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# Tracing Control for a Tracked Vehicle Based on Prediction Model of Virtual Wheeled Robot

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**Abstract**—This paper presents a tracking control method for tracked vehicles. An instantaneous motion of a tracked vehicle with slip motion can be regarded kinematically as a motion of a virtual mobile robot with two independent driving wheels. The proposed method is constructed based on a model of the virtual robot. A control law is derived by applying a differential feedback control method for wheeled robots to the virtual robot. The control law guarantees that the virtual robot follows the virtual desired trajectory which is derived from a given desired trajectory of the original tracked vehicle. Simulation results show the effectiveness of the proposed method. An on-line prediction method for the virtual model is also discussed.

## 1. Introduction

Tracked vehicle enables stable straightforward movements on rough terrain due to a high road-hugging property of its crawler. To the contrary, when the vehicle has a rotating motion, many regions of the crawler have to slip against the contact ground. Since the slip motion is different according to contact conditions and is unpredictable, there is no fixed kinematic model for the tracked vehicle such as the one of wheeled mobile robot. Hence, it is difficult to realize a autonomous driving control of the tracked vehicle.

This paper presents a control method for tracked vehicles to track a given desired trajectory. The proposed method is constructed based on a model of a virtual wheeled robot.

An instantaneous motion of the tracked vehicle with slip motion can be regarded kinematically as a motion of a virtual mobile robot with two independent driving wheels. The virtual robot is derived from the vehicle motion as the kinematic model. A control law for tracked vehicles is derived by applying a differential feedback control method for wheeled robots to the virtual robot. The virtual desired trajectory for the virtual robot is also derived from the given desired trajectory of the original tracked vehicle under the assumption that the virtual model is fixed. The control law guarantees that the virtual robot follows the virtual desired trajectory. Since the virtual model varies according to movement, some modification of the model is considered for practical use. Simulation results show the effectiveness of the proposed method. An on-line prediction method for the virtual model is also discussed.

## 2. Related work

Some tracking control methods with slip compensation for tracked vehicle have been proposed. Ahmadi presented a path tracking control algorithm with feed forward friction compensation[1]. The compensation was derived based on the dynamic model of the vehicle including the force-slip relationship. Although their method has good performance in the simulation, the controller requires some soil parameters as prior knowledge. Martinez described wheeled differential drive vehicle models kinematically equivalent to tracked vehicles exist[2]. Moreover, the approximate position of wheel contact points was identified and optimized off-line, and were utilized for online odometry. Performance of the method may lower in the environment different from the identification environment.

The feature of our method is to need no prior knowledge about the ground. It may be continuously applicable to different characteristics of the ground.

## 3. Kinematic model

Figure 1 shows a typical model of the mobile robot. In the figure, $\Sigma_R(O_R - X_R Y_R)$ is the coordinate frame fixed to the robot with its origin at the intersection between the wheel axile and the center line of the robot. $Y_R$ axis of the frame is on the wheel axile. The tread width, which is the distance between the two wheels, is denoted by $T$. As known well, in constructing a kinematic model of the mobile robot, it is usual to make following assumptions:

**A.1** Each wheel is in contact with the ground at a point,

**A.2** No slip will occur at the point.

Under these assumptions, the kinematic model of the wheeled robot is represented by the following equation:

$$\begin{bmatrix} V_o \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2T} & -\frac{1}{2T} \end{bmatrix} \begin{bmatrix} v_r \\ v_l \end{bmatrix}, \quad (1)$$

where $V_o$ and $\omega$ are the translational and rotational velocity with respect to the robot respectively. Also, $v_r$ and $v_l$ are peripheral velocities of right and left wheels respectively.

Now, we consider a rotational motion of the vehicle with the center of rotation(COR) on the $Y_R$ axis, as shown in Figure 2(a). In this case, the velocities at the points on $Y_R$ axis are parallel to $X_R$ axis. However, it is easy to find from kinematic constraints that velocities at another points

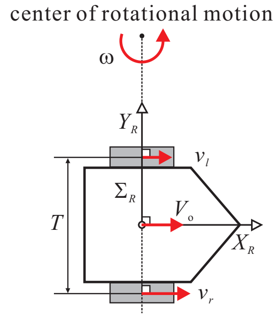
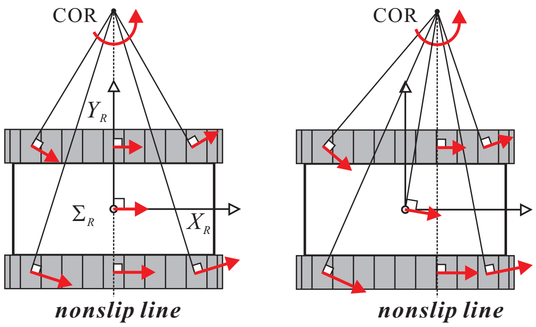



Figure 1: Typical model of wheeld mobile robot




(a) Case 1 (b) Case 2

Figure 2: Rotaitional motions of the tracked vehicle

on the crawlers are not parallel to $X_R$ axis. This means that the crawlers surely have the points on which slips occur. Therefore, on any motion of the tracked vehicle, the assumption **A.1** and **A.2** are not realized. Here after, the line which includes COR and is parallel to $Y_R$ axis is called "*nonslip line*". The location of *nonslip line* may change according to the contact conditions between the crawlers and ground, as shown in Figure 2(b). Moreover, it is difficult to predict the location. This is the reason that the tracked vehicle is difficult to be controled autonomously.

Figure 3(a) shows a case that *nonslip line* is not on $Y_R$ axis. Let ${}^R\boldsymbol{v}_o = [{}^Rv_{ox}\ {}^Rv_{oy}]^T$ denote the translational velocity at the center of the vehicle, respectively. Describing the position of COR as ${}^R\boldsymbol{p}_c = [{}^Rx_c\ {}^Ry_c]^T$, following equations can be obtained:

$$ {}^Rx_c = -\frac{{}^Rv_{oy}}{\omega},\quad {}^Ry_c = \frac{{}^Rv_{ox}}{\omega}. \tag{2} $$

Note that upper subscript $R$ means the values are expressed with respect to $\Sigma_R$. From the kinematic constraints, we can find the point $\bar{O}$ which is on the *nonslip line* and has following propaties:

Describing the position of $\bar{O}$ as ${}^R\boldsymbol{p}_{\bar{o}} = [{}^Rx_{\bar{o}}\ {}^Ry_{\bar{o}}]^T$, the velocity at $\bar{O}$ as ${}^R\boldsymbol{v}_{\bar{o}} = [V_{\bar{o}}\ 0]^T$, following constraints are fulfilled:

$$ {}^Rx_{\bar{o}} = {}^Rx_c, \tag{3} $$

$$ V_{\bar{o}} = -({}^Ry_{\bar{o}} - {}^Ry_c)\omega, \tag{4} $$

$$ \begin{bmatrix} V_{\bar{o}} \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2\bar{T}} & -\frac{1}{2\bar{T}} \end{bmatrix} \begin{bmatrix} v_r \\ v_l \end{bmatrix}. \tag{5} $$

Supposing that the velocity $({}^Rv_{ox}, {}^Rv_{oy}, \omega)$ and $(v_r, v_l)$ can be measured, from equation (3)~(5), ${}^R\boldsymbol{p}_{\bar{o}}$ and $\bar{T}$ are given by

$$ \begin{cases} {}^Rx_{\bar{o}} = -\dfrac{{}^Rv_{oy}}{\omega}, \\ {}^Ry_{\bar{o}} = -\dfrac{v_r + v_l - 2\,{}^Rv_{ox}}{2\omega}, \\ \bar{T} = \dfrac{v_r - v_l}{\omega}. \end{cases} \tag{6} $$

At this time, the motion of the tracked vehicle can be regarded as the one of the virtual wheeled robot whose center is at the point $\bar{O}$ and whose tread width is $\bar{T}$, as shown in Figure 3(b). The kinematic model of a tracked vehicle is described by equation (5) and (6). This model is the similar one derived in the reference [1].

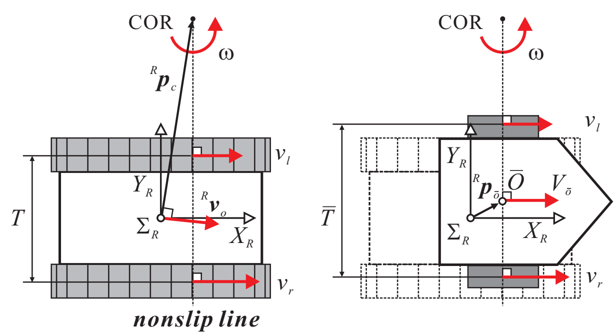
Our tracking control method is based on this virtual robot. The parameter set of the virtual robot $({}^Rx_{\bar{o}}, {}^Ry_{\bar{o}}, \bar{T})$ is varied according to the contact state between the crawlers and the ground. An adaptive prediction of the parameters is required in order to realize the tracking control. The prediction method which is next step of our research will be discussed in Section 5.

## 4. Tracking control for tracked vehicle

### 4.1. Desired Trajectory

Since some tracking control methods for wheeled mobile robots have been proposed in the research fields, the vehicle with slip may be able to control via a certain proposed control method by using the model of virtual mobile robot. However, as one can see easily from kinematic restrictions, in a rotational motion with slip it is impossible to realize the original desired velocity at the center of the vehicle, because the *nonslip line* is shifted (see Figure 4). This means the vehicle with slip can not no longer track the given trajectory strictly. Therefore, it is necessary to alter the desired posture of the vehicle slightly so that the center of the vehicle can follow the trajectory.

An univarsal coordinate frame $\Sigma_U(O_U - X_U Y_U)$ is attached to the work ground. We consider that the desired trajectory for a tracked vehicle is given by the position vector ${}^U\boldsymbol{p}_{od}(t) = [{}^Ux_{od}(t)\ {}^Uy_{od}(t)]^T$, which is second-order differentiable function about time $t$, as shown in figure 5. Then, the




(a) Tracked vehicle (b) Virtual mobile robot

Figure 3: Kinematic constraints of the tracked vehicle

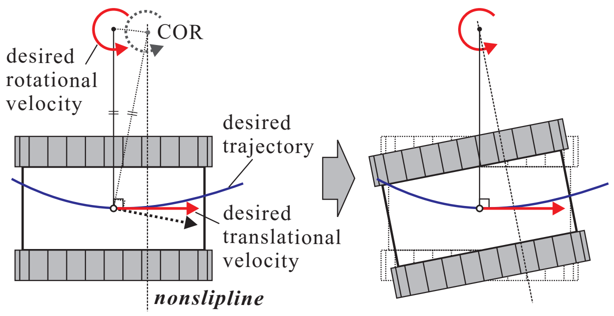



Figure 4: Alteration of desired posture

desired orientation $\theta_{od}(t)$, translational velocity $V_{od}(t)$, and rotational one $\omega_d(t)$ are given by,

$$\begin{aligned} \theta_d(t) &= \mathrm{atan2}({}^U\dot{y}_{od}(t),\ {}^U\dot{x}_{od}(t)), &(7)\\ V_{od}(t) &= {}^U\dot{x}_{od}(t)\cos\theta_{od}(t) + {}^U\dot{y}_{od}(t)\sin\theta_{od}(t), &(8)\\ \omega_d(t) &= {}^U\dot{x}_{od}(t){}^U\ddot{y}_{od}(t) - {}^U\ddot{x}_{od}(t){}^U\dot{y}_{od}(t). &(9) \end{aligned}$$

Also, we assum $V_{od}(t) > 0\ \ (t > 0)$.

Now, we assume that the model of virtual robot is fixed to $({}^Rx_{\bar{o}}, {}^Ry_{\bar{o}}, \bar{T})$. The angular displacement of orientation $\alpha(t)$ at ${}^U\boldsymbol{p}_{od}(t)$ is given by

$$\alpha(t) = \sin^{-1}\left(\frac{{}^Rx_{\bar{o}}\omega_d(t)}{V_{od}(t)}\right). \tag{10}$$

### 4.2. Control Algorithm

In order to control the tracked vehicle with slip, we apply a differential feedback control method for wheeled mobile robots proposed by Kanayama[3] to the virtual robot. Since the center of the virtual robot, about which the control rule shoul be adopted, is different from the center of the original vehicle, the desired trajectory for the virtual robot is needed.

First, we derive the virtual desired trajectory ${}^U\boldsymbol{p}_{\bar{o}d}(t)$ for the virtual robot from the desired trajectory and altered posture. Since the displacement of the vertual center from the origin of $\Sigma_R$ is ${}^R\boldsymbol{p}_{\bar{o}} = [{}^Rx_{\bar{o}}\ {}^Ry_{\bar{o}}]^T$, the vertual desired trajectory is given by

$${}^U\boldsymbol{p}_{\bar{o}d}(t) = {}^U\boldsymbol{p}_{od}(t) + \boldsymbol{R}(\theta_{od}(t) + \alpha(t)){}^R\boldsymbol{p}_{\bar{o}}, \tag{11}$$

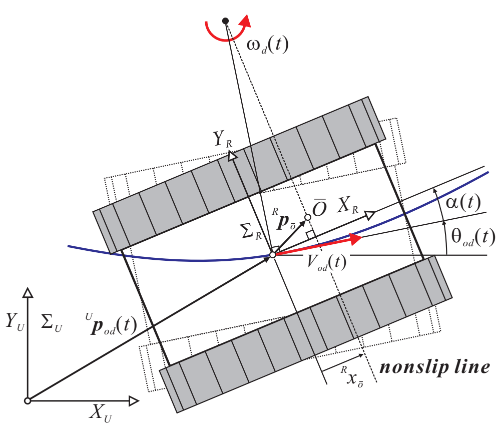



Figure 5: Desired trajectory and configuration of vehicle

where $\boldsymbol{R}(\theta)$ is the rotation matrix described as follows

$$\boldsymbol{R}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \tag{12}$$

The desired orientation is obtained by $\theta_{\bar{o}d}(t) = \theta_{od}(t) + \alpha(t)$, and the desired translational velocity is also given by $V_{\bar{o}d}(t) = V_{od}(t) - {}^Ry_{\bar{o}}\omega_d(t)$.

Next, we derive the feedback control rule for tracked vehicles. We consider the position error of the virtual center expressed in $\Sigma_R$ with respect to the virtual trajectory, denoted by ${}^R\boldsymbol{p}_e(t) = [{}^Rx_e(t), {}^Ry_e(t)]^T$. Denoting the actual position of the virtual center expressed in $\Sigma_U$ by ${}^U\boldsymbol{p}_{\bar{o}}(t)$, which is given by,

$${}^U\boldsymbol{p}_{\bar{o}}(t) = {}^U\boldsymbol{p}_o(t) + \boldsymbol{R}(\theta_o(t)){}^R\boldsymbol{p}_{\bar{o}}, \tag{13}$$

the error ${}^R\boldsymbol{p}_e(t)$ is obtained by

$$\begin{aligned} {}^R\boldsymbol{p}_e(t) &= \boldsymbol{R}(-\theta_{\bar{o}}(t))({}^U\boldsymbol{p}_{\bar{o}d}(t) - {}^U\boldsymbol{p}_{\bar{o}}(t)) \\ &= \boldsymbol{R}(-\theta_{\bar{o}}(t))({}^U\boldsymbol{p}_{\bar{o}d}(t) - {}^U\boldsymbol{p}_o(t)) \\ &\quad + \{\boldsymbol{R}(\theta_e(t)) - \boldsymbol{I}\}{}^R\boldsymbol{p}_{\bar{o}}, \end{aligned} \tag{14}$$

where $\theta_e(t) = \theta_{od}(t) + \alpha(t) - \theta_o(t)$ is the orientation error and $\boldsymbol{I}$ is the unit matrix. Applying the differential feedback rule described by

$$\begin{bmatrix} V_{\bar{o}ref} \\ \omega_{ref} \end{bmatrix} = \begin{bmatrix} V_{\bar{o}d}\cos\theta_e + K_x{}^Rx_e \\ \omega_d + V_{\bar{o}d}(K_y{}^Ry_e + K_\theta\sin\theta_e) \end{bmatrix}, \tag{15}$$

it is guaranteed that $\boldsymbol{e}_{\bar{o}}(t) = [{}^R\boldsymbol{p}_e^T(t)\ , \theta_e(t)]^T$ converges uniformly asymptotically to $\boldsymbol{0}$ as far as $V_{\bar{o}d} > 0$. In equation (15), $K_x$, $K_y$, and $K_\theta$ are feedback parameters, and $V_{\bar{o}ref}$, $\omega_{ref}$ are reference velocities of the virtual robot. When the desired trajectory is a linear motion with constant velocity given by ${}^U\boldsymbol{p}_{\bar{o}d}(t) = [Vt, 0]^T$, $\theta_{\bar{o}d}(t) = 0$, and initial condition $\theta_e(0) = 0$, $1/K_x$ corresponds to the time constant of the exponential decay concerning the initial position error ${}^Rx_e(0)$. Also, $K_\theta/(2\sqrt{K_y})$ and $V\sqrt{K_y}$ correspond to the damping coefficient and natural circular frequency of ${}^Uy_{\bar{o}}(t)$ respectively. (For more information, see the reference 3.)

Finally, substituting equation (15) to inverse function of equation (5), the reference velocities for the crawlers of actual tracked vehicle, $v_{r\,ref}$ and $v_{l\,ref}$, are obtained. Consequently, if we set $v_{r\,ref}$ and $v_{l\,ref}$ to the both crawlers as control inputs, we can expect to make the tracked vehicle follow the desired trajectory.

### 4.3. Modification for practical use

In the previous subsection, the tracking control law was derived under the assumption the model of virtual robot is fixed. However, in real environment, the model parameters $({}^Rx_{\bar{o}}, {}^Ry_{\bar{o}}, \bar{T})$ will varies according to vehicle movements. So, we have to get on-line these parameters from equation (6) using the observed variables $v_r$, $v_l$, ${}^Rv_{ox}$, ${}^Rv_{oy}$, and $\omega$. Here, we assume these variables can be measured by using gyro sensor, GPS, and so on. As known well, the measurement data from these sensors is usually very noisy, and also may be sensitive to the effect of the slip caused by feedback control. From the above consideration, the measurement data passed through an adequate low-pass filter will be utilized as the observed variables. Also, following modification is made for practical use:

1) The range of virtual center is limited by

$$\left(\frac{2\,{}^{R}x_{\bar{o}}}{L}\right)^2 + \left(\frac{2\,{}^{R}y_{\bar{o}}}{T}\right)^2 \leq 1, \tag{16}$$

2) Virtual tread width is restricted by $\frac{T}{2} \leq \bar{T} \leq 4T$,

where $L$ is the track length. If the virtual center appears outside of the range, the distance from the center of vehicle is reduced without change of the direction.

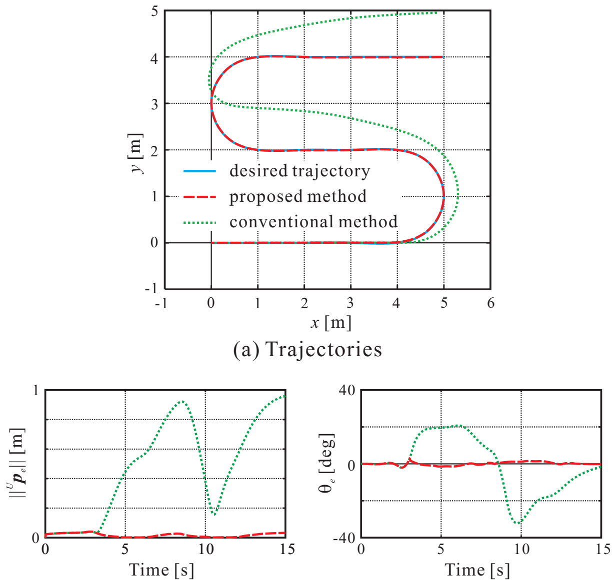
### 4.4. Simulation

We verify the availability of our method by simulations. Table 1 shows the parameters of vehicle used in the simulations. These parameters are the same as ones of the miniature vehicle robot which will be used on our next work. Motion of the vehicle is calculated by use of the dynamic friction model described in reference[1]. The ground is assumed to be "*dry clay*", for which, soil parameters, coefficients of friction, and so on, are the same as ones used in the reference. Control period is 1ms, and cut-off frequency of the filters mentioned in the previous subsection is set to 10Hz. Desired trajectories are made of natural spline curves. Feedback gains are chosen as $K_x$=1, $K_y$=1, and $K_\theta$=0.81, which correspond to the time constant 1, the damping coefficient 0.9, and the natural circular frequency 1.

Table 1: Parameters of tracked vehicle

| | | | |
|---|---|---|---|
| Mass | 40 kg | Tread width | 0.34 m |
| Moment of inertia | 0.55 kgm$^2$ | Track length | 0.55 m |
| | | Track width | 0.08 m |

Figure 6 shows an example results. The simulation is started with a initial state (${}^{U}x_o(0)$, ${}^{U}y_o(0)$, $\theta(0)$, $V_o(0)$, $\omega(0)$)=(0, 0, 0, 0, 0). From the figure, one can see that the proposed method keeps the tracking errors small, although the conventional one cannot compensate the effect of slip.




(a) Trajectories

(b) Position errors (c) Orientation errors

Figure 6: Simulation results

### 5. Prediction for the model of virtual mobile robot

In order to get on-line the model parameters, we assume that the information from internal sensors mounted on the vehicle, such as gyro sensor, GPS, and so on, is utilized. Note that the measurement frequency of GPS is usually 1~10Hz, that is, it is very low in comparison with the control frequency. Therefor, estimation and prediction of the parameters are required. Reference [4] presents an algorithm for estimation of the vehicle position and the slip rate by using the Kalman-filter technique based on the dynamic model including force-slip relationship. We will construct a similar system for parameter estimation and prediction by utilizing the proposed algorithm combined with the information data from encoders, gyro sensors, accelerometers, and GPS. This is the next step of our research.

### 6. Conclusions

This paper presented a tracking control method for a tracked vehicle. The proposed method is constructed by using a virtual wheeled mobile robot. The simulation results show the availability of proposed method. The feature of the presented method is to need no prior knowledge about the ground. Therefore, the method is continuously applicable to different characteristics of the ground To the contrary, In order to sufficiently bring out its capability, the parameters of the virtual model are necessary to be obtained with high frequency. Consequently, it is required to construct a system of parameter estimation and prediction. That is the next step of our research.

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