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Relaxation Oscillation in Single-Electron Transistor with Resistively-Shunted Gate

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Abstract—The author presents that a single-electron transistor with a resistively-shunted gate (RSG-SET) has an operation mode exhibiting relaxation oscillation. A shunting resistor added parallel to the gate capacitance is the key. Single-electron tunneling carries electric charge discretely into the island, while the dissipation in the shunting resistor reduces the island charge continuously. The combination of discrete and continuous charge transfer eventually results in the relaxation oscillation of the island charge. Numerical Monte Carlo simulation demonstrates the relaxation oscillation. Conditions for the bias voltage are also derived.

1. Introduction

The recent development of nanofabrication techniques has enabled us to manipulate single electrons in solid-state circuits, leading to a new field called "single electronics" [1]. Single-electron (SE) devices are essentially composed of arrays of tiny (submicron to nm order) tunnel junctions. The operation of SE devices is based on the Coulomb blockade that appears if the charging energy of a single electron, $E_C = e^2/2C$, is much larger than the thermal energy k_BT, where e, C, k_B, and T are the fundamental charge, tunnel junction capacitance, Boltzmann constant, and absolute temperature, respectively.

Among various SE devices, a single-electron transistor (SET) shown in Fig. 1(a) is the most important device in single electronics. It comprises two small tunnel junctions connected in series. The island electrode between the tunnel junctions has a gate capacitor. The current through the junctions is controlled by sub-electron extenal charge, and hence, an SET is the most sensitive device to electric charge. So far, the charge sensitivity as high as $10^{-6}e/\sqrt{Hz}$ has been demonstrated [2].

In this paper, the author presents that a single-electron transistor with a resistively-shunted gate (RSG-SET) has an operation mode exhibiting relaxation oscillation. In the RSG-SET, a shunting resistor is added parallel to the gate capacitor as shown in Fig. 1(b). Its operation principle and numerical wavefomrs are demonstrated below.

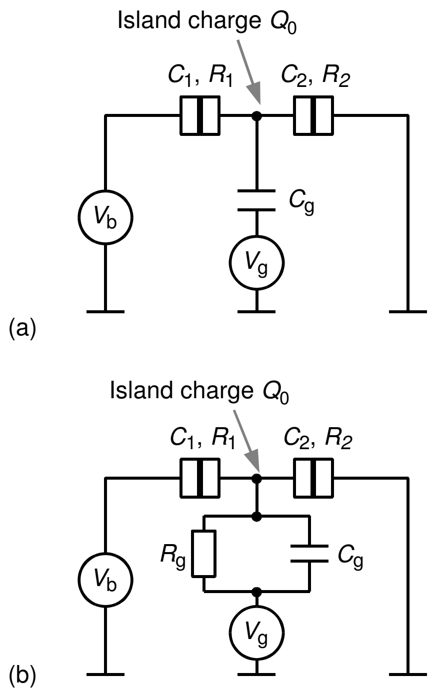

Figure 1: (a) Configuration of a single-electron transistor (SET). The current through the two small tunnel junctions are controlled by electric charge in a gate capacitor (C_g). (b) SET with a resistively-shunted gate (RSG-SET). A shunting resistor (R_g) is added parallel to the gate capacitor. Dissipation in the shunting resistor is one of the keys for relaxation oscillation.

2. Operation Principle of Relaxation Oscillation

The electric charge Q_0 and potential V_0 of the island electrode in the conventional SET shown in Fig. 1(a) are given as

$$Q_0 = ne \tag{1}$$

and

$$V_0 = \frac{Q_0 + C_1V_b + C_gV_g}{C_1 + C_2 + C_g}, \tag{2}$$

where n is an integer. Its Coulomb blockade conditions are expressed as follows for the zero initial island charge, i.e.,

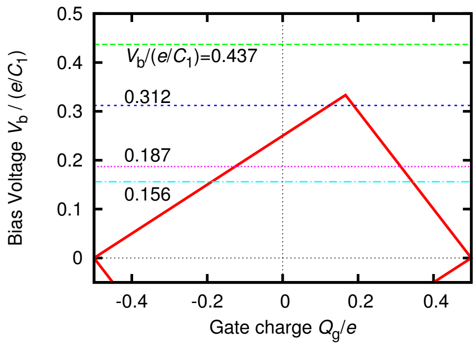

Figure 2: Part of the Coulomb diamond of the SET for the zero island charge, $n = 0$ (solid lines). The conditions of $C_1 = C_2 = C_g$ are assumed. Bias voltages $V_b/(e/C_1)$ of 0.437, 0.312, 0.187, and 0.156 are presented as dashed lines, which are used in numerical simulation.

$n = 0$ [3].

$$V_b > (Q_g - e/2)/(C_2 + C_g) \quad (3)$$

$$V_b < (Q_g + e/2)/(C_2 + C_g) \quad (4)$$

$$V_b > -(Q_g + e/2)/C_1 \quad (5)$$

$$V_b < -(Q_g - e/2)/C_1 \quad (6)$$

Here Q_g is the external charge given by $C_g V_g$. The inequalities (3) and (4) are the Coulomb blockade conditions of the left tunnel junction, whereas the inequalities (5) and (6) are those of the right junction. The boundaries of these conditions are commonly referred to as the "Coulomb diamond." Note again that the island charge Q_0 changes discretely in units of e.

Part of the Coulomb diamond is presented in Fig. 2. Here $C_1 = C_2 = C_g$ are assumed. Inside of the diamond, electron tunneling in the junctions is prohibited, and the island charge Q_0 is fixed to zero.

SE tunneling in the SET occurs outside the Coulomb diamond. Below, the external charge $Q_g = C_g V_g$ is assumed to be zero. When the bias voltage V_b is increased slightly beyond the boundary of the condition (4), an electron tunnels from the island to the bias voltage source through the left junction. Then, the island charge Q_0 becomes $+e$, which breaks the Coulomb blockade of the right junction and an electron enters the island through the right junction. So, the island charge Q_0 returns to zero, that is, the initial condition. Such sequential SE tunneling repeats, resulting in continuous electric current through the SET.

The RSG-SET shown in Fig. 1(b) operates similarly if the time constant $C_g R_g$ is much greater than the time constants $C_1 R_1$ and $C_1 R_2$. Differently from the conventional SET, however, the island charge Q_0 is not discrete but continuous in the RSG-SET. Since V_0 is given as $-R_g(dQ_0/dt)$, Eq. (2) is rewritten as

$$-R_g \frac{dQ_0(t)}{dt} = \frac{Q_0(t) + C_1 V_b + C_g V_g}{C_1 + C_2 + C_g}. \quad (7)$$

Besides, SE tunneling through the junctions makes a discrete shift of $\pm e$ in Q_0.

Figure 3 provides a simplified explanation for the first cycle of sequential SE tunneling in the RSG-SET. When the bias voltage V_b is increased slightly beyond the boundary of the condition (4), an electron tunnels from the island to the bias voltage source through the left junction. Then, the island charge Q_0 becomes $+e$. Unlike the conventional SET, due to the dissipation in the shunting resistor, the island charge Q_0 decays from $+e$ to $+e - \Delta$ before the subsequent SE tunneling occurs in the right junction. As a result, the residual charge of $-\Delta$ remains in the island after the SE tunneling in the right junction. That is, one cycle of sequential SE tunneling in the junctions leaves finite charge of $-\Delta$ in the island.

Repeating such sequential SE tunneling accumulates the residual charge in the island. Finally, even after the SE tunneling in the left junction, the accumulated charge prevents the subsequent SE tunneling in the right junction, and the sequential tunneling is suspended. In other words, the accumulated residual charge equivalently shifts the extenal charge until the operation point enters the Coulomb diamond.

During the intermittent of SE tunneling, the island charge decays due to the dissipation in the shunting resistor. When the island charge decreases enough to start again the SE tunneling in the left junction, the sequential SE tunneling in the RSG-SET resumes.

To obtain the relaxation oscillation, the bias voltage V_b must satisfy two conditions. Firstly, V_b must be smaller than the peak of the Coulomb diamond, that is,

$$V_b < e/(C_1 + C_2 + C_g). \quad (8)$$

Secondly, SE tunneling must occur in the left junction even when the island potential is as low as the ground level, which gives the condition of $V_b > (-C_1 V_b + e/2)/(C_2 + C_g)$. That is,

$$V_b > e/2(C_1 + C_2 + C_g). \quad (9)$$

3. Numerical Simulation

3.1. Simulation Conditions

To verify the relaxation oscillation in the RSG-SET, the author simulated the device operation using a simulator for SE devices and circuits (SIMON) on the basis of the Monte Carlo method [4]. In the simulation, the zero temperature condition was assumed and higher-order co-tunneling processes were excluded. The device parameters were set as follows: $C_1 = C_2 = C_g$, $R_1 = R_2$, $R_g = 100R_1$, and $V_g = 0$. The time constant of the resistively-shunted gate is

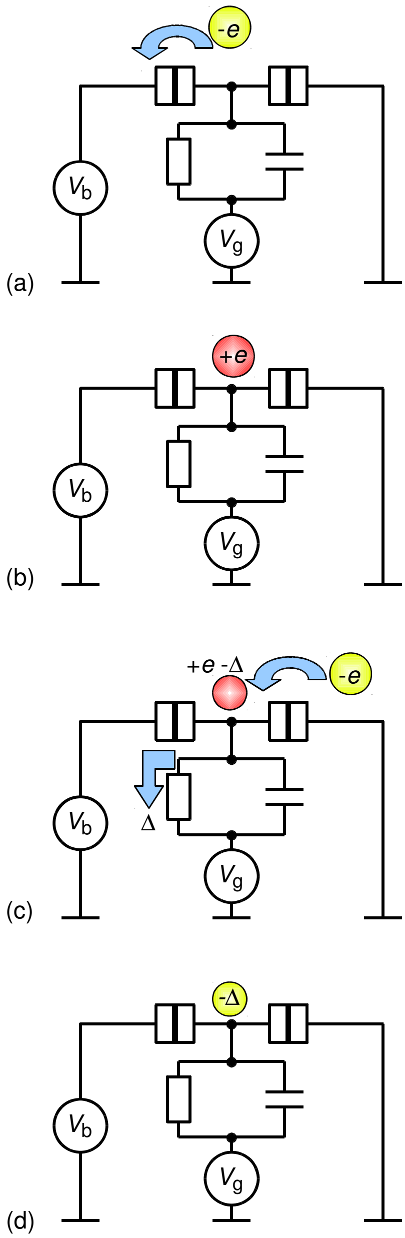

Figure 3: Schematic drawing of the simplified first cycle of sequential SE tunneling in the RSG-SET. V_b is assumed to be slightly beyond the boundary of the Coulomb blockade condition. (a) An electron tunnels through the left junction. (b) The island charge Q_0 becomes $+e$. (c) Before the subsequent SE tunneling occurs in the right junction, the island charge Q_0 decreases from $+e$ to $+e - \Delta$ due to the dissipation in the shunting resistor. (d) After the SE tunneling in the right junction, the residual charge of $-\Delta$ remains in the island.

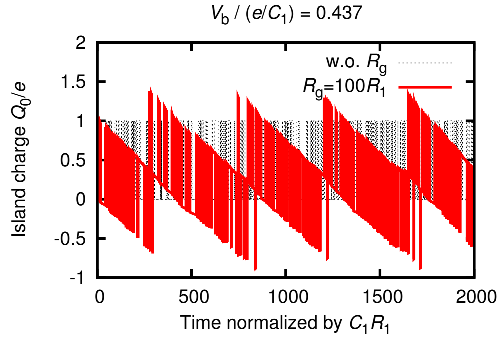

Figure 4: Simulated waveforms for $V_b = 0.437e/C_1$. The bias voltage is greater than the peak of the Coulomb diamond of $0.333e/C_1$. Dashed and solid curves are the results for the conventional SET and the RSG-SET, respectively.

100 times larger than those of the tunnel junctions, that is, $C_g R_g = 100 C_1 R_1$.

Besides the operation of the RSG-SET shown in Fig. 1(b), the operation of the conventional SET in Fig. 1(a) was also simulated for reference.

3.2. Simulated Waveforms of Island Charge Q_0

3.2.1. Results for $V_b > e/(C_1 + C_2 + C_g)$

The condition of $V_b > e/(C_1 + C_2 + C_g)$ means that the bias voltage is greater than the peak of the Coulomb diamond. Figure 4 shows the simulated waveforms of the island charge Q_0 for $V_b = 0.437e/C_1$, which is greater than the peak of the Coulomb diamond of $0.333e/C_1$. (See Fig. 2.)

The island charge Q_0 oscillates continuously in both the conventional SET and the RSG-SET. Q_0 in the conventional SET has two values of 0 and $+e$, whereas the Q_0 curve of the RSG-SET is tilted due to the dissipation in the shunting resistor.

3.2.2. Results for $e/2(C_1+C_2+C_g) < V_b < e/(C_1+C_2+C_g)$

The conditions $e/2(C_1+C_2+C_g) < V_b < e/(C_1+C_2+C_g)$ are necessary for the relaxation oscillation in the RSG-SET. Because of $C_1 = C_2 = C_g$, the conditions are given as $e/(6C_1) < V_b < e/(3C_1)$. Results for $V_b = 0.312e/C_1$ and $V_b = 0.187e/C_1$ are shown in Fig. 5.

In Fig. 5(a), the island charge Q_0 in the SET oscillates continuously, whereas the Q_0 in the RSG-SET exhibits relaxation oscillation. Two lines of $Q_0 = 0.188e$ and $0.124e$ in Fig. 5(a) presents the boundaries of Coulomb diamond for $V_b = 0.312e/C_1$. It can be seen that the Q_0 oscillation occurs only outside the Coulomb diamond.

In Fig. 5(b), the relaxation oscillation is demonstrated more clearly, while the island charge Q_0 in the conventional SET stays at zero. No oscillation occurs in the conventional

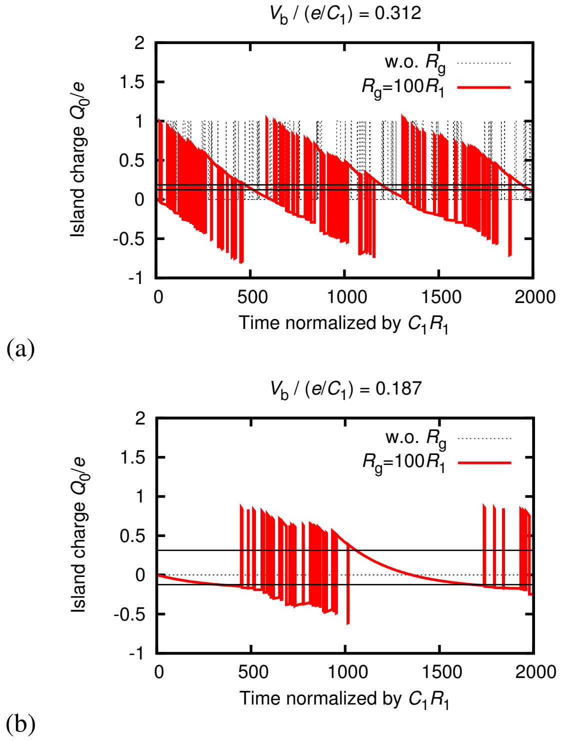

Figure 5: Simulated waveforms for (a) $V_b = 0.312e/C_1$ and (b) $V_b = 0.187e/C_1$. The both bias voltages satisfy the necessary conditions of $e/(6C_1) < V_b < e/(3C_1)$. Dashed and solid curves are the results for the conventional SET and the RSG-SET, respectively. Two lines of $Q_0 = 0.188e$ and $0.124e$ in (a), and two lines of $Q_0 = 0.313e$ and $-0.125e$ in (b) are the thresholds of the Coulomb blockade.

SET, because $V_b = 0.187e/C_1$ is smaller than the threshold of $0.250e/C_1$.

3.2.3. *Results for $V_b < e/2(C_1 + C_2 + C_g)$*

The island charge Q_0 does not oscillate under the small bias condition of $V_b < e/2(C_1 + C_2 + C_g)$, of which results are shown in Fig. 6. The island charge Q_0 in the conventional SET stays at zero. Although Q_0 in the RSG-SET approaches to $-C_1V_b$, it does not exceed the threshold.

4. Conclusion

In this paper, the author demonstrated the relaxation oscillation in the RSG-SET. The RSG-SET has a shunting resistor parallel to the gate capacitor. Because of the dissipation in the shunting resistor, the island charge Q_0 becomes continuous, while the SE tunneling carries electric charge into the island discretely in units of e. Thus, even when the operation point is initially set outside the Coulomb diamond, sequential SE tunneling moves the operation point eventually into the Coulomb diamond, resulting in relaxation oscillation. After the operation principle

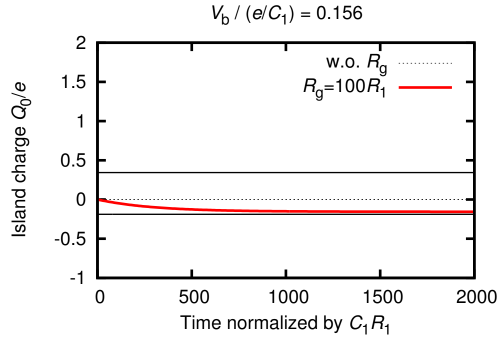

Figure 6: Simulated waveforms for $V_b = 0.156e/C_1$. The bias voltage is smaller than the critical value of $0.167e/C_1$. Dashed and solid curves are the results for the conventional SET and the RST-SET, respectively. Two lines at $Q_0 = 0.344e$ and $-0.188e$ are the thresholds of the Coulomb blockade.

was schematically explained, several numerical waveforms were demonstrated. The conditions of the bias voltage for the relaxation oscillation were also derived.

Acknowledgments

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