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Stochastic Resonance in a Simple Electric Circuit having a Double-Well Potential ∼Circuit Experiments with a Single Operational Amplifier∼

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Abstract—In this report, we propose a double-well potential system that can easily be implemented by a single operational amplifier. The system is described by the same dynamics as traditional analog neurons. We first introduce a potential function obtained from the proposed dynamics and explain the bistable conditions. Then we describe how we constructed an electronic SR system that is implemented by a single operational amplifier, show experimental results of the proposed circuit, and demonstrate that the circuit exhibits the same SR behavior demonstrated in traditional double-well potential systems [1].

1. Background

Stochastic resonance (SR) is a phenomenon where a system can stochastically detect a weak input signal with the help of external noise when the input signal is below the system's threshold [1]. SR has been observed in many systems such as threshold systems [2], monostable systems [3], and bistable systems (double-well potential systems) [4]. Moreover, the SR has also been found in electrical applications such as threshold circuits [5], bistable circuits [6], and semiconductor laser [7]. SR may be utilized for weak signal detection in electric circuits [8]. Especially, SR in double-well potential electronic systems, i.e., bistable electronic systems, can be used not only for weak signal detection, but also for logic memories.

SR-utilizing logic memory circuits may be useful when the supply voltage of the circuit is extremely low. Because electric power consumption of digital circuits is proportional to the square of the supply voltage, decreasing the supply voltage is very effective to reduce power consumption. However, decreasing the supply voltage causes data writing to fail or stored data to be lost due to threshold voltage deviation of MOSFETs. If the threshold deviation becomes the dominant factor, SR would decrease SRAM cell failure rates. When threshold deviation disturbs data writing, noise and fluctuations change the potential barrier of double-well potential of a memory cell, and the data writing would succeed. Noise sources exist everywhere. For instance, power supply noise in LSIs [9] can be used as a noise source.

Memory cells of SRAMs must be designed with minimal size because integration density of the cell must be high. The most widely used memory cell is a latch circuit composed of two inverter circuits. In order to construct SR-utilizing memory cells, obtaining potential function of the latch circuit is required to evaluate SR characteristics. However, obtaining the potential function is not easy. So, we propose a mathematical model whose potential function can be obtained theoretically and construct an electric circuit which is equivalent quantitatively to the latch circuit.

2. Stochastic resonance in double-well potential system

In this paper, we will use the following dynamics,

$$\tau \frac{du}{dt} = -u + f_\beta(u - I), \tag{1}$$

where $f_\beta(\cdot)$ the sigmoid function whose slope factor is β and I the external input signal. Suppose that β is large enough, so $u \to 1$ when $u > I$, wheareas $u \to 0$ when $u < I$. Thus one can be convinced that this system is a bistable system.

Next, potential function of this system H will be given. When the following equation,

$$\frac{\partial H}{\partial t} = \frac{du}{dt} \cdot \frac{\partial H}{\partial u} < 0, \tag{2}$$

is satisfied, the system is considered to be stable. One can easily notice that

$$\frac{\partial H}{\partial u} = -\tau \frac{du}{dt}, \tag{3}$$

is a candidate that satisfies the condition. By substituting Eq. 1 to the above equation, the following equation,

$$\frac{\partial H}{\partial u} = u - f_\beta(u - I), \tag{4}$$

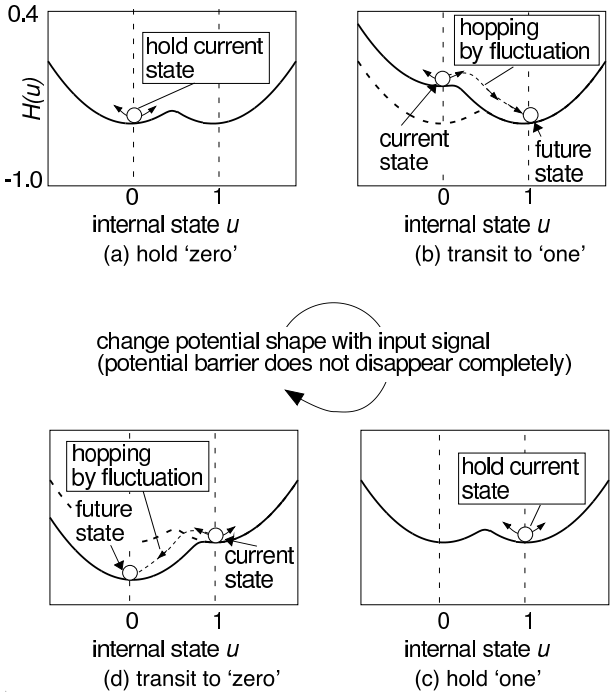


Figure 1: Double-well potential function of proposed system

is obtained. Integrating this by u leads to the following potential function,

$$H = \frac{1}{2}u^2 - \frac{1}{\beta}\ln\big(\exp(\beta u) + \exp(\beta I)\big) + C, \quad (5)$$

where C the integral constant.

Figure 1 plots the obtained potential function ($\beta = 30$, $C = 0$). Here consider $I = A\sin(t) + B$, where A the amplitude and B the offset. When $I = 0.5$ [Fig. 1(a), (c)], the middle potential barrier becomes the highest. In this condition, the system holds current state $u = 1$ or $u = 0$. Setting the external input I to 0 (or 1) changes the internal state to 1 (or 0). Figure 1(b) and (c) show the potential curves when I is set to 0.2 and 0.8. When the system accepts noises, the transition may occur. This small barrier can be surpassed by applying moderate noises. Small amounts of noise cannot cause the transition, whereas excessive amounts of noise cause uncorrelated transitions of the state with the input signal. This phenomenon is called "stochastic resonance in the double-well potential system" [1].

3. Electric circuits having double-well potential

The double-well potential system proposed in the previous section was implemented in electric circuits with only one operational amplifier. A fundamental property of the amplifier is reviewed briefly here. The

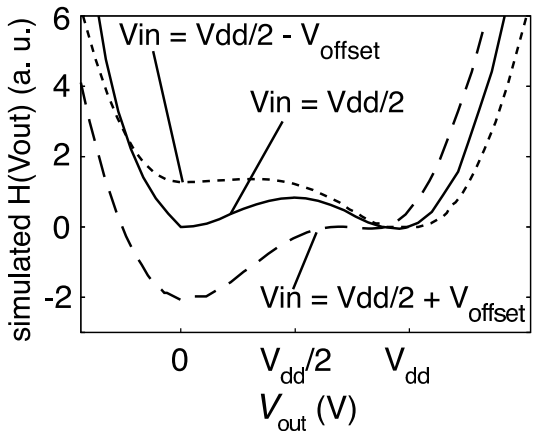


Figure 2: Potential function of proposed circuit $H(V_{\rm out})$.

operational amplifier has 2-input (V_+ and V_-, for example) and it amplifies the voltage difference between V_+ and V_- with gain A_v. The output $A_v \cdot (V_+ - V_-)$ is clamped at supply voltages ($V_{\rm dd}$, $V_{\rm ss}$). Thus, when the gain is large enough and $V_{\rm ss} = 0$, the amplifier's output is approximated by $V_{\rm dd} \cdot \theta(V_+ - V_-)$, where $\theta(\cdot)$ is the step function.

Suppose the time constant τ in Eq. (1) is small enough, then we obtain $u \approx f_\beta(u - I)$ from Eq. (1). When β is large enough, we can say $f_\beta(\cdot) \approx \theta(\cdot)$. When we consider $V_{\rm out}$ as $u \cdot V_{\rm dd}$ and $V_{\rm in}$ as $v \cdot V_{\rm dd}$, we obtain the approximate equation,

$$V_{\rm out} \approx V_{\rm dd} \cdot \theta(V_{\rm out} - V_{\rm in}). \quad (6)$$

The right side of the above equation is equal to the estimated output voltage of the amplifier where "+" node is set to $V_{\rm out}$ and "–" node is set to $V_{\rm in}$. Thus, by connecting the output node of the amplifier and "+" node, the system for Eq. (1) is implemented in electric circuits.

4. Simulation and experimental results

Figure 2 shows the potential function of the proposed circuit. Time courses of $V_{\rm out}$ was obtained from SPICE simulations (TSMC 0.18 μm CMOS parameter, $V_{\rm dd} = 1.8$ V, $V_{\rm offset} = 0.3$ V) and the potential function in Fig. 2 was numerically obtained by calculating $dV_{\rm out}/dt$ from the simulation results and integrating $dV_{\rm out}/dt$ by $V_{\rm out}$.

Figure 3 shows the circuit configuration. A CMOS full-swing (Rail-To-Rail) operational amplifier (National semiconductor, LMC6482) was used and supply voltage $V_{\rm dd}$ was set to 3.0 V. A input signal applied to the amplifier was $V'_{\rm in} = V_{\rm in} + V_{\rm n}$, where $V_{\rm in}$ the sinusoidal input and $V_{\rm n}$ the noise voltage. $V'_{\rm in}$

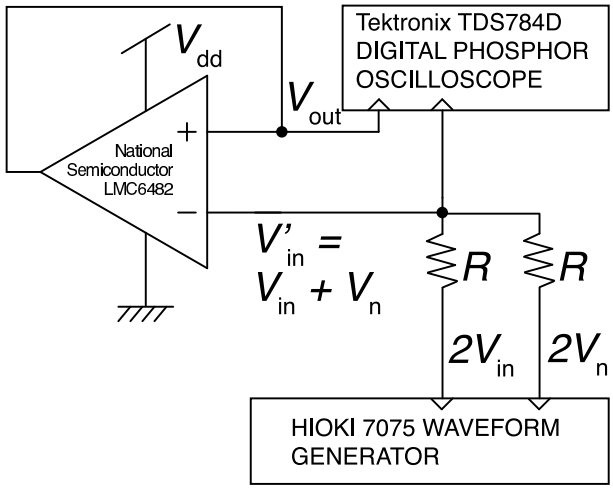
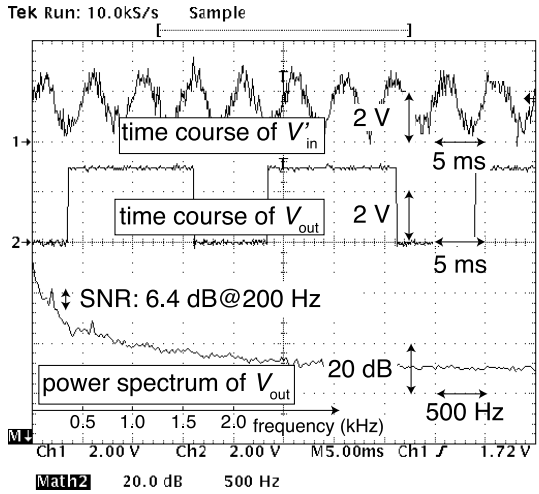


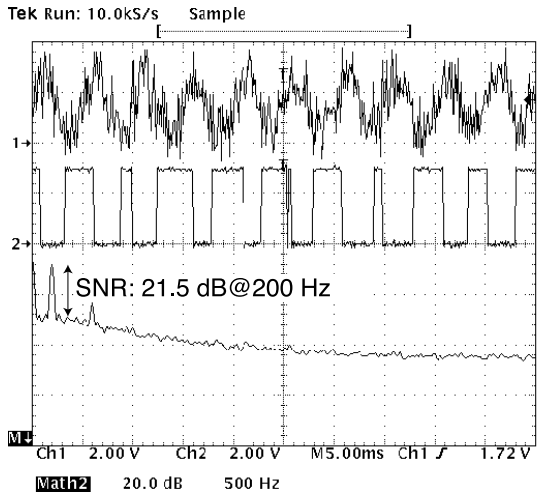
Figure 3: Electric circuit having using Double-well potential using single operational amplifier.

was generated by a resistive voltage divider (R = 1 kΩ). $V'_{\rm in}$ was applied to "–" node (an input node of the proposed circuit). Furthermore, $V_{\rm in}$ was given by $V_{\rm cm} + V_A \cdot \sin(2\pi f_0 t)$, where $V_{\rm cm}$ was 1.5 V, V_A was 1 V, and f_0 was 200 Hz. $V_{\rm n}$ was the time varying Gaussian noise voltage whose average and standard deviation were 0 V and σ V. Both $V_{\rm in}$ and $V_{\rm n}$ were given by a waveform generator (HIOKI, 7075). Pseudo-random sequences were generated by using the Box-Muller method from a computer simulation. $V_{\rm n}$ was the Pseudo-random sequence that was imported to the waveform generator with frequency limitation of 19 kHz. Note that our circuit has the simpler structure than the double-well potential electronic system proposed in [10]. We observed waveforms of the input and output voltages ($V'_{\rm in}$ and $V_{\rm out}$) by a oscilloscope (Techtronix, TDS784D) and sampling rate was 100 kHz . We also observed power spectrum of the output voltage ($V_{\rm out}$) by a FFT module, which is equipped with the oscilloscope (averaged over 1000 times in frequency domain), and obtained SNR by subtracting the background noise level on f_0 from the signal level on f_0.

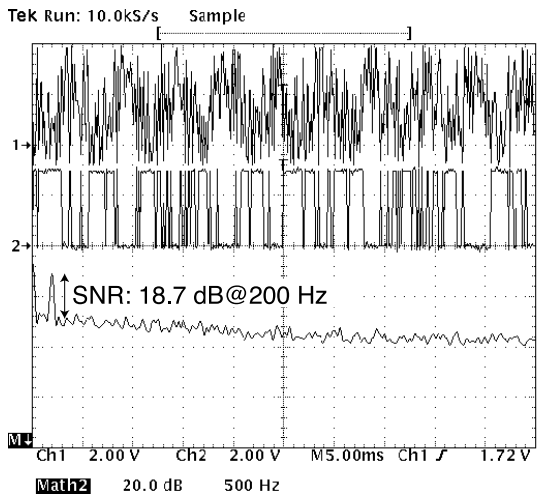
Figure 4 shows experimental results (waveform screens of the oscilloscope) where σ was set to 0.3 V, 0.75 V, and 1.5 V. Each figure, (a), (b), and (c), contains time courses of $V'_{\rm in}$ (upper), time courses of $V_{\rm out}$ (middle), and power spectrum of $V_{\rm out}$ (lower). When σ = 0.3 V, probability of $V_{\rm out}$ transition was small [Fig. 4(a)]. Because the offset and amplitude of $V_{\rm in}$ were 1.5 V and 2 Vpp ($V_{\rm in}$ = 0.5–2.5 V), the minimal voltage of $V_{\rm n}$ for transitions from '1' to '0' and from '0' to '1' are +0.5 V (when $V_{\rm in}$ is 2.5 V) and -0.5 V (when $V_{\rm in}$ is 0.5 V), respectively. This indicates that when σ were 0.3 V, a possibility of $V_{\rm n}$ > 0.5V (or



(a) standard deviation of noise σ = 0.3 V.



(b) σ = 0.75 V.



(c) σ = 1.5 V.

Figure 4: Experimental results. Top waveform in each figure: input of operational amplifier $V'_{\rm in}$, middle: output of operational amplifier $V_{\rm out}$, bottom: power spectrum of $V_{\rm out}$.

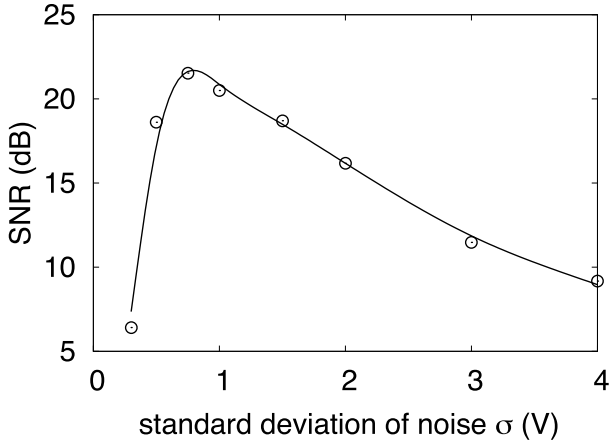


Figure 5: Stochastic resonance curve in electric circuit using operational amplifier.

$V_{\rm n} < -0.5$ V) and the transition possibility were small [Fig. 4(a)]. We measured signal and background level at 200 Hz in power spectrum and found that SNR was 6.4 dB. When $\sigma = 0.75$ V, possibility of $V_{\rm n} > 0.5$V (or $V_{\rm n} < -0.5$V) was higher than the transition possibility of $\sigma = 0.3$ V, and transition possibility was also higher [Fig. 4(b)]. The SNR in this case was 21.5 dB. The important fact is that the possibility of the transition from 0 to $V_{\rm dd}$ became high when $V_{\rm in}$ is low, and the possibility of the transition from $V_{\rm dd}$ to 0 became high when $V_{\rm in}$ was high. In other words, although $V_{\rm in}$ didn't have the amplitude required for the transition, the noise stochastically helped the transition of $V_{\rm out}$ to $V_{\rm dd}$ (or 0) depending on $V_{\rm in}$. The experimental results in $\sigma = 1.5$ V are shown in Fig. 4(c). In this case, the SNR was 18.7 dB. The SNR didn't decrease suddenly as it seems, but the frequent transition of $V_{\rm out}$ occured. This is because the noise level almost always surpassed the signal level required for transition.

Figure 5 shows the experimental SNR curve that was obtained by varying standard deviation of noise σ from 0 to 4 V. The maximum SNR was 21.5 dB ($\sigma = 0.75$ V).

5. Conclusion

We proposed a double-well potential system that can easily be implemented by a single operational amplifier. We first obtained a potential function of the system and its bistable conditions. Then, we constructed a simple electric circuit based on the system. And we conducted experiments of the circuit. We confirmed the same SR behavior observed in conventional double-well potential systems [1].

Acknowledgments

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