each obtained coordinate value as a time series. Then we analyze the transformed time series using the nonlinear time series prediction.

In this paper, we use the method of analogues [13] to evaluate transformed time series from complex networks. After predicting time series using the method of analogues, we evaluate nonlinear prediction accuracies [14] to distinguish the transformed time series.

4. Experiments

At first, as the initial network for the WS and NW models [11, 12], we generate the regular network in which the number of nodes is 1,000 and each node has 10 edges. Next, we generate three types of networks from the WS and NW models with the probability $p = 0.0, 0.1$, and 1.0. Then, we apply our proposed method to these three networks and analyze using the nonlinear prediction [13]. In the following experiments, we use the first half of the time series as a database to construct a predictor. We use the eigenvector corresponding to the 10-th eigenvalue as the time series because the time series has a sufficient number of period and the 10-th eigenvalue has a high contributing rate.

Next, we apply the nonlinear prediction to the obtained time series to precisely quantify the time series. In this experiment, we generate three networks from the WS and NW models with the probability $p = 0.0001, 0.001$, and 0.01. Then, we examine whether the structural difference between two different models are distinguishable or not, because C and L of the two models are very similar and indistinguishable with these probabilities.

5. Result

In Fig. 3, we show the time series transformed from the networks generated from the WS and NW models. From

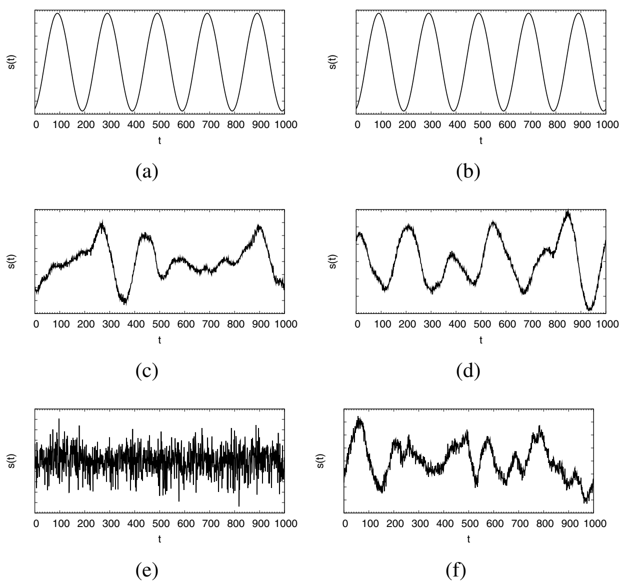


Figure 3: Time series obtained from the WS model with the probability $p =$ (a) 0.0, (c) 0.1, and (e) 1.0. Time series obtained from the NW model with the probability $p =$ (b) 0.0, (d) 0.1, and (f) 1.0.

Fig. 3, the time series transformed from the regular networks (Figs. 3(a) and (b)) show periodic behavior and those from the random networks (Fig. 3(e)) show random behavior. In addition, the time series transformed from the small-world networks exhibit noisy periodicity. The time series becomes noisy depending on the probability p. If the probability increases, the strength of noise becomes strong. Comparing the time series of the WS model with the time series of the NW model, we can confirm that the time series of the WS model are more noisy than the time series of the NW model. The reason comes from the fact that the NW model includes a regular structure completely.

Next, we show the results of nonlinear predictions for the time series transformed from the networks of the WS and NW models by using various values of probability p. In Fig. 4, we show the results for the time series transformed from the networks with $p = 0.0, 0.1$, and 1.0.

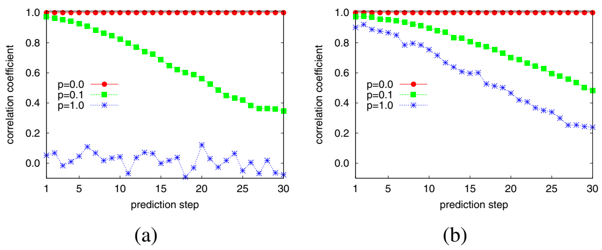


Figure 4: Correlation coefficients r between original time series and predicted time series of (a) the WS model and (b) the NW model shown in Fig. 3. The vertical and horizontal axes represent the correlation coefficient and the prediction step, respectively.

In Fig. 4 (a), it is clearly shown the time series of the WS model shows a phase shift from periodic to random phase as the increase of the probability p. However, different from the WS model, the results of nonlinear prediction for the NW model show that the time series of $p = 1.0$ shows periodic behavior. We think that he difference of prediction results between these two models comes from the difference of production process in these models, as the rewiring and addition of edges.

Finally, we show the prediction results for the WS and NW models with $p = 0.0001, 0.001$, and 0.01 (Fig. 5). As previously mentioned, although networks of the two models are almost indistinguishable by the measures, C and L (see Figs. 1 and 2), the network structures of these two models are different each other. In Fig. 5, we show the prediction results of the time series generated from these networks, where the prediction results are averaged over 50 simulations.

From the prediction results of the NW model, it appears that the prediction accuracy is better than that of the WS model. This reason might come from the fact that the networks generated from the NW model preserve the structure of the initial regular network. These results indicate that even if a structure of two networks is very similar from the viewpoint of the conventional measures, the clustering coefficient C and the characteristic path length L,

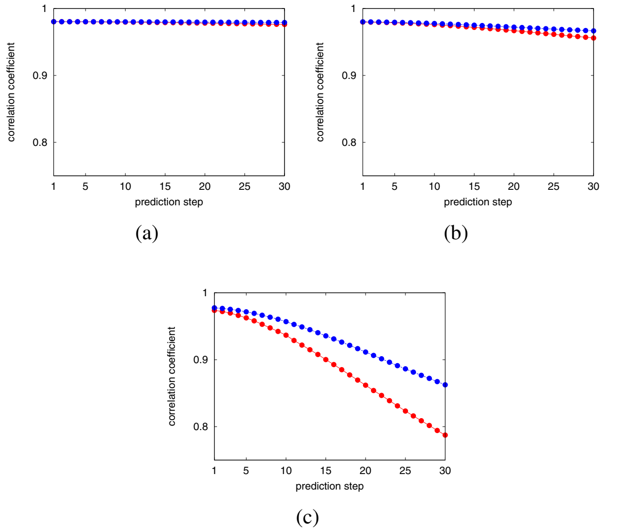


Figure 5: Prediction results for the time series obtained from the WS model and the NW model with different probabilities: (a) $p = 0.0001$, (b) $p = 0.001$, and (c) $p = 0.01$. In each plot, red circles are the results for the WS model and blues ones for the NW model.

our method could distinguish the small structural difference among these networks.

6. Conclusion

We proposed a method in which a complex network is transformed to a time series. We applied the method to two mathematical models; the WS model and the NW model. As a result, the time series transformed from regular networks show periodic property. In addition, our method transforms the network generated with the edge-rewiring or edge-adding processes into noisy periodic or random time series.

Next, we applied the nonlinear prediction method to the transformed time series. The prediction results indicated that the time series transformed from regular networks are periodic and those from random networks are random. On the other hand, the time series generated from neither regular nor random networks are noisy periodic. Even if the networks have very similar C and L, we can predict which model is the origin of the time series. From these results, the time series obtained by our method well reflects original network structure even if the difference between two networks is very small.

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