New Idea of the Pseudo-Inverse Maps in Optimal Pre-Correction of Nonlinear Systems as the Result of Modeling and Optimal Past-Correction

Grzegorz Ciesielski† and Paulina Sobanska‡

†Division of Theoretical Electrotechnics, Institute of Theoretical Electrotechnics, Metrology and Materials Sciences, Technical University of Lodz, Ul. B. Stefanowskiego 18/22, 90-924 Lodz, Poland
‡INWAT Ltd., Research and Design Company, Ul. H. Sienkiewicza 101/109, 90-301 Lodz, Poland
Emails: gregulus@p.lodz.pl, gregulus@toya.net.pl or gregulusus@gmail.com and pola.sob@gmail.com

Abstract – This paper presents the new idea of the pseudo-inverse maps applied to the optimal pre-corrections of nonlinear systems. This concept is a result of search for optimal models and optimal past-correctors of nonlinear systems from perspective of the Functional Theory of Nonlinear Systems which is discussed in this article. Considered systems are multidimensional, all of input and output signals are real or complex valued and their sets are finally equipped with the structure of the Hilbert spaces. All maps used in this paper are functions for the static systems, convolutions for the linear time-invariant systems, and nonlinear operators for the nonlinear systems. It is shown, that the nonlinear system past- and pre-corrections can be reduced to the modeling tasks of some systems which can be reduced further to the generalized least mean square (LMS) approximations.

1. Introduction

The beginning of the Theory of Nonlinear Systems dates back to 1887 when Vito Volterra announced Theory of his Series which was developed in the course of subsequent research. The Volterra Series, although inconvenient from the numerical point of view, play the role in the Theory of Nonlinear Systems as significant as Convolutions in Modeling of Time-Invariant Linear Systems. Another giant step in the Theory of Nonlinear Systems took place around 1920, when Norbert Wiener, the creator of Cybernetics, conducted research on Orthogonal Operators with random inputs, nowadays known as Wiener Operators. These operators are magnificent from the numerical point of view due to their orthogonality. Later, since about 1962, the significant progress in the Theory of Nonlinear Systems has been made by Martin Schetzen, the brilliant follower of Wiener (M. Schetzen [9]). In 1970s an equally great contribution to the Theory of Nonlinear Systems was made by Russians K. A. Pupkov, W. I. Kapalin and A. S. Jushtchenko (K. A. Pupkov *et al.* [8]). Afterwards outstanding papers by Irvin W. Sandberg, Rudi J. P. de Figueiredo *et al.* appeared. One of the authors of this article has dealt with past-correction of static systems since 1977, initially in a purely electronic way and then numerically with the use of the Splines. His further work led him eventually to the Theory of Nonlinear Systems in which he applied methods resulting from advanced Functional Analysis, Nonlinear Analysis, Theory of Lebesgue Measure and Integral, Algebra and Topology, although he is not a mathematician by education. In 1994 this work led him to the Functional Theory of Nonlinear Systems (G. Ciesielski [4]), which is a generalization of the classical approach used in the Theory of Nonlinear Systems. This uniform generalized Theory shows among other things that there is no need to treat Volterra and Wiener Theories of Nonlinear Systems as two separate theories as wrote Martin Schetzen in his book. The second author of this article has recently begun her scientific experience, starting with a successful application of neural networks to modeling of highly complex real systems, such as steam turbines (P. Sobanska, P. Szczepaniak [10]).

2. Optimal Modeling

Let us assume further in the whole of article that $\boldsymbol{F}$ is the real or complex number field, $\boldsymbol{U_F}$ is the normed space over the number field $\boldsymbol{F}$ of some maps with the norm $\|\cdot\|_{\boldsymbol{U}}$ and $\upsilon_{[k]} := (\upsilon_i \in \boldsymbol{U_F})_1^k$ is a system of some maps from $\boldsymbol{U_F}$. For the given *modeled system* described by the map $\phi \in \boldsymbol{U_F}$ we search for its *model* $\phi_k \in \operatorname{span} \upsilon_{[k]} \subset \boldsymbol{U_F}$ as the linear combination of elements of the system $\upsilon_{[k]}$ not necessarily linearly independent, so $\phi_k = c_{[k]}^T \upsilon_{[k]} := \sum_{i=1}^k c_i \upsilon_i$. Let on $\boldsymbol{U_F}$ the functional $\sigma_{mod\ \operatorname{span} \upsilon_{[k]}}(\phi) := \min_{\gamma \in \operatorname{span} \upsilon_{[k]}} \|\phi - \gamma\|_{\boldsymbol{U}}$. The coefficients $c_{[k]}$ of the *optimal model* ϕ_k should satisfy the equation: $\|\phi - \phi_k\|_{\boldsymbol{U}} = \sigma_{mod\ \operatorname{span} \upsilon_{[k]}}(\phi)$. As we can see now, this modeling task leads us to the problem of approximation of the system ϕ by the linear combination of elements of the system $\upsilon_{[k]}$ what can be very easily computed.

3. Optimal Past-Correction

Let us assume further in the whole of article that $\boldsymbol{V_F}$ and $\boldsymbol{W_F}$ are the normed spaces of some maps with the norms $\|\cdot\|_{\boldsymbol{V}}$ and $\|\cdot\|_{\boldsymbol{W}}$ adequately. These spaces satisfy the condition: $\forall u \in \boldsymbol{U_F}\ \forall v \in \boldsymbol{V_F}\ (v \circ u \in \boldsymbol{W_F})$. Let $\nu_{[k]} := (\nu_i \in \boldsymbol{V_F})_1^k$ is a system of some maps from $\boldsymbol{V_F}$. For the given *corrected system* $\phi \in \boldsymbol{U_F}$ and *required system* $\eta \in \boldsymbol{W_F}$ we search for the *past-corrector* $\gamma_k \in \operatorname{span} \nu_{[k]} \subset \boldsymbol{V_F}$ as the linear combination of elements of the system $\nu_{[k]}$ not necessarily linearly independent, so $\gamma_k = c_{[k]}^T \nu_{[k]} := \sum_{i=1}^k c_i \nu_i$. Similarly as previously, let the functional $\sigma_{past\ \mathrm{span}\, \nu_{[k]}}(\phi,\eta) := \min_{\gamma\in\mathrm{span}\, \nu_{[k]}} \|\eta-\gamma\circ\phi\|_{\boldsymbol{W}}$ on $\boldsymbol{U_F}\times\boldsymbol{W_F}$. The coefficients $c_{[k]}$ of the *optimal past-corrector* γ_k satisfy the equation: $\|\eta-\gamma_k\circ\phi\|_{\boldsymbol{W}} = \sigma_{past\ \mathrm{span}\, \nu_{[k]}}(\phi,\eta)$.

This is the *unconstructive form of determination of the past-correction of the system* ϕ by the linear combination of the maps $\nu_{[k]}$ for the given required system η. The unconstructiveness of this form arises from the fact that there is no well known mathematical method of solving this problem. But after a bit of consideration, we can notice that this approach can be rearranged to the *constructive form of determination of the past-correction of the system* ϕ. This equivalent constructive form consists in the modeling of the required system $\eta\in\boldsymbol{W_F}$ by the linear combination of the maps $\omega_{[k]} := (\omega_i\in\boldsymbol{W_F})_1^k := (\nu_i\circ\phi\in\boldsymbol{W_F})_1^k$. Therefore, for the given required system η we search for its model $\eta_k\in\mathrm{span}\,\omega_{[k]}$ as the linear combination of elements of system $\omega_{[k]}$, perhaps linearly dependent, so $\eta_k = c_{[k]}^T\omega_{[k]} := \sum_{i=1}^k c_i\omega_i$. The coefficients $c_{[k]}$ of the optimal past-corrector γ_k are the same as for the optimal model η_k of the required system η and satisfy equation: $\|\eta-\eta_k\|_{\boldsymbol{W}} = \sigma_{mod\ \mathrm{span}\,\omega_{[k]}}(\eta)$. Obtained proceeding can be easily computed.

4. Pseudo-Inverse System

Let us assume that the *identity map* $\mathrm{id}\in\boldsymbol{W_F}$ or some its approximation $\widetilde{\mathrm{id}}\in\boldsymbol{W_F}$ which is chosen by us. We can notice from the optimal past-correction task that if $\eta=\mathrm{id}$ in the exact case we get $\gamma_k=\phi^{-1}$, if inverse map ϕ^{-1} exists, because as we well know $\phi^{-1}\circ\phi=\mathrm{id}$. From mathematical analysis we know that *the inverse map* ϕ^{-1} *exists if and only if the map* ϕ *is a bijection.* This theorem limits us to such a simple approach to find inverse system ϕ^{-1} in the exact case. But, if we make approach similar to the optimal past-correction task previously discussed we can obtain the pseudo-inverse system ϕ^+. So, let us define on $\boldsymbol{U_F}$ the functional $\sigma_{inv\,\boldsymbol{V_F}}(\phi) := \sigma_{past\,\boldsymbol{V_F}}(\phi,\mathrm{id})$. The system denoted by ϕ^+ which satisfies the equation: $\|\mathrm{id}-\phi^+\circ\phi\|_{\boldsymbol{W}} = \sigma_{inv\,\boldsymbol{V_F}}(\phi)$ we will call the *pseudo-inverse system for the system* ϕ *in* $\boldsymbol{V_F}$. We will denote the compound map $\phi^+\circ\phi$ as $\widetilde{\mathrm{id}}$.

In approximate approach, for the given *system* $\phi\in\boldsymbol{U_F}$ we search for the *pseudo-inverse system* $\phi_k^+\in\mathrm{span}\,\nu_{[k]}$ as the linear combination of elements of system $\nu_{[k]}$ not necessarily linearly independent, so $\phi_k^+ = c_{[k]}^T\nu_{[k]} := \sum_{i=1}^k c_i\nu_i$. The coefficients $c_{[k]}$ of the *optimal pseudo-inverse system* ϕ_k^+ satisfy equation: $\|\mathrm{id}-\phi_k^+\circ\phi\|_{\boldsymbol{W}} = \sigma_{inv\ \mathrm{span}\,\nu_{[k]}}(\phi)$. Let us denote the compound map $\phi_k^+\circ\phi$ as $\mathrm{id}_k = c_{[k]}^T\omega_{[k]} := \sum_{i=1}^k c_i\omega_i$. Similarly as previously, the coefficients $c_{[k]}$ of the optimal pseudo-inverse system ϕ_k^+ are the same as for the optimal model id_k of the identity map id and should satisfy equation: $\|\mathrm{id}-\mathrm{id}_k\|_{\boldsymbol{W}} = \sigma_{mod\ \mathrm{span}\,\omega_{[k]}}(\mathrm{id})$. Obtained proceeding can be easily computed.

<u>This result is especially interesting *e.g.* from the electrical circuit diagnostics or metrology perspective.</u>

5. Optimal Pre-Correction

For the given *corrected system* $\phi\in\boldsymbol{V_F}$ and *required system* $\eta\in\boldsymbol{W_F}$ we search for the *pre-corrector* $\gamma_k\in\mathrm{span}\,\upsilon_{[k]}\subset\boldsymbol{U_F}$ as the linear combination of elements of system $\upsilon_{[k]} := (\upsilon_i)_1^k$ not necessarily linearly independent, so $\gamma_k = c_{[k]}^T\upsilon_{[k]} := \sum_{i=1}^k c_i\upsilon_i$. Let the functional $\sigma_{pre\ \mathrm{span}\,\upsilon_{[k]}}(\phi,\eta) := \min_{\gamma\in\mathrm{span}\,\upsilon_{[k]}}\|\eta-\phi\circ\gamma\|_{\boldsymbol{W}}$ on $\boldsymbol{V_F}\times\boldsymbol{W_F}$. The coefficients $c_{[k]}$ of the *optimal pre-corrector* γ_k satisfy the equation:

$$\|\eta-\phi\circ\gamma_k\|_{\boldsymbol{W}} = \sigma_{pre\ \mathrm{span}\,\upsilon_{[k]}}(\phi,\eta)\,. \qquad (*)$$

We obtain the *unconstructive form of determination of the pre-correction of the system* ϕ by the linear combination of elements of system $\upsilon_{[k]}$ for the given required system η. The unconstructiveness of this form arises from the fact that there is no well known mathematical method of solving this difficult problem. After due consideration, we can notice that this approach can be rearranged to the *constructive form of determination of the pre-correction of the system* ϕ. From the left side of the equation (*) we have $\|\phi^+\circ\eta-\phi^+\circ\phi\circ\gamma_k\|_{\boldsymbol{U}} \cong \|\phi^+\circ\eta-\gamma_k\|_{\boldsymbol{U}}$. The system $\phi^+\circ\phi =: \widetilde{\mathrm{id}}$ is some approximation of the identity map id. If this approximation is good then we can assume that $\widetilde{\mathrm{id}}\cong\mathrm{id}$. So, the coefficients $c_{[k]}$ of the optimal pre-corrector γ_k are approximately the same as for the optimal model γ_k of the system $\phi^+\circ\eta$ which satisfy the equation: $\|\phi^+\circ\eta-\gamma_k\|_{\boldsymbol{U}} = \sigma_{mod\ \mathrm{span}\,\upsilon_{[k]}}(\phi^+\circ\eta)$. Obtained proceeding can be relatively easily computed.

<u>This result is especially interesting from *e.g.* the nonlinear control perspective.</u>

6. Generalized Theorem on the LMS Approximation

One of the main problems which came out here is the linear dependence of the sets $\upsilon_{[k]}$, $\nu_{[k]}$ or especially $\omega_{[k]}$ what can arise in general case. Let us denote for any set $\upsilon_{[k]}$ by $G(\upsilon_{[k]}) := (\langle\upsilon_i,\upsilon_j\rangle)_{i,j=1}^k$ the *Gramian matrix* and for any matrix A by A^+ let us denote the *pseudo-inverse matrix of Moore-Penrose* (F. R. Gantmacher [5], p. 33 and J. Stoer, R. Bulirsch [11], p. 220). Now, the remedy for this problem is the following theorem.

GENERALIZED THEOREM ON THE LMS APPROXIMATION. *If* $\boldsymbol{U_F}$ *is the unitary space with the norm* $\|\cdot\|$ *determined by the inner product* $\langle\cdot,\cdot\rangle$ *as* $\|\cdot\| := \sqrt{\langle\cdot,\cdot\rangle}$, $\upsilon_{[k]} := (\upsilon_i)_1^k$ *is some set of elements from* $\boldsymbol{U_F}$ *, such that* $\dim\mathrm{span}\,\upsilon_{[k]}\le k$ *, and* $c_{[k]} := G^+(\upsilon_{[k]})\langle\upsilon_{[k]},u\rangle := G^+(\upsilon_{[k]})(\upsilon_i,u)_1^k$*, then there exists exactly one LMS approximation* $v_k := c_{[k]}^*\upsilon_{[k]}$ *of element* $u\in\boldsymbol{U_F}$ *in the linear space* $\mathrm{span}\,\upsilon_{[k]}$ *and the LMS error is* $\|u-v_k\| = \sqrt{\|u\|^2-\|v_k\|^2}$.

P r o o f. The existence of the exactly one element v_k results from the fact that for each matrix exactly one pseudo-inverse matrix exists (F. R. Gantmacher [5], p. 32). If $\dim \operatorname{span} v_{[k]} = 0$ then of course $v_k = 0$. So, let us consider the case when $\dim \operatorname{span} v_{[k]} > 0$. From well known features of the LMS approximation, it will be enough to show that $(u - v_k) \perp \operatorname{span} v_{[k]}$, which means that $\langle u - v_k, v_{[k]} \rangle = 0 \in \boldsymbol{F}^k$. To do this, let us notice that

$$\begin{aligned}\langle u - v_k, v_{[k]} \rangle &= \langle u - c_{[k]}^* v_{[k]}, v_{[k]} \rangle = \\ &= \langle u - \left(G^+(v_{[k]})\langle v_{[k]}, u\rangle\right)^* v_{[k]}, v_{[k]} \rangle = \\ &= \langle u, v_{[k]} \rangle - \langle \left(G^+(v_{[k]})\langle v_{[k]}, u\rangle\right)^* v_{[k]}, v_{[k]} \rangle = \\ &= \langle u, v_{[k]} \rangle - \langle \left(G^+(v_{[k]})\langle v_{[k]}, u\rangle\right)^* v_{[k]}, v_i \rangle_1^k = \\ &= \langle u, v_{[k]} \rangle - (v_j, v_i)_{i,j=1}^{k} \overline{G^+(v_{[k]})\langle v_{[k]}, u\rangle} = \\ &= \langle u, v_{[k]} \rangle - G^T(v_{[k]}) \overline{G^+(v_{[k]})\langle v_{[k]}, u\rangle} = \\ &= \langle u, v_{[k]} \rangle - \overline{G(v_{[k]})}\, \overline{G^+(v_{[k]})\langle v_{[k]}, u\rangle} = \\ &= \langle u, v_{[k]} \rangle - \overline{G(v_{[k]}) G^+(v_{[k]})}\langle u, v_{[k]} \rangle .\end{aligned}$$

As we know, *the Gramian matrix $G(v_{[k]})$ is invertible if and only if the system $v_{[k]}$ is linearly independent* (V. A. Ilyin, E. G. Poznyak [6], p. 216), but the theorem assumptions do not make this certain, so we have $l \coloneqq \dim \operatorname{span} v_{[k]} \le k$. Then, we can choose such elements of the set $v_{[k]}$ which form the base $\omega_{[l]} \coloneqq (\omega_i)_1^l$ of the linear space span $v_{[k]}$. Let $A \in \boldsymbol{M}_{\boldsymbol{F}k\times l}$ be such matrix that $v_{[k]} = A\omega_{[l]}$. Let us denote $\Gamma \coloneqq (A^*)^+ G^{-1}(\omega_{[l]})A^+$, then from fundamental features of the pseudo-inverse matrices (J. Stoer, R. Bulirsch [11], p. 220) we get

$$\begin{aligned}G(v_{[k]})\Gamma &= AG(\omega_{[l]})A^*(A^*)^+ G^{-1}(\omega_{[l]})A^+ = \\ &= AG(\omega_{[l]})A^+ A G^{-1}(\omega_{[l]})A^+ .\end{aligned}$$

Because the rank of the matrix A denoted by $\operatorname{rk} A = l$, then let $B \in \boldsymbol{M}_{\boldsymbol{F}k\times l}$ and $C \in \boldsymbol{M}_{\boldsymbol{F}l\times l}$ be decomposition matrices of the matrix A such that rank $\operatorname{rk} A = \operatorname{rk} B = \operatorname{rk} C$ and $A = BC$. It can be proved (F. R. Gantmacher [5], p. 33) that $A^+ = C^*(CC^*)^{-1}(B^*B)^{-1}B^*$ and from here, assuming $B = A$ and $C = I_{[l,l]}$, we get

$$\begin{aligned}A^+A &= I_{[l,l]}^*\left(I_{[l,l]}I_{[l,l]}^*\right)^{-1}(A^*A)^{-1}A^*A = \\ &= (A^*A)^{-1}A^*A = I_{[l,l]} ,\end{aligned}$$

which means that the product

$$G(v_{[k]})\Gamma = AG(\omega_{[l]})I_{[l,l]}G^{-1}(\omega_{[l]})A^+ = AA^+ .$$

Presented relations and the pseudo-inverse matrices features (J. Stoer, R. Bulirsch [11], p. 220) make the following conditions be fulfilled:

$$\begin{aligned}G(v_{[k]})\Gamma G(v_{[k]}) &= AA^+AG(\omega_{[l]})A^* = \\ &= AG(\omega_{[l]})A^* = G(v_{[k]}) , \end{aligned} \quad (1)$$

$$\begin{aligned}\Gamma G(v_{[k]})\Gamma &= (A^*)^+G^{-1}(\omega_{[l]})A^+AA^+ = \\ &= (A^*)^+G^{-1}(\omega_{[l]})A^+ = \Gamma, \end{aligned} \quad (2)$$

$$\begin{aligned}\Gamma G(v_{[k]}) &= (A^*)^+G^{-1}(\omega_{[l]})A^+AG(\omega_{[l]})A^* = \\ &= (AA^+)^* = AA^+ = \left(\Gamma G(v_{[k]})\right)^* \text{ and} \end{aligned} \quad (3)$$

$$G(v_{[k]})\Gamma = AA^+ = (AA^+)^* = \left(\Gamma G(v_{[k]})\right)^* . \quad (4)$$

It lets us conclude (J. Stoer, R. Bulirsch [11], p. 221) that $\Gamma = G^+(v_{[k]})$. Next, from the inner product features, we get $G(v_{[k]}) = G(A\omega_{[l]}) = AG(\omega_{[l]})A^*$ and $\langle u, v_{[k]} \rangle = \overline{\langle v_{[k]}, u \rangle} = \overline{A\langle \omega_{[l]}, u \rangle} = \bar{A}\langle u, \omega_{[l]} \rangle$. Therefore, the considered inner product

$$\begin{aligned}\langle u - v_k, v_{[k]} \rangle &= \langle u, v_{[k]} \rangle - \overline{G(v_{[k]})G^+(v_{[k]})}\langle u, v_{[k]} \rangle = \\ = \langle u, v_{[k]} \rangle - \overline{AA^+}\langle u, v_{[k]} \rangle &= \langle u, v_{[k]} \rangle - \overline{AA^+}\,\bar{A}\langle u, \omega_{[l]} \rangle = \\ = \langle u, v_{[k]} \rangle - \overline{AA^+A}\langle u, \omega_{[l]} \rangle &= \langle u, v_{[k]} \rangle - \bar{A}\langle u, \omega_{[l]} \rangle = \\ &= \langle u, v_{[k]} \rangle - \langle u, v_{[k]} \rangle = 0 \in \boldsymbol{F}^k .\end{aligned}$$

Finally, in the LMS error formula we can notice the Generalized Pythagoras Theorem what ends the proof. ■

This is a very strong and novel theorem on the *LMS approximation*.

7. Obtained Results

Some results obtained by presented theory for the linear time-invariant systems (R. Wojciechowski [12]) are shown in fig. 1, for the static system (A. Albrecht [1]) are shown in fig. 2 and for the nonlinear time-invariant system (G. Ciesielski [4]) are shown in fig. 3.

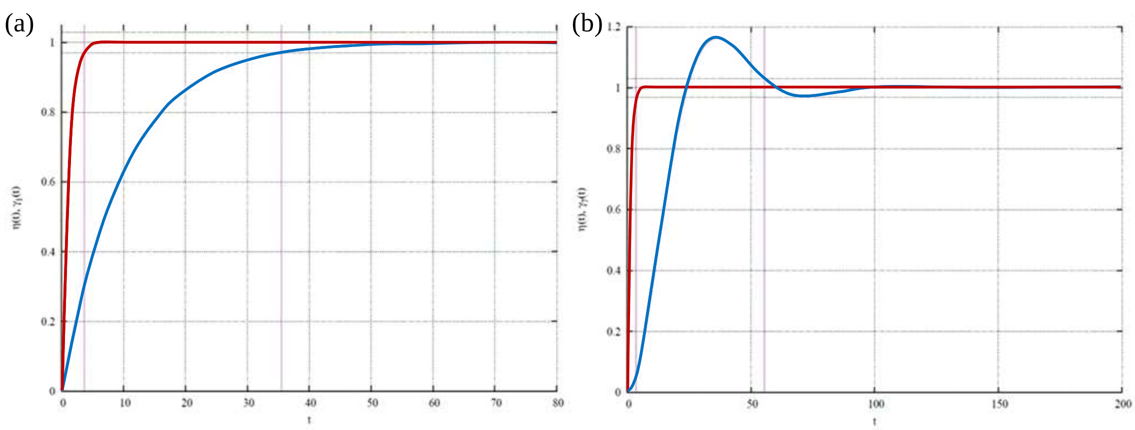

Fig. 1. The step responses of the first order inertial system (a) and oscillatory system (b) before ■ and after ■ the optimal past- and pre-correction with the use of 8 Laguerre Functions (R. Wojciechowski [12]).

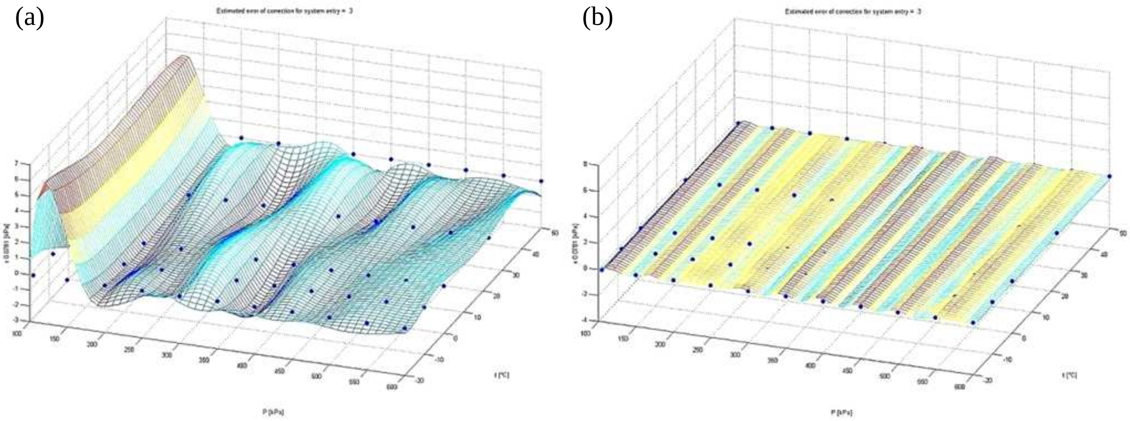

Fig. 2. The error functions of the pressure measurements system with the use of the DRUCK Sensor PDCR 901 and some classic past-correction (a) and after the optimal past-correction (b) with the use of the Cubic *B*-Splines tensor product for the breakpoints `p_appr = [100:500/20:600]` kPa and `t_appr = [-20:70/7:50]` °C (A. Albrecht [1]).

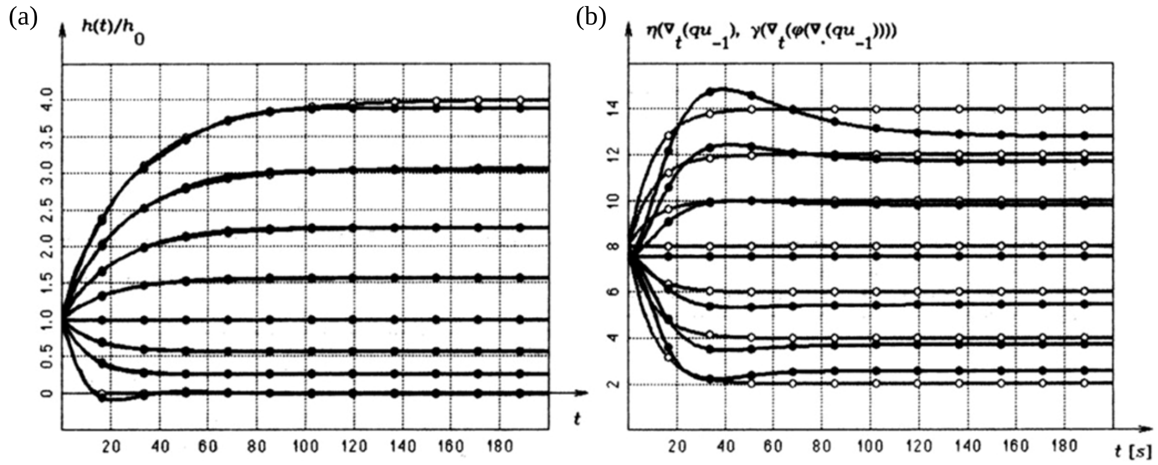

Fig. 3. The step responses of the Danaide (orifice gauging tank) volumetric flow sensor ○ together with its optimal model ● (a) and its step responses after optimal past-correction ● for required system ○ (b) with the use of 27 Wiener Operators (G. Ciesielski [4]).

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