# Chaos control of chaotic Hindmarsh-Rose models for neurons

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**Abstract** – Master-slave and mutual method for synchronization and amplification (attenuation) of chaos for two chaotic systems are presented. Numerical results are given for the synchronization of two Hindmarsh-Rose models for neurons.

### 1. Introduction

One of the possible applications of the synchronization is in the neuroscience because chaotic regimens have been observed experimentally in neurons. The time evolution of the neuronal activity is commonly describe by nonlinear diferential equations. The Hindmarsh-Rose model, one of the most popular neuron models exhibiting complicated dynamics, is a three-variable model for the bursting of neurons and a variant of the FitzHugh-Nagumo model [1]. Hindmarsh and Rose work was initiated by the discovery of a neuronal cell in the brain of pond snail Lymnae which, when it was depolarized by a short pulse, generated an action potential followed by slow depolarizing afterpotential. In invertebrates there are networks of neurons in which every neuron has reciprocal connections to other members. The Hindmarsh-Rose equations are developed to study synchronization of firing of two snail neurons without the need to use the full Hodgkin-Huxley equations. The natural choice was to use the FitzHugh-Nagumo model, which is more or less a simplification of the Hodgkin-Huxley equations. FitzHugh and Nagumo observed independently that in the Hodgkin-Huxley equations, the membrane potential as well as sodium activation evolves on similar time-scales during an action potential, while sodium inactivation and potassium activation change on similar, although slower time scales. Starting with the two-variable subsystem which describes the action potential, they add a third variable which is governed by a simple linear equation, and note that the resulting autonomous system admits aperiodic behavior. Then the complicated current-voltage relationship of the conductance models has been replaced by polynomials in the dynamical variables.

The dynamical behavior of the Hindmarsh–Rose model has been intensively investigated in the last two decades. In recent years, more and more authors have devoted to investigating the synchronization of the Hindmarsh–Rose models from numerical simulation studies [2]. Recently Wu et al. [3] investigated the issues of control and synchronization of the chaotic Hindmarsh–Rose models via impulsive control with varying impulsive intervals and Ma at al. [4] the role of noise in the biological and neuronal system because the neurons often are sensitive to the external noise. Jackson and Grosu [5], [6] developed a powerful method of control: the open-plus-closed-loop (OPCL) method. This method gives precise driving for any continuous system in order to reach any desired dynamics and it has been applied to synchronization of two identical systems by Lerescu et al. [7], and Oancea [8]. More than this it can be extends to 3 systems and 4 systems [9], [10].

The main objective of this paper is to investigate the synchronization and amplification (attenuation) of chaos for Hindmarsh–Rose models of neurons.

## 2. Master-slave Synchronization

Let's consider a general master system:	
$d\mathbf{X}/dt = \mathbf{F}(\mathbf{X}); \mathbf{X} \in \mathbf{R}^n$	(1)
then the slave system:	
$d\mathbf{x}/dt = \mathbf{F}(\mathbf{x}) + \mathbf{D}(\mathbf{x},\mathbf{X})$	(2)
where $D(x,X)=(A - \partial F / \partial x x=X)(x - \partial x)(x - \partial x)(x$	X)-
$\frac{1}{2}(\partial^2 \mathbf{F}/\partial \mathbf{x}^2)(\mathbf{x}-\mathbf{X})^2 - \frac{1}{6}(\partial^3 \mathbf{F}/\partial \mathbf{x}^3)(\mathbf{x}-\mathbf{X})^3 + \dots$	

assures  $\mathbf{x}(t) \rightarrow \mathbf{X}(t)$  for any  $\| \mathbf{x}(0) - \mathbf{X}(0) \|$  small enough.

A is a constant Hurwitz matrix with negative real part eigenvalues. The matrix A should be chosen in such a manner in order that the coupling to be as simple as possible.

### 3. Mutual Synchronization

Let's consider two identical general oscillators: dx/dt=F(x): dv/dt=F(y):

 $d\mathbf{x}/dt = \mathbf{F}(\mathbf{x}) + \mathbf{u}(\mathbf{x},\mathbf{y}); \quad d\mathbf{y}/dt = \mathbf{F}(\mathbf{y}) + \mathbf{u}(\mathbf{x},\mathbf{y}); \quad (4)$ 

(3)

where  $\mathbf{u}(\mathbf{x},\mathbf{y})=(\mathbf{A}-\mathbf{dF}(\mathbf{s})/\mathbf{ds})^*(\mathbf{x}-\mathbf{y})/2$ ,  $\mathbf{s}=(\mathbf{x}+\mathbf{y})/2$  and A is the Hurwitz matrix.

The present method has been applied to all systems from the Sprott collection [11].

### 4. Amplification of chaos

Grosu et al. [12] designed the coupling for stable synchronization and antisynchronization in chaotic systems under parameter mismatch. The driver is:

$$\frac{dy}{dt} = F(y) + \Delta F(y); \tag{5}$$

 $\mathbf{y} \in \mathbf{R}^{n}$ , unde  $\Delta \mathbf{F}(\mathbf{y})$  contains mismatch parameters.

The driven system is given by:  $dx/dt = F(x) + D(x, \alpha y)$  (6)

dx/dt = F(x) + D(x, dy)

where  $D(x,\alpha y) = \alpha dy/dt - F(\alpha y) - (A - J F(\alpha y)(x - \alpha y) (7)$ J being the Iacobian and A the arbitrary constant Hurwitz matrix.

## 5. Numerical results

The Hindmarsh-Rose model has three variables  $X_1$ ,  $X_2$ ,  $X_3$ , satisfying the following polynomial equations:

 $dX_{1}/dt = X_{2} - aX_{1}^{3} + bX_{1}^{2} - X_{3} + I;$   $dX_{2}/dt = c - dX_{1}^{2} - X_{2};$  $dX_{3}/dt = r [s(X_{1} - e) - X_{3}]$ (8)

Here  $X_1$  is membrane action potential,  $X_2$  is potential of the ionic channels subserving accommodation,  $X_3$  the slow adaptation current which moves the voltage in and out of the inherent bistable regime and which terminates spike discharges, *I*, external direct current and a, b, c, d, e, r, s are constants.

Depending on the values of above parameters neurons can be in a steady state, they can generate a periodic lowfrequency repetitive firing, chaotic bursts, or high frequency discharges of action potentials. Just as proposed in [1], we consider the system (8) with the constant values:

a=1; b=3; c=1; d=5; I=3.25; e=- $(1+\sqrt{5})/2$ ; r=0.005; s=4 The Hindmarsh-Rose system is chaotic (Fig. 1):



Fig.1. Phase portrait for Hindmarsh-Rose system with initial conditions  $(X_1(0) = -1; X_2(0) = -0.1 X_3(0) = -0.01)$ 

We can choose the Hurvitz matrix having a two constant parameters and the Routh-Hurwitz conditions give for these parameters:

 $0.02 > 0.05(p_1+p_2)$ 

0.025>1.005p1+p2

The slave system (of the master system (8)) with  $p_1=p_2 = -10$  is:  $dx_1/dt=x_2-x_1^3+3x_1^2-x_3+3.25+(-10+3X_1^2-6X_1)(x_1-X_1)$ ;  $dx_2/dt=1-5x_1^2-x_2+(-10+10X_1))(x_1-X_1)$ :

$$\frac{dx_2}{dt} = 0.005[4(x_1 + 1.618) - x_3]$$
(9)

Fig.2 and 3 presents the fast synchronization of two Hindmarsh-Rose systems for  $p_1=p_2=-10$ ]



Fig.2 X<sub>1</sub>(t)-green x<sub>1</sub>(t)-black  $[X_1(0) = -1; X_2(0) = -0.1 X_3(0) = -0.01; x_1(0) = 1; x_2(0) = 0.1 x_3(0) = 0.01]$ 



Fig.3.  $X_2(t)$ -red,  $x_2(t)$ -black  $[X_1(0) = -1; X_2(0) = -0.1 X_3(0) = -0.01; x_1(0) = 1; x_2(0) = 0.1 x_3(0) = 0.01; p_1=p_2 = -10]$ 



Fig.4. Synchronization errors between (8) and (9)  $[X_1(t)-x_1(t), green; X_2(t)-x_2(t), red; X_3(t)-x_3(t), black]$  and  $p_1=p_2=-10; (X_1(0)=-1; X_2(0)=-0.1; X_3(0)=-0.01; x_1=1; x_2=0.1; x_3=0.01)$ 

The mutual synchronization systems are:

 $\begin{array}{ll} dx_1/dt &=& x_2 \cdot x_1^3 & + 3x_1^2 \cdot x_3 + 3.25 + \left[-10 + 3 (x_1 + y_1)^2 / 4 - 6 (x_1 + y_1) / 2\right] (x_1 - y_1) / 2\,; \\ dx_2/dt = & 1 - 5x_1^2 - x_2 + \left[-10 + 10 (x_1 + y_1) / 2\right] (x_1 - y_1) / 2\,; \\ dx_3/dt &=& 0.005 \left[4 (x_1 + 1.618) \cdot x_3\right] \\ dy_1/dt &=& y_2 \cdot y_1^3 & + 3y_1^2 \cdot y_3 + 3.25 & + \left[-10 + 3 (x_1 + y_1)^2 / 4 - 6 (x_1 + y_1) / 2\right] (-x_1 + y_1) / 2; \\ dy_2/dt = & 1 - 5y_1^2 - y_2 + \left[-10 + 10 (x_1 + y_1) / 2\right] (-x_1 + y_1) / 2\,; \\ dy_3/dt &=& 0.005 \left[4 (y_1 + 1.618) \cdot y_3\right] \end{array}$ 

Numerical results for the mutual synchronization are given in Fig.5, 6 and 7.



Fig.5  $x_1(t)$ -green,  $y_1(t)$ -black  $[x_1(0) = -1; x_2(0) = -0.1 x_3(0) = -0.01; y_1(0) = 1; y_2(0) = 0.1 y_3(0) = 0.01; p_1=p_2 = -10]$ 



Fig.6  $x_2(t)$ -red,  $y_2(t)$ -black  $[x_1(0) = -1; x_2(0) = -0.1 x_3(0) = -0.01; y_1(0) = 1; y_2(0) = 0.1 y_3(0) = 0.01; p_1=p_2 = -10]$ 



Fig. 7 Synchronization errors  $[(x_1(t) - y_1(t), \text{green}; x_2(t) - y_2(t), \text{red}; x_3(t) - y_3(t), \text{black})]$  oscillators  $[x_1(0) = -1;x_2(0) = -0.1; x_3(0) = -0.01; y_1(0) = 1; y_2(0) = 0.1; y_3(0) = 0.01; p_1=p_2=-10]$ 

The driver system 5 and the driven system (6) with  $\alpha$ =-2 are:

$$\begin{array}{l} dy_1/dt =& y_2 \cdot y_1^3 + 3 y_1^2 \cdot y_3 + 3.25 \cdot 0.5 \ y_1 + 0.1 y_2 - 0.1 y_3; \\ dy_2/dt =& 1 \cdot 5 y_1^2 - y_2; \\ dy_3/dt =& 0.005 \ [4(y_1 + 1.618) \cdot y_3] \\ dx_1/dt =& x_2 \cdot x_1^3 + 3 x_1^2 - x_3 + 3.25 + y_1^3 \cdot 0.2 \ y_2 + 0.2 y_3 \cdot 6 \ y_1^3 \cdot 18 \\ y_1^2 + [-10 \cdot 12 \ y_1^2 - 12 y_1](x_1 + 2 y_1) \\ dx_2/dt =& 1 \cdot 5 x_1^2 - x_2 + 30 \ y_1^2 + [-10 \cdot 20 y_1] \ (x_1 + 2 y_1); \\ dx_3/dt =& 0.005 \ [4(x_1 + 1.618) \cdot x_3] \end{array}$$

Amplification of chaos for Hindmarsh-Rose system is given in figures 8-11.



Fig. 8  $2y_1(t)$ -green,  $x_1(t)$ -black, for  $\alpha$ =-2;  $(y_1(0)=0.001; y_2(0)=0.001; y_3(0)=0.001; x_1(0)=-0.001; x_2(0)=0.001; x_3(0)=0.001)$ 



Fig. 9  $2y_2(t)$ -red,  $x_2(t)$ -black, for  $\alpha$ =-2;  $(y_1(0)=0.001; y_2(0)=0.001; y_3(0)=0.001; x_1(0)=-0.001; x_2(0)=0.001; x_3(0)=0.001)$ 



Fig. 10  $2y_3(t)$ -blue  $x_3(t)$ -black, for  $\alpha$ =-2;  $(y_1(0)=0.001; y_2(0)=0.001; y_3(0)=0.001; x_1(0)=-0.001; x_2(0)=0.001; x_3(0)=0.001)$ 



Fig. 11 Phase portrait of  $(y_1, y_2)$ -black and  $(x_1, x_2)$ -red, for p=-10 and  $\alpha$ =-2;  $(y_1(0)=0.001; y_2(0)=0.001; y_3(0)=0.001; x_1(0)=-0.001; x_2(0)=0.001; x_3(0)=0.001)$ 

#### 6. Conclusions

In this paper we applied the master-slave, mutual synchronization and amplification of chaos methods to the chaotic systems that are known as Hindmarsh-Rose oscillators. The transient time until synchronization depends on initial conditions of two systems and on the values of negative part of eigenvalues. Generally speaking, synchronization properties of dynamical neural networks essentially depend on the coupling configuration, the number of cells, and the type of coupling.

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